

**PRECISION AG 2.0 Conference**  
*February 11-12, 2014 — Calgary, Alberta, Canada*

## **Returning the Scientific Horse to in Front of the Technical Cart**



Taking **Traditional Science**...



...is the **Technical Cart** in front of the **Scientific Horse**?



... to a **New Level**

...this presentation suggests that is a new “map-ematics” that **extends traditional math/stat concepts and procedures for the quantitative analysis of map variables** (spatial data) which promises to change STEM education, science and innovation in general, and the agricultural sciences in particular—

**SpatialSTEM** the “new math” of the 21<sup>st</sup> Century

Breakout session by **Joseph K. Berry**

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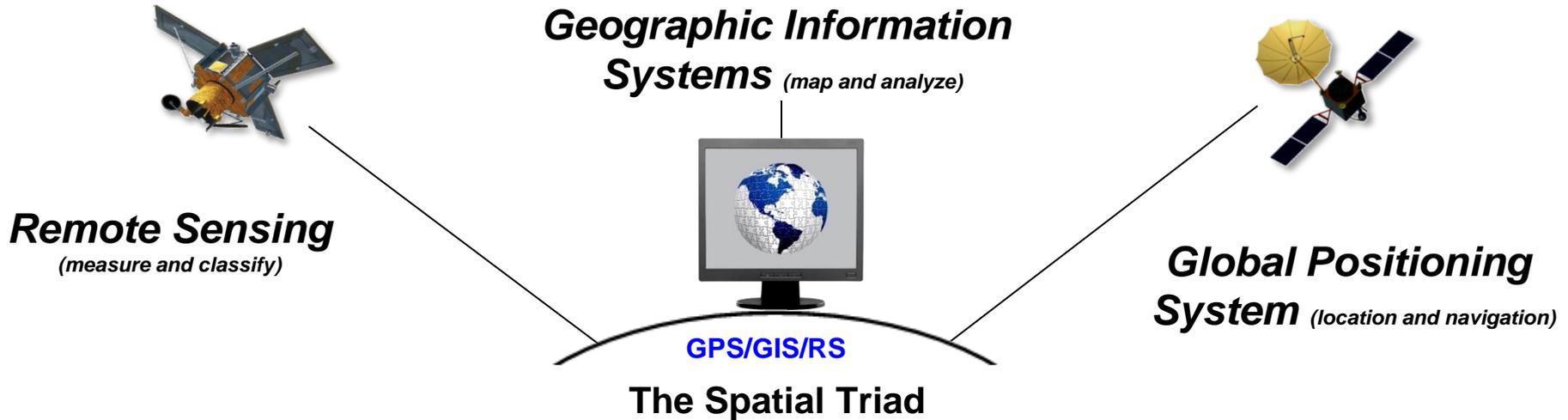
(See [http://www.innovativegis.com/basis/present/PAconf\\_Calgary2014/](http://www.innovativegis.com/basis/present/PAconf_Calgary2014/) to access support materials including PowerPoint)

# Geotechnology as a Mega-Technology

**Geotechnology** is one of the three "mega -technologies" for the 21st century and promises to forever change how we conceptualize, utilize and visualize spatial relationships in scientific research and commercial applications (U.S. Department of Labor)

*...recall from the earlier overview discussion*

## Geographic Information Systems (map and analyze)



Computer Mapping (70s)  
Spatial Database Management (80s)

Map Analysis (90s)  
GeoWeb Interactions (00s)

### Technological Tool

### Where

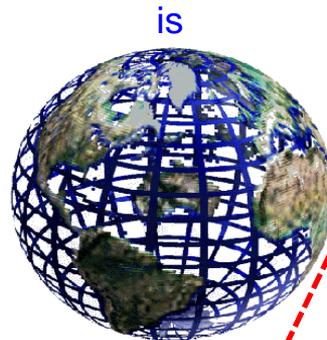
### What

### Analytical Tool

**Mapping** involves precise placement (delineation) of physical features (graphical inventory)



Descriptive Mapping



Prescriptive Modeling



**Modeling** involves analysis of spatial patterns and relationships (map analysis/modeling)

Why So What and What If

# Is GIS Technology Ahead of Science?



...is the **Technical Cart** in front of the **Scientific Horse**?

*Five critical questions underlying Precision Agriculture...*

1) Is the “**scientific method**” relevant in the data-rich age of knowledge engineering?

2) Is the “**random thing**” pertinent in deriving mapped data?

3) Are **geographic distributions** a natural extension of numerical distributions?

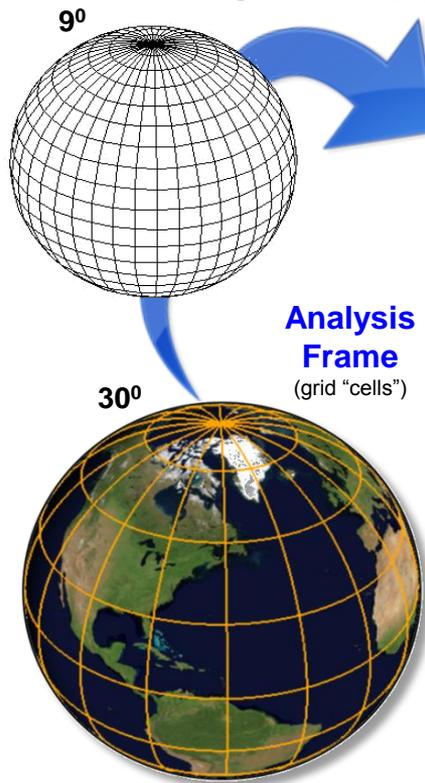
4) Can **spatial dependencies** be modeled within a map variable (spatial autocorrelation) and among map variables (spatial correlation)?

5) How can “site-specific” analysis and on-farm studies contribute to the **scientific body of knowledge**?

