

Beyond Mapping III

Topic 5: Analyzing Accumulation Surfaces



[Map Analysis](#) book with companion CD-ROM for hands-on exercises and further reading

[Building Accumulation Surfaces](#) — reviews how proximity analysis and effective distance is used to construct accumulation surfaces

[Analyzing Accumulation Surfaces](#) — describes how two surfaces can be analyzed to determine the relative travel-time advantages

[Determining Optimal Path Corridors](#) — describes a technique for determining the set of the set of best paths between two points

[Analyzing Stepped Accumulation Surfaces](#) — describes a technique for forcing an optimal path through a series of points

Note: The processing and figures discussed in this topic were derived using MapCalc™ software. See www.innovativegis.com to download a free MapCalc Learner version with tutorial materials for classroom and self-learning map analysis concepts and procedures.

<[Click here](#)> for a printer-friendly version of this topic (.pdf).

[\(Back to the Table of Contents\)](#)

Building Accumulation Surfaces

(GeoWorld, October 1997, pg. 26)

[\(return to top of Topic\)](#)

“You can’t get there from here,” is often the flippant response when you ask directions. In many cases it is a perfectly valid answer, as movement in geographic space is rarely as straight forward as a straightedge. Often there are several possible contorted paths that twist and turn in route to a destination. However, from the perspective of a ruler there is only one—the “shortest, straight line connecting two points.” Several Beyond Mapping columns¹ have addressed the concepts and procedures behind GIS’s capability to derive effective distance and optimal paths that respect intervening barriers. This article begins a series on the nature and analysis of accumulation surfaces, the basis of modeling real world movement.

Before we get too far, a quick review might be prudent. Recall that there are primarily two ways to represent distance in a GIS; the Pythagorean theorem which mathematically mimics a ruler and the “splash” algorithm which mimics a drunken sailor weaving through a maze of barriers. There’s a good chance you remember Pythagorean’s equation of $c^2 = a^2 + b^2$ from the brow-beating you received about right triangles in high

school geometry. The GIS simply uses its X and Y coordinates in solving the equation for simple distance between two points.

The splash technique tracking simple proximity is a bit less familiar, but conceptually easy. Imagine what happens to a still pond when you toss in a rock—splash, then one ripple away, then another, and another, until there's a whole set of concentric rings about the starting point. If the conditions are the same throughout the pond, the effect is similar to nailing a ruler at the starting point and spinning it, while scribing the set of circles formed by dragging the ruler's tic marks. In a raster GIS, the width of each "wavelength ring" is one grid space. The result isn't a single value identifying the length of the shortest, straight line between two points; it's a map of values identifying all of the shortest, straight line distances between everywhere and the starting point.

Now imagine tossing a handful of rocks—splash, one ripple, two ripple, three ripple, and more radiate out from each of the starting points. When two wave-fronts meet, they stop, with the point of interference identifying the halfway location between starting points. The same holds true if you toss in a set of sticks or pieces of plywood, with the ripple patterns conforming to the irregular shapes of the objects. The end result of all the simulated splashing and crashing in a GIS is a map of the shortest, straight line distance from everywhere to the nearest starting location; be it a point, line or polygon.

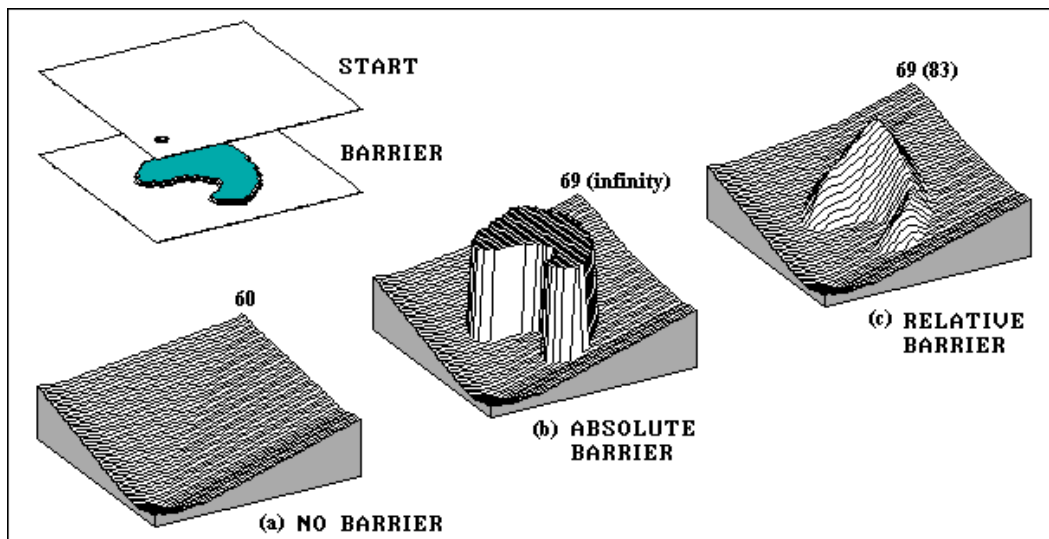


Figure 1. Accumulation surfaces showing the effects of relative and absolute barriers in mapping proximity.