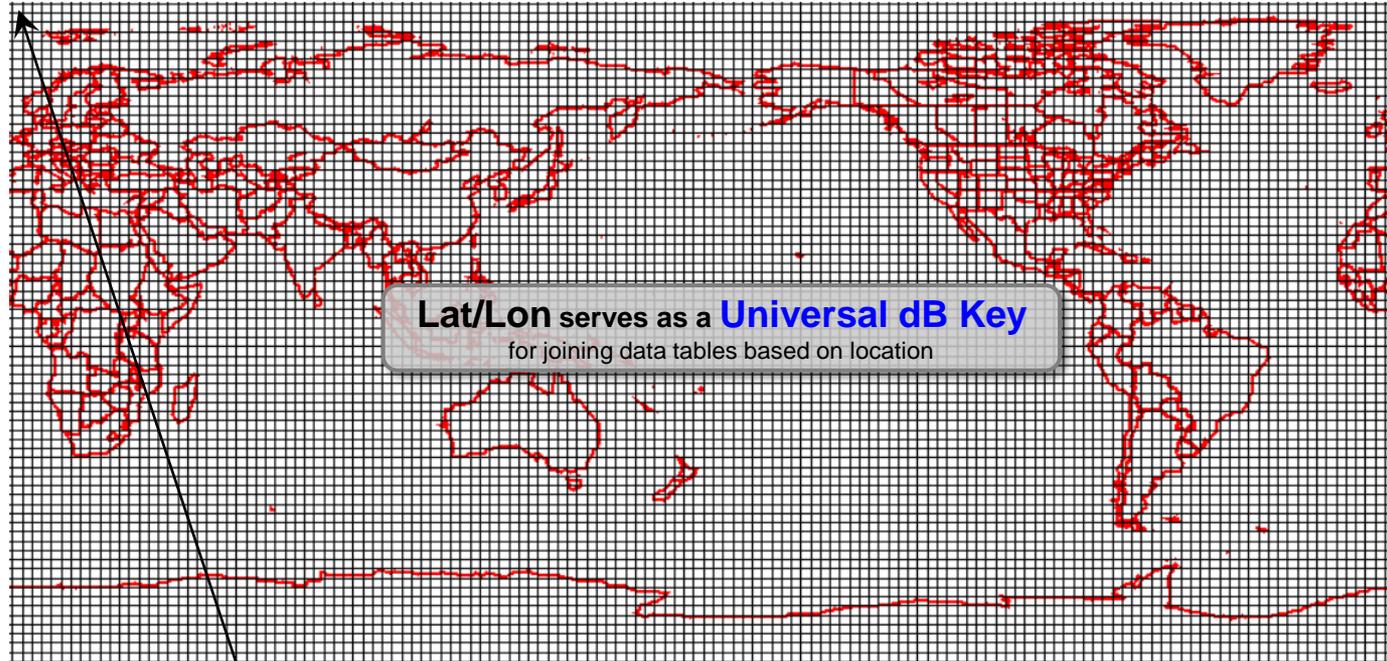
...turning traditional science on its head by extending math/stat operations for [quantitative analysis of mapped data](#)



# Grid-based Map Data *(geo-registered matrix of map values)*



2.5° Latitude/Longitude Grid (140mi grid cell size)



Coordinate of first grid cell is 90° N 0° E #Rows= 73 #Columns= 144 = 10,512 grid cells

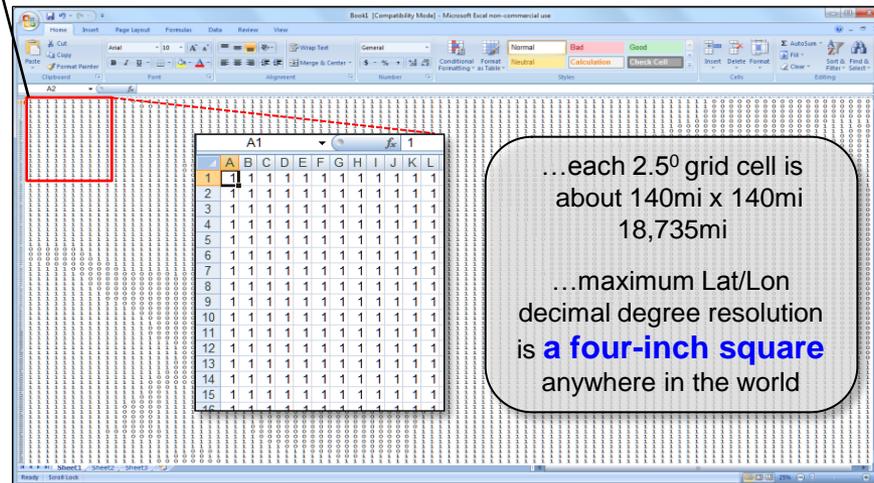
↑ The **Latitude/Longitude** grid forms a continuous surface for geographic referencing where each grid cell represents a given portion of the earth' surface.

The easiest way to conceptualize a grid map is as an Excel spreadsheet with each **cell** in the table corresponding to a Lat/Lon grid space (location) and each **value** in a cell representing the characteristic or condition (information) of a mapped variable occurring at that location. →

...the bottom line is that...

All **spatial topology** is *inherent in the grid.*

Conceptual Spreadsheet (73 x 144)



Lat/Lon

...from Lat/Lon  
"crosshairs to grid cells"

that contain map values indicating characteristics or conditions at each location

# A Mathematical Structure for Map Analysis/Modeling

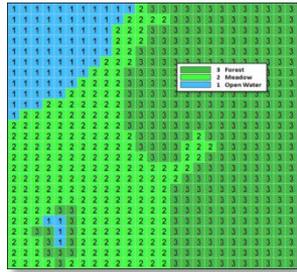
## Technological Tool

Geotechnology → RS — **GIS** — GPS

## Analytical Tool

Mapping/Geo-Query (*Discrete, Spatial Objects*) (*Continuous, Map Surfaces*)

**Map Analysis/Modeling**



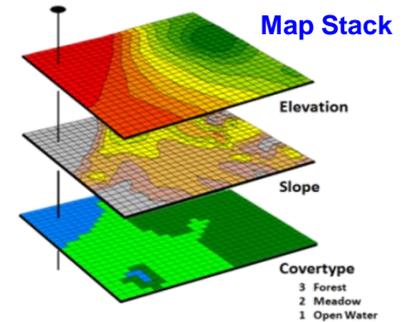
...organized set of numbers

Geo-registered  
Analysis Frame → **Matrix**  
of Numbers

“Map-ematics”

## Maps as Data, not Pictures

Vector & **Raster** — Aggregated & **Disaggregated**  
Qualitative & **Quantitative**



## Spatial Analysis Operations

Grid-based  
Map Analysis  
Toolbox

## Spatial Statistics Operations

### GISer's Perspective:

**Reclassify** and **Overlay**  
**Distance** and **Neighbors**

### GISer's Perspective:

**Surface Modeling**  
**Spatial Data Mining**

### Mathematician's Perspective:

**Basic GridMath & Map Algebra**  
**Advanced GridMath**  
**Map Calculus**  
**Map Geometry**  
**Plane Geometry Connectivity**  
**Solid Geometry Connectivity**  
**Unique Map Analytics**



## The SpatialSTEM Framework

Traditional math/stat procedures can be extended into **geographic space** to stimulate those with diverse backgrounds and interests for...

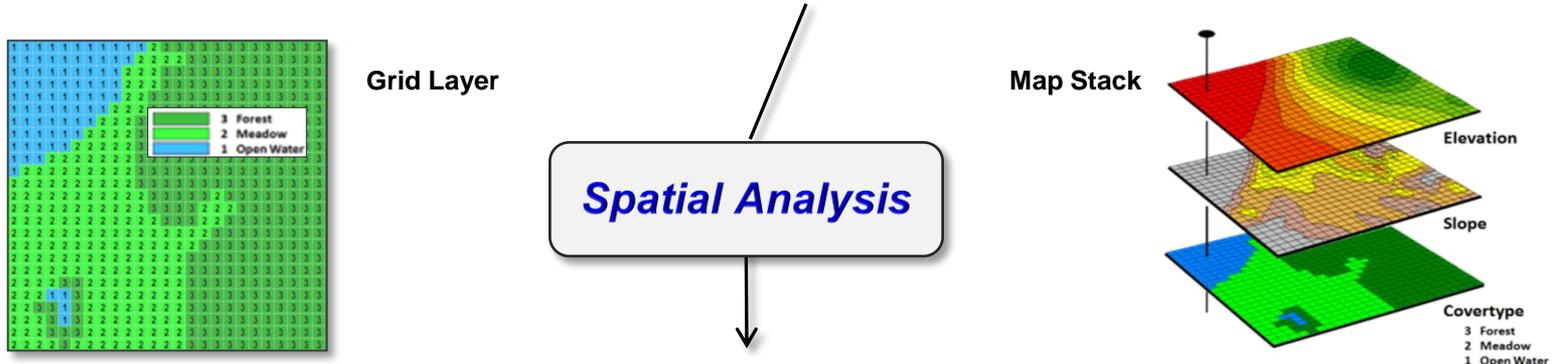
“**thinking analytically with maps**”

### Statistician's Perspective:

**Basic Descriptive Statistics**  
**Basic Classification**  
**Map Comparison**  
**Unique Map Statistics**  
**Surface Modeling**  
**Advanced Classification**  
**Predictive Statistics**

# Spatial Analysis Operations *(Geographic Context)*

GIS as “Technical Tool” (*Where is What*) vs. “**Analytical Tool**” (*Why, So What and What if*)



**Spatial Analysis** extends the basic set of discrete map features (points, lines and polygons) to map **surfaces** that represent continuous geographic space as a set of contiguous grid cells (matrix), thereby providing a **Mathematical Framework** for *map analysis* and *modeling* of the

**Contextual Spatial Relationships** within and among grid map layers

Map Analysis Toolbox



✓ Unique spatial operations

## **Mathematical Perspective:**

**Basic GridMath & Map Algebra** (+ - \* /)

**Advanced GridMath** (Math, Trig, Logical Functions)

**Map Calculus** (Spatial Derivative, Spatial Integral)

**Map Geometry** (Euclidian Proximity, Effective Proximity, Narrowness)

**Plane Geometry Connectivity** (Optimal Path, Optimal Path Density)

**Solid Geometry Connectivity** (Viewshed, Visual Exposure)

✓ **Unique Map Analytics** (Contiguity, Size/Shape/Integrity, Masking, Profile)

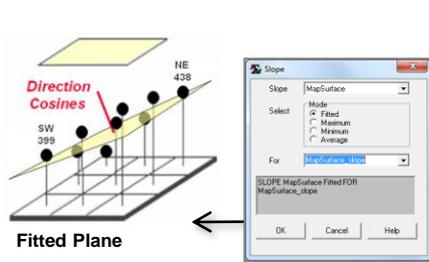
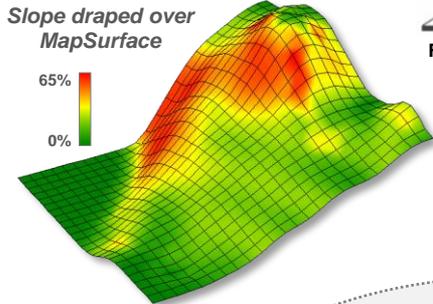
# Spatial Analysis Operations (Math Examples)



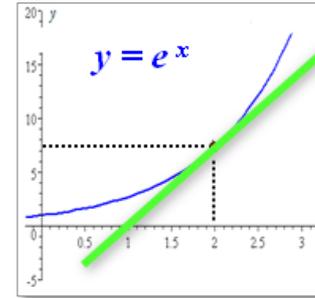
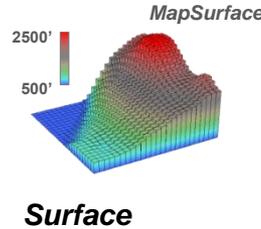
**Advanced Grid Math** — Math, Trig, Logical Functions  
**Map Calculus** — Spatial Derivative, Spatial Integral

## Spatial Derivative

...is equivalent to the slope of the **tangent plane** at a location



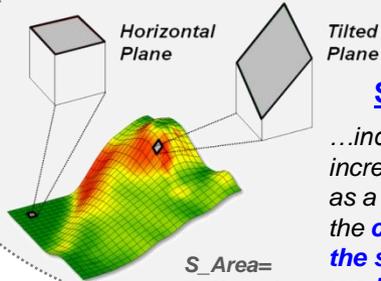
SLOPE MapSurface Fitted FOR MapSurface\_slope



The **derivative** is the instantaneous "rate of change" of a function and is equivalent to the slope of the **tangent line** at a point

$D_{z,xy}$  Elevation

## Advanced Grid Math



**Surface Area**

...increases with increasing inclination as a Trig function of the cosine of the slope angle

$$S\_Area = Fn(Slope)$$

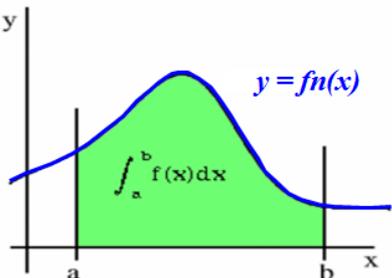
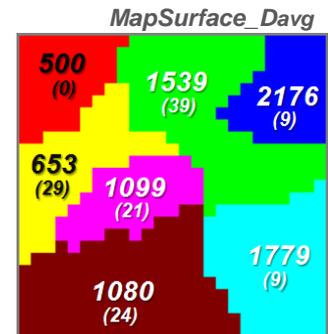
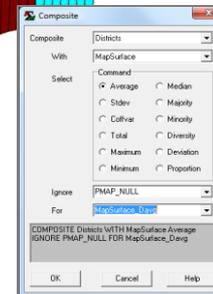
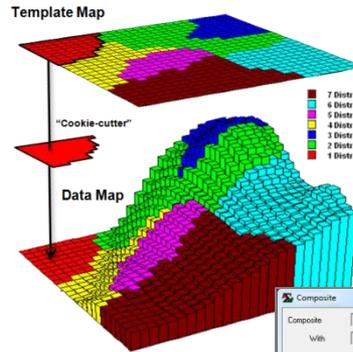
$$S\_area = cellsize / \cos(D_{z,xy} \text{ Elevation})$$

$\int$  Districts\_Average Elevation

## Spatial Integral

...summarizes the values on a surface for specified map areas (Total= **volume** under the surface)

COMPOSITE Districts WITH MapSurface Average FOR MapSurface\_Davg



The **integral** calculates the **area** under the curve for any section of a function.

Surface  
Curve

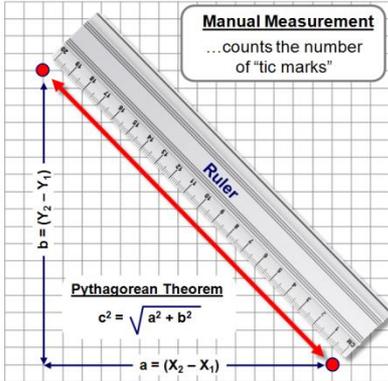
# Spatial Analysis Operations *(Distance Examples)*

**Map Geometry** — (Euclidian Proximity, Effective Proximity, Narrowness)

**Plane Geometry Connectivity** — (Optimal Path, Optimal Path Density)

**Solid Geometry Connectivity** — (Viewshed, Visual Exposure)

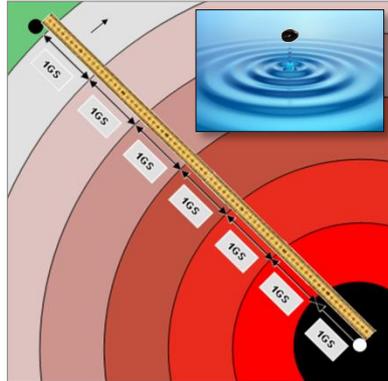
## Distance



**Mathematical Measurement**  
...solves the Pythagorean Theorem

Shortest straight line between two points...