Inset (a) in figure 1 is a 3-D plot of a simple proximity surface radiating from a single point. The lowest point on the surface contains the value 0 denoting it is "0 grid spaces away" from the start. Note that the surface is shaped like a bowl with increasing values (1 away, 2 away, etc.). The farthest location is in the upper right corner at a distance of 60 grid spaces * 100 meters/grid space = 6000 meters. The slight depressions along the orthogonal and diagonal are a result of the slight directional variations in distances

computed by the “splash” algorithm.

But continuous, straight line movement forming a perfect proximity bowl is rarely the case in the real world. Effective proximity respects movement around and through barriers—not “as the crow flies,” but as the crow might walk. Suppose there’s a lake in the way. Inset (b) identifies the absolute barrier itself as being infinitely far away (fear/reality of drowning). It assigns a value of 69 grid spaces to the farthest accessible location, indicating that the distance is a 900 meters farther as a result of going around the lake. The set of all map values indicate the “shortest, but not necessarily straight” distances between the starting point and all of the other locations.

However, in winter the lake freezes and can be crossed, though at a much slower pace on the slippery ice. It represents a relative barrier that impedes movement, but doesn’t totally restrict movement. Inset (c) shows the accumulation surface assuming you walk 5 times slower on the ice. The results show that it’s still 6900 meters to the opposite corner by going around the lake. However, if you gingerly trek to the center of the lake, it’s equivalent to traveling 8300 meters on open land.

Previous Beyond Mapping columns described how the computer finds the “steepest, downhill path” along an accumulation surface to locate the “not necessarily straight” optimal path. It’s analogous to a rain drop’s route along the surface, which effectively retraces the wave front that got there first. In fact, the rain drop paths from all locations identify the “shortest, but not necessarily straight set of lines connecting the origin to everywhere.” Information you with a ruler, or Pythagorus with a calculator, could never derive. But it’s chicken-feed compared to what insights you get by analyzing the surfaces themselves—see you next time.

¹ See *GIS World* issues September 1989 through May 1990 for discussion of the fundamental concepts in measuring effective distance and October 1992 through December 1992 for discussion of the procedures involved. These columns have been compiled into Topics 3 and 9, respectively, in *Beyond Mapping: Concepts, Algorithms and Issues in GIS* (Berry, 1993; GIS World Books).

Analyzing Accumulation Surfaces

(GeoWorld, November 1997, pg. 30-32)

[\(return to top of Topic\)](#)

The previous article in this series described the nature of accumulation surfaces and how they are built. Recall that the “splash” algorithm measures distance from a starting location like waves that spread out when a rock is tossed into a pond. The result is a 3-dimensional surface with increasing distance reflected by the increasing Z values stored in a matrix of grid cells. If absolute and relative barriers to movement are introduced, these surfaces form unique shapes with ridges and peaks similar to terrain surfaces.

However, unlike terrain surfaces, accumulation surfaces are always increasing (no “false-bottoms”) from point, line and areal features designated as starting locations. Areas with

absolute barriers are identified as infinitely far away and form sheer walls on an accumulation surface. Relative barriers form hills and ridges as they identify areas that are passable, but at an increased “cost” (e.g., more time) per grid space. The valleys emanating from the starting locations locate corridors of minimal resistance to movement along the accumulation surface.