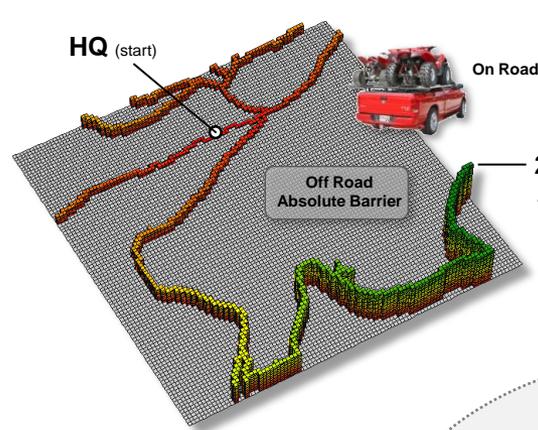
## Euclidean Proximity



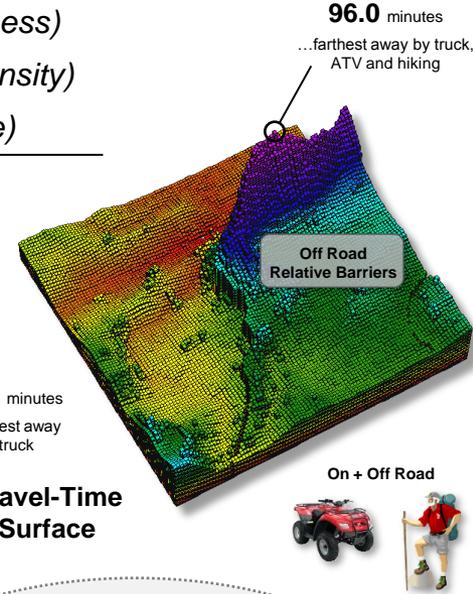
**Splash Algorithm**  
...like tossing a rock into a pond—ripples

...from a point to everywhere...

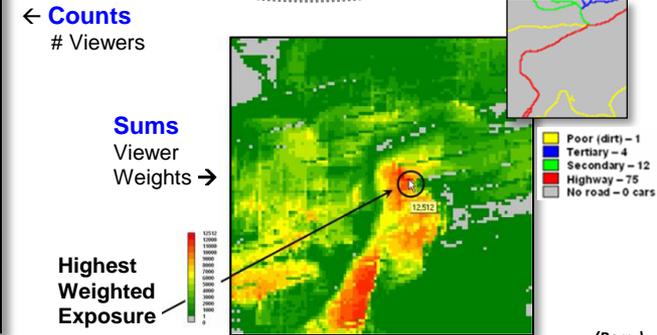
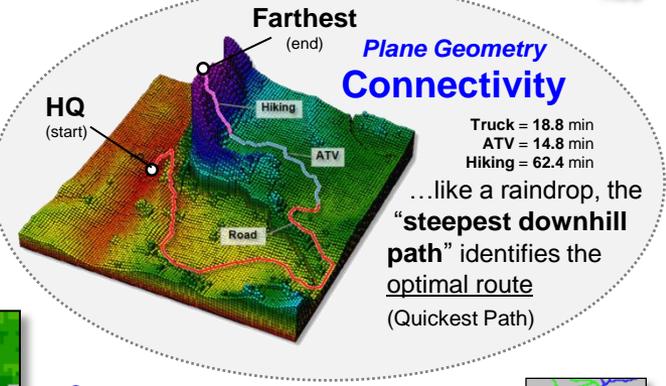
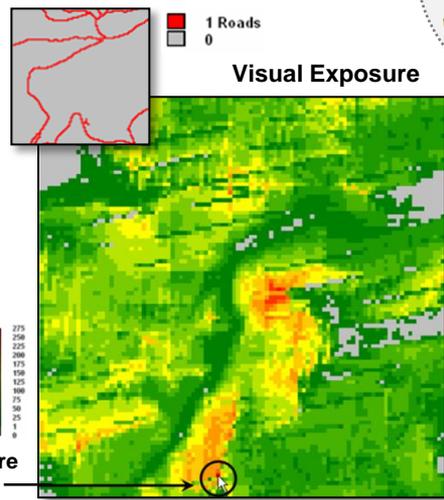
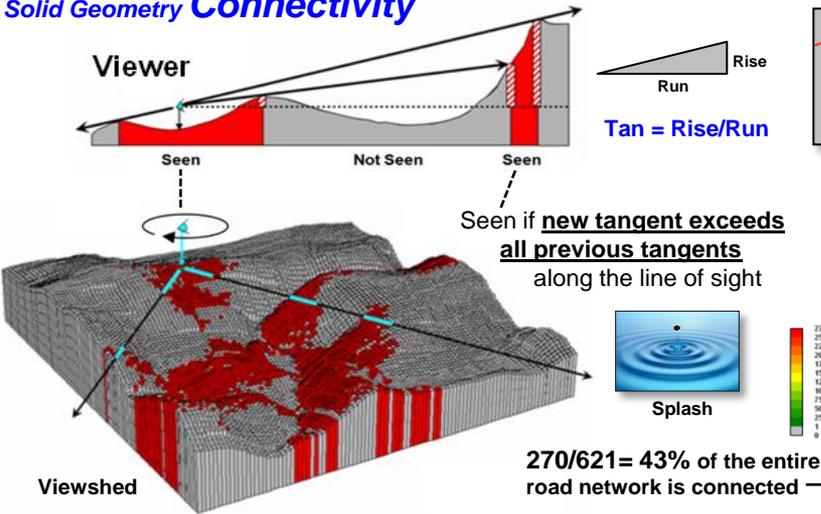
## Effective Proximity



...not necessarily straight lines (movement)

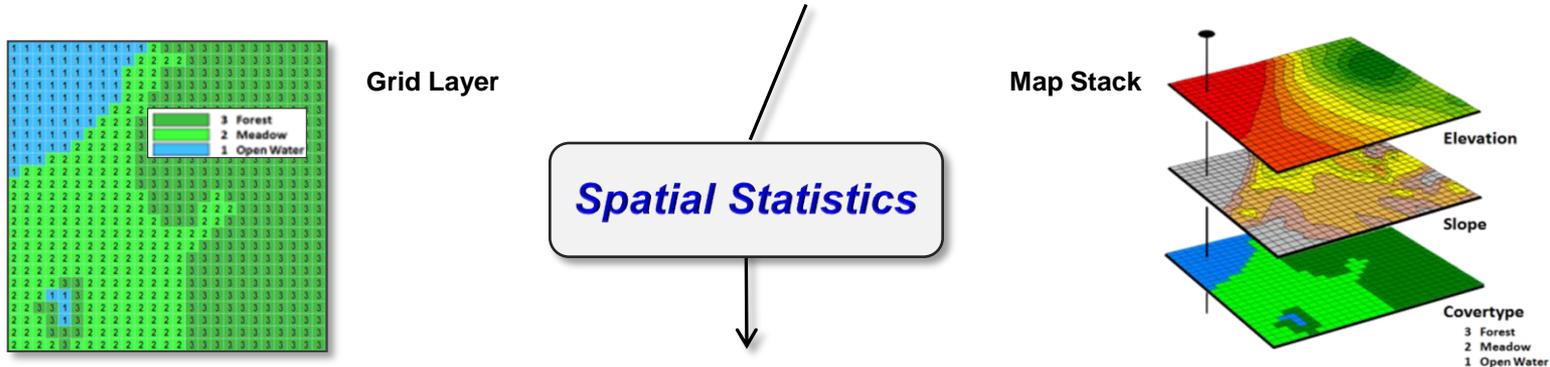


## Solid Geometry Connectivity



# Spatial Statistics Operations (*Numeric Context*)

GIS as “Technical Tool” (*Where is What*) vs. “**Analytical Tool**” (*Why, So What and What if*)



**Spatial Statistics** seeks to map the variation in a data set instead of focusing on a single typical response (central tendency), thereby providing a **Statistical Framework** for *map analysis* and *modeling* of the

Numerical Spatial Relationships within and among grid map layers

## Statistical Perspective:

**Basic Descriptive Statistics** (*Min, Max, Median, Mean, StDev, etc.*)

**Basic Classification** (*Reclassify, Contouring, Normalization*)

**Map Comparison** (*Joint Coincidence, Statistical Tests*)

✓ **Unique Map Statistics** (*Roving Window and Regional Summaries*)

✓ **Surface Modeling** (*Density Analysis, Spatial Interpolation*)

**Advanced Classification** (*Map Similarity, Maximum Likelihood, Clustering*)

**Predictive Statistics** (*Map Correlation/Regression, Data Mining Engines*)

Map Analysis Toolbox

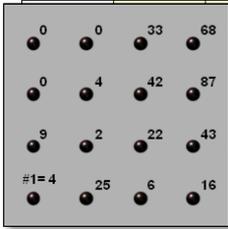


✓ **Unique spatial operations**

# Spatial Statistics *(Linking Data Space with Geographic Space)*

## Geo-registered Sample Data

Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
	1000	1000	10	6
	1500	17	22	
	2000	34	42	
	2500	22	33	
	1000	20	16	
	1500	28	43	
	2000	42	87	
	2500	34	68	



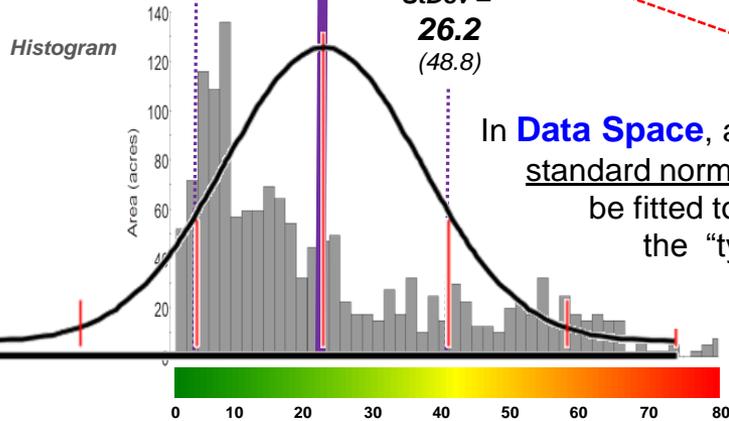
Non-Spatial Statistics

## Standard Normal Curve

Average = 22.6

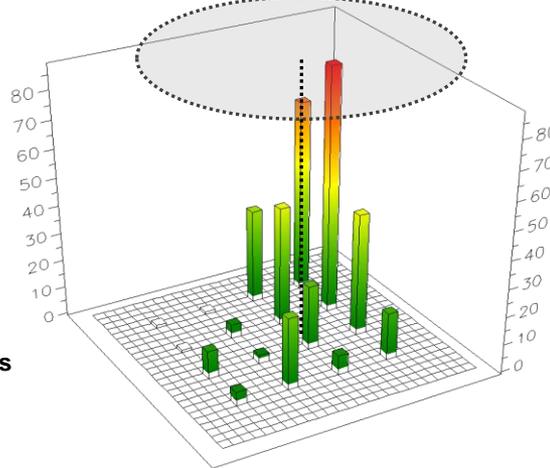
StDev = 26.2 (48.8)

In **Data Space**, a standard normal curve can be fitted to the data to identify the "typical value" (average)



## Numeric Distribution

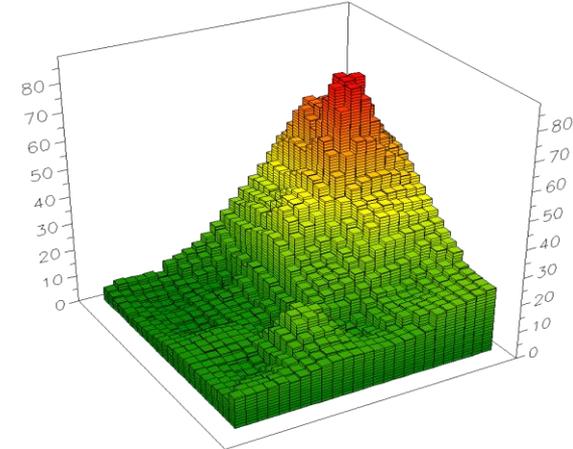
## Roving Window (weighted average)



## Discrete Sample Map

Spatial Statistics

## Spatial Distribution



## Continuous Map Surface

**Surface Modeling** techniques are used to derive a continuous map surface from discrete point data— fits a Surface to the data (maps the variation).

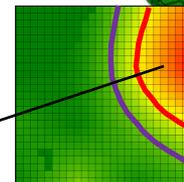
In **Geographic Space**, the typical value forms a horizontal plane implying the average is everywhere to form a horizontal plane

$\bar{X} = 22.6$

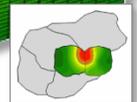
...lots of NE locations exceed Mean + 1Stdev

$$\begin{aligned} \bar{X} + 1\text{StDev} &= 22.6 + 26.2 \\ &= 48.8 \end{aligned}$$