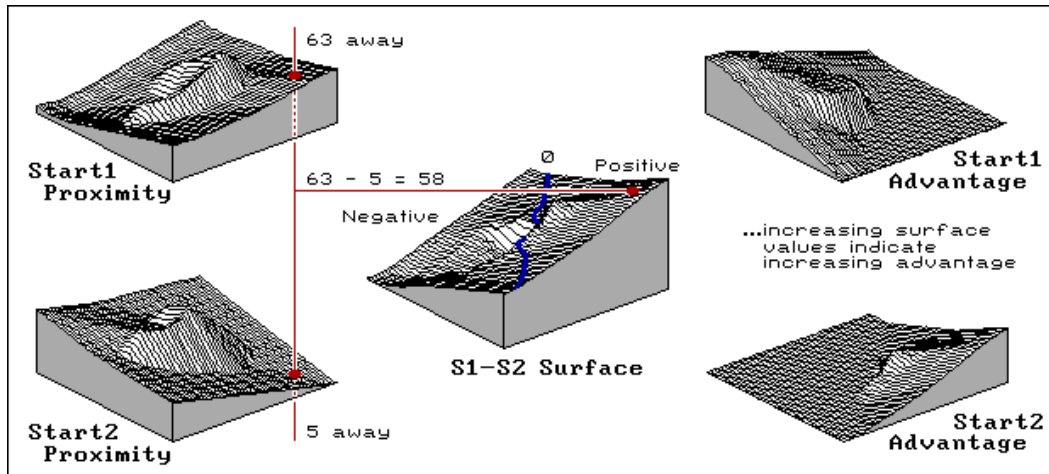


Figure 1. The difference between two proximity surfaces identifies the relative geographic advantage between two locations.

For example, the two surfaces on the extreme left of figure 1 characterize movement from opposing corners through the horseshoe-shaped relative barrier described in the previous article. The proximity from Start1 generally increases from left to right, while the increase from Start2 is in the opposite direction. Both surfaces show an abrupt increase when the relative barrier is encountered, however the shape of the resulting “hill” is different due to the different directions of the distance waves and the shape of the barrier.

Since the horseshoe ends of the barrier face Start1, the waves easily move into the center before interacting with the increased impedance of the barrier. The formation of a ridge indicates that some of the waves moved around the barrier, then penetrated the barrier from the back side. Any location along the ridge is equally distant from the start by moving to either side of hill. The locations on the back side of the ridge, however, are optimally connected to Start1 by moving down the hill to the right and around the barrier... a counter-intuitive move. Neither ruler nor Pythagorous’ theorem suggest that you must initially move away from Start1 to begin the optimal route connecting the location to Start1. That’s because they simply assume that all movement is in a straight line connecting two points—an extremely limiting assumption in the real world of complex barriers.

The vertical line intersecting both surfaces identifies a map location that is 63 grid spaces from Start1 and only 5 from Start2. Since these “shortest, but not necessarily straight” distances are stored at the same column, row position in the two proximity matrices, they

can be easily retrieved and their difference computed ($63 - 5 = 58$). If this is done for all locations, a difference surface (S1-S2 Surface inset in the middle of the figure) is generated identifying the relative advantage between Start1 and Start2 access for all locations throughout the project area.

The “0” line identifies locations that are equally distant between the two starting locations. It’s similar to the “perpendicular bisector” you might remember from high school geometry, except it is bent and twisted reflecting the effect of the intervening barrier on actual movement. The sign of the difference indicates who has the advantage—negative values identify locations where Start1 has an advantage; positive values indicate a Start2 advantage. The magnitude of the value identifies the strength of the advantage. The two surfaces on the right isolate the relative advantages for both starting locations. Similar advantage surfaces can be derived for additional starting locations, keeping in mind that the “starters” can be any combination of point, line or areal features.

In the example, a +58 denotes a location with a strong advantage for Start2 access... it would be stupid to trek all the way to Start1. If you were a thirsty animal (or pub patron), why would you travel the extra distance? The liquid libations would have to be a lot better, or the ambiance and other thirsty organisms much more to your liking. If that were the case, then the relative attractiveness of starter locations can be incorporated into the derivation of the accumulation surfaces (termed “gravity” modeling).

Accumulation surface analysis provides valuable information for a wide array of applications. Natural resource managers use the technique to identify “home ranges” and “corridors of movement” based on the arrangement of landscape features. Instead assuming a simple distance of “within a two mile radius” of an animal’s burrow, an effective distance home range based on absolute (e.g., river) and relative (e.g., cover type preferences) barriers can be modeled.

Similarly, a retail market analyst can model the “home range” of a particular breed of shopper by characterizing the road network (e.g., speed limit) and ancillary attractions (e.g., areas of interest). Or in-store shopper habitat can be modeled using the aisles like roads and department fixtures as attractions; even “blue-light specials” could be modeled as dynamic features in a real time system. Traffic flows, whether in-store, across town or in the wild are similar beasts from a GIS analysis perspective. The absolute and relative barriers might change, but the basic application model (GIS procedure) remains the same. Next time we will consider other accumulation surface analyses... what do you think would happen if you added two accumulation surfaces