Unusually high values

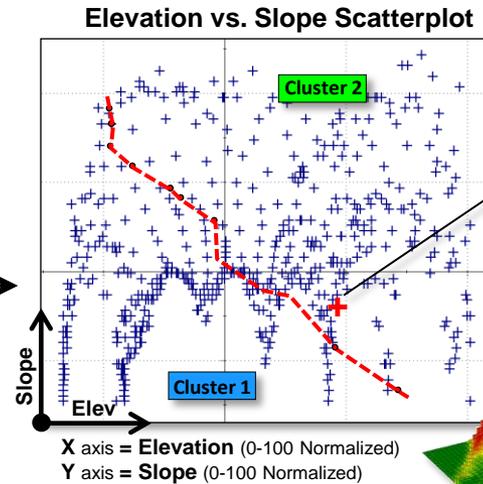
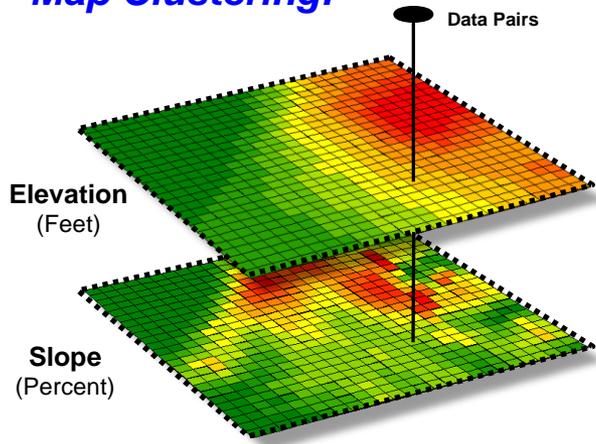


+StDev  
Average



# Spatial Statistics Operations (Data Mining Examples)

## Map Clustering:



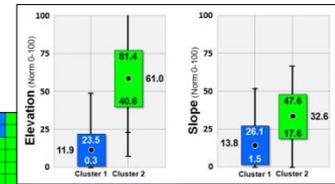
Plots here in...

Data Space

Geographic Space

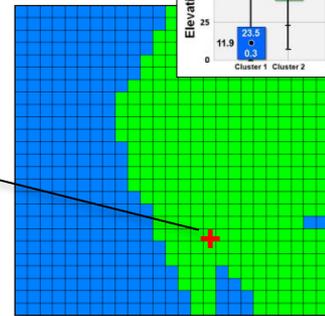
Slope draped on Elevation

### Box and Whisker



Two Clusters

Cluster 1  
Cluster 2

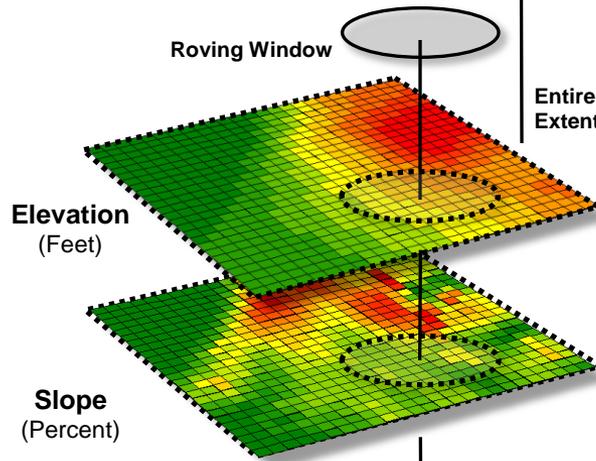


## Geographic Space

## Advanced Classification (Clustering)

## Data Space

## Map Correlation:



### Spatially Aggregated Correlation

Scalar Value – one value represents the overall non-spatial relationship between the two map surfaces

...1 large data table  
with 25 rows x 25 columns =  
625 map values for map wide summary

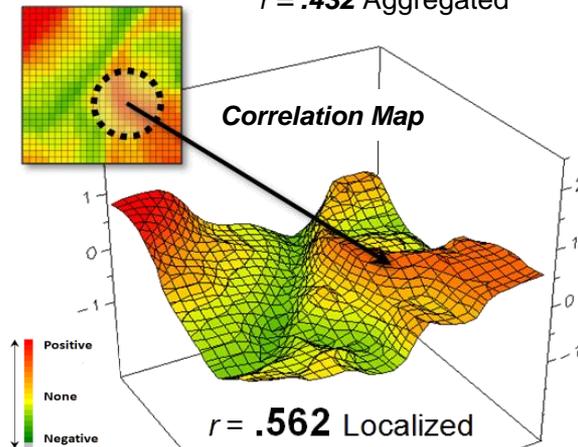
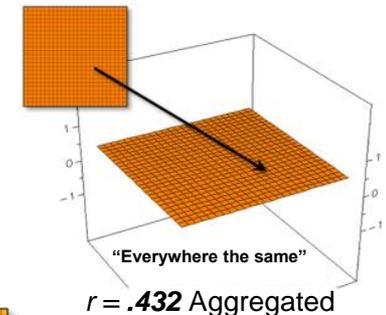
$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{([n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2])}}$$

...where **x** = Elevation value and **y** = Slope value and **n** = number of value pairs

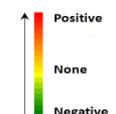
...625 small data tables  
within 5 cell reach =  
81 map values for localized summary

### Localized Correlation

Map Variable – continuous quantitative surface represents the localized spatial relationship between the two map surfaces



## Predictive Statistics (Correlation)



# Spatial Statistics Operations *(Data Mining Examples)*

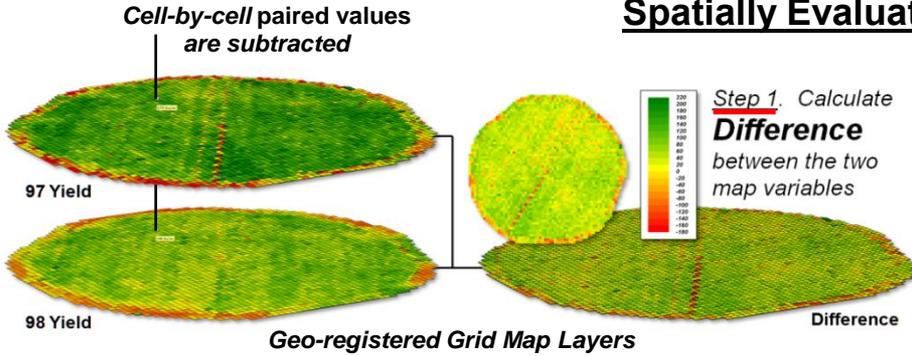
## Spatially Evaluating the "T-Test"



Yield Monitor

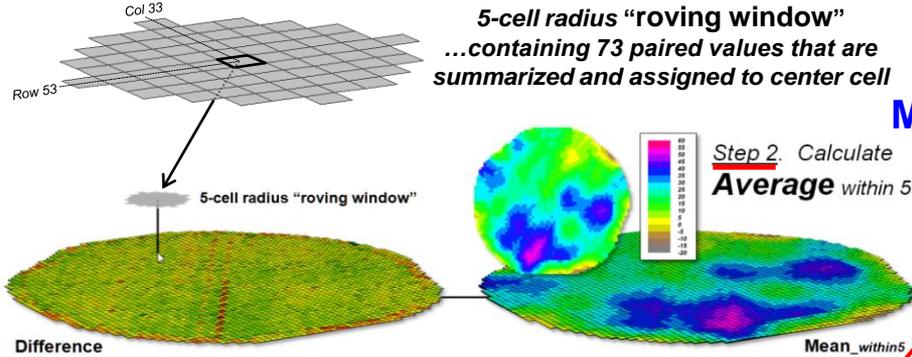


GPS

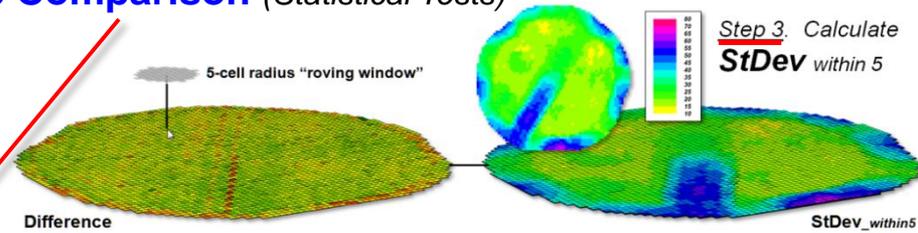


The **T-statistic** equation is evaluated by first calculating a map of the **Difference** (Step 1) and then calculating maps of the **Mean** (Step 2) and **Standard Deviation** (Step 3) of the Difference within a "roving window." The **T-statistic** is calculated using the derived Mean and StDev maps using the standard equation (step 4) — spatially localized solution.

$$T\text{-statistic} = \frac{\text{Mean}_{\text{difference}}}{\text{StDev}_{\text{difference}} / \text{Sqrt}(73)}$$



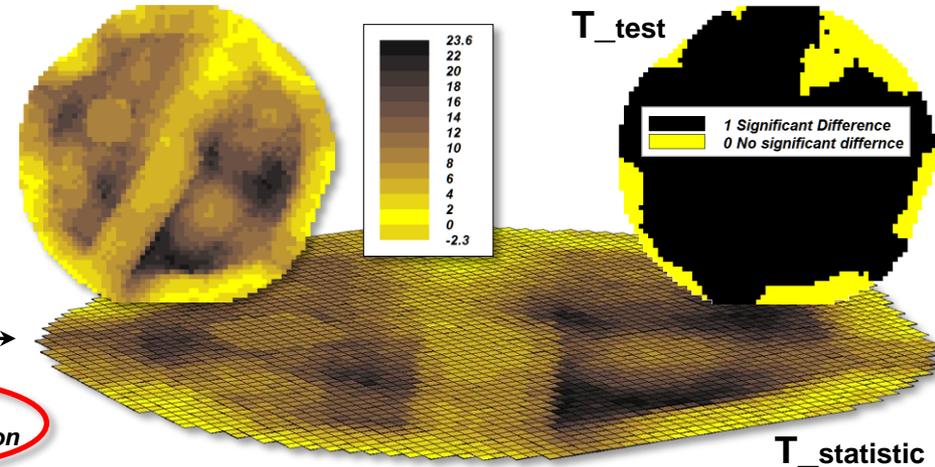
## Map Comparison (Statistical Tests)



Step 4. Calculate the "Localized" T-statistic (using a 5-cell roving window) for each grid cell location

...the result is map of the **T-statistic** indicating how different the two map variables are throughout geographic space and a **T-test** map indicating where they are significantly different.

$$T\text{-statistic} = \frac{\text{Mean}_{\text{within5}}}{[\text{StDev}_{\text{within5}} / \text{sqrt}(73)]} \rightarrow \text{Spatially Evaluate T-statistic Equation}$$

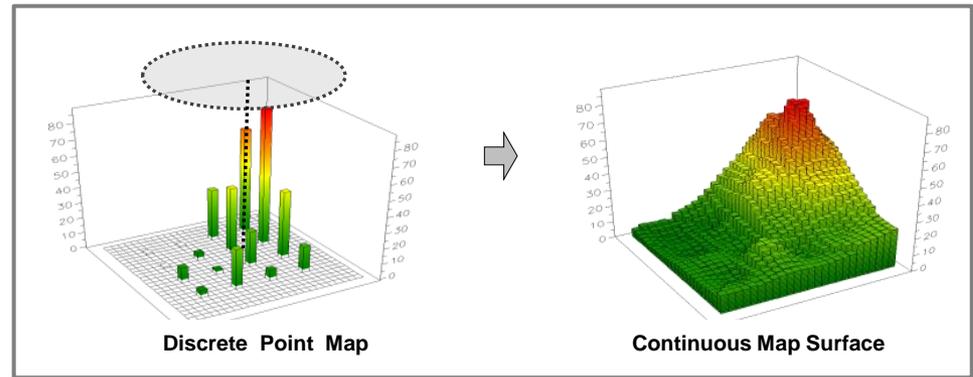


# Two Types of Spatial Variable Dependence

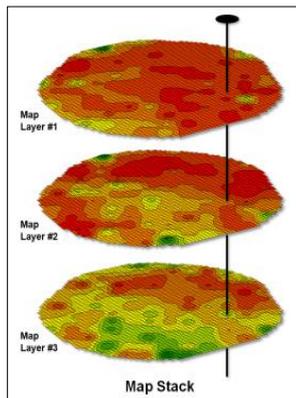
...keep in mind that all Spatial Statistics used in Precision Ag is based upon the condition that “what occurs at a location in geographic space is related to”—

1) ...the **conditions of that variable at nearby locations**, termed **Spatial Autocorrelation** (*intra-variable* dependence; *within* a map layer)

**Surface Modeling** – identifies the continuous spatial distribution implied in a set of discrete point samples



2) ...the **conditions of other variables at that location**, termed **Spatial Correlation** (*inter-variable* dependence; *among* map layers)



**Statistical Analysis** – investigates spatial relationships among multiple map layers by spatially evaluating traditional statistical procedures

**Map Stack** – relationships among maps are investigated by aligning grid maps with a common configuration— same **#cols/rows**, **cell size** and **geo-reference**

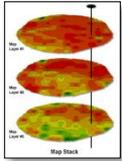
**Data Shish Kebab** – within a statistical context, each map layer represents a **Variable**; each grid space a **Case**; and each value a **Measurement** with all of the rights, privileges, and responsibilities of non-spatial mathematical, numerical and statistical analysis

# Techniques for Accessing Multi-layered Data

Characterization of the **Spatial Correlation** among two or more map layers involves:

**Descriptive Spatial Statistics** involves summarizing/presenting mapped data in metric, tabular and graphic forms

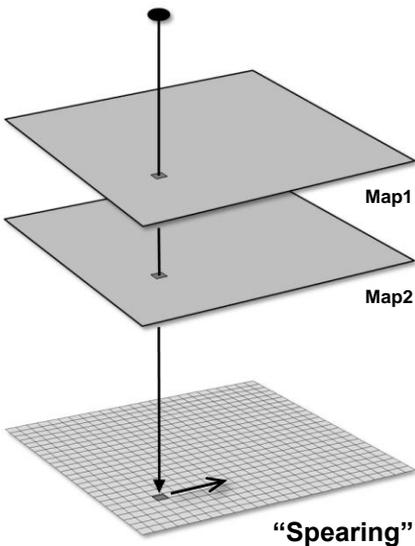
**Predictive Spatial Statistics** involves inferential analysis, e.g., hypothesis testing, regression and spatial modeling



Assessing the spatial relationships among map layers involves spatially evaluating traditional statistical procedures using one of four basic techniques for

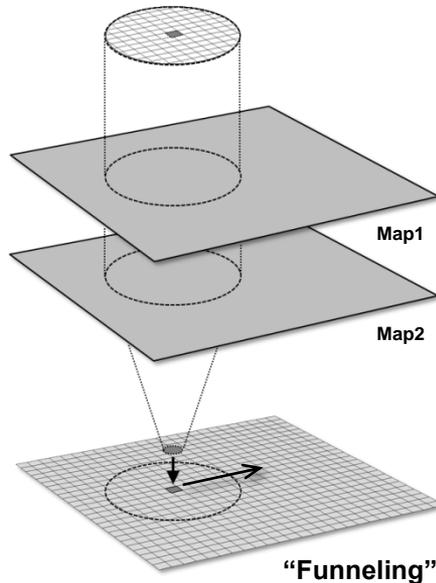
accessing and organizing geo-registered data for multi-layer spatial statistics—

## Local



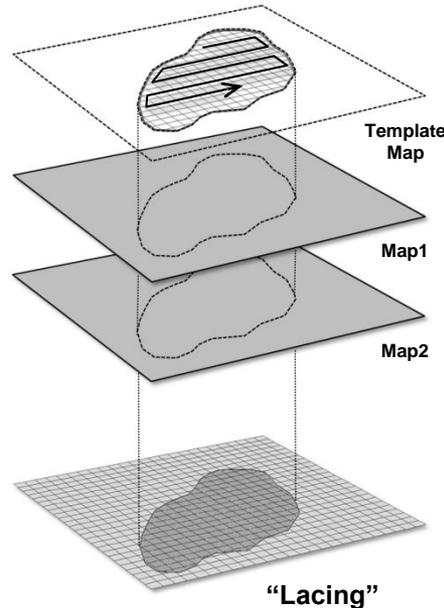
...collects data on a cell-by-cell basis and reports a single value on a cell-by-cell basis

## Focal



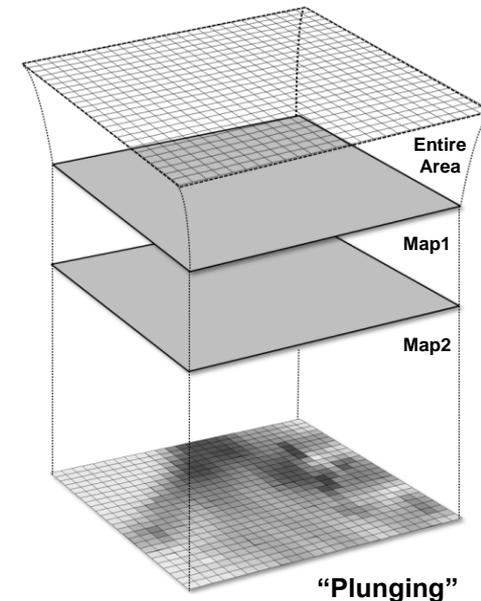
...collects data on a neighborhood basis and reports a single value on a cell-by-cell basis

## Zonal



...collects data on a region-wide basis and reports summary on a region-wide basis

## Global



...collects data on a map-wide basis and reports results on a map-wide or cell-by-cell basis



# Additional Information *(live links by slide #)*

Slide 1, Title – a URL link to more information on SpatialSTEM approach is posted online at—

[www.innovativegis.com/Basis/Courses/SpatialSTEM/](http://www.innovativegis.com/Basis/Courses/SpatialSTEM/)

The following links are to the online book [Beyond Mapping III](#) posted at [www.innovativegis.com](http://www.innovativegis.com)

- Slide 2, **Geotechnology** – [Overview of Spatial Analysis and Statistics](#); [Is it Soup Yet?](#) ; [What's in a Name?](#) ; [Melding the Minds of the “-ists” and “-ologists”](#)
- Slide 3, **Is GIS Technology Ahead of Science?** – [Making a Case for SpatialSTEM](#); [A Multifaceted GIS Community](#); [GIS Education's Need for “Hitchhikers”](#); [Questioning GIS in Higher Education](#)
- Slide 4, **Grid-based Mapped Data (Matrix of Map Values)** – [Organizing Geographic Space for Effective Analysis](#); [To Boldly Go Where No Map Has Gone Before](#); [Beware the Slippery Surfaces of GIS Modeling](#); [Explore Data Space: The Universal Key for Unlocking GIS's Full Potential](#); [Thinking Outside the Box](#)
- Slide 5 **A Mathematical Structure for Map Analysis/Modeling** – [Moving Mapping to Map Analysis](#); [Use Map-ematical Framework for GIS Modeling](#); [Getting the Numbers Right](#)
- Slide 6, **Spatial Analysis Operations (Geographic Context)** – [Simultaneously Trivializing and Complicating GIS](#); [SpatialSTEM Has Deep Mathematical Roots](#); [Understanding Grid-based Data](#); [Suitability Modeling](#)
- Slide 7, **Spatial Analysis Operations (Math Examples)** – [Map-ematically Messing with Mapped Data](#); [Characterizing Micro-terrain Features](#); [Reclassifying and Overlaying Maps](#) ; [Use Map-ematical Framework for GIS Modeling](#)
- Slide 8, **Spatial Analysis Operations (Distance Examples)** – [Bending Our Understanding of Distance](#); [Calculating Effective Distance and Connectivity](#); [E911 for the Backcountry](#); [Routing and Optimal Paths](#); [Deriving and Using Travel-Time Maps](#); [Applying Surface Analysis](#); [Deriving and Using Visual Exposure Maps](#); [Creating Variable-Width Buffers](#)
- Slide 9, **Spatial Statistics Operations (Numeric Context)** – [Infusing Spatial Character into Statistics](#); [Paint by Numbers Outside the Traditional Statistics Box](#); [Use Spatial Statistics to Map Abnormal Averages](#)
- Slide 10, **Spatial Statistics Operations (Linking Data Space with Geographic Space)** – [Spatial Interpolation Procedures and Assessment](#); [Linking Data Space and Geographic Space](#); [Babies and Bath Water](#); [Making Space for Mapped Data](#)
- Slide 11, **Spatial Statistics Operations (Data Mining Examples)** – [Characterizing Patterns and Relationships](#); [Analyzing Map Similarity and Zoning](#); [Discover the “Miracle” in Mapping Data Clusters](#)
- Slide 12, **Spatial Statistics Operations (Data Mining Examples)** – [Spatially Evaluating the T-test](#); [Depending on Where is What](#); [Recasting Map Analysis Operations](#)
- Slide 13, **Techniques for Assessing Multi-layered Data**– [Finding Common Ground in Paper and Digital Worlds](#); [Maps Are Numbers First, Pictures Later](#); [Multiple Methods Help Organize Raster Data](#); [VtoR and Back! \(Pronounced “V-tore”\)](#)
- Slide 14, **Techniques for Assessing Multi-layered Data**– [Finding Common Ground in Paper and Digital Worlds](#); [Maps Are Numbers First, Pictures Later](#); [Multiple Methods Help Organize Raster Data](#); [VtoR and Back! \(Pronounced “V-tore”\)](#)
- Slide 15, **So What's the Point?** – [Is GIS Technology Ahead of Science?](#); [GIS Evolution and Future Trends](#); [Spatial Modeling in Natural Resources](#); [Lumpers and Splitters Propel GIS](#); [The Softer Side of GIS](#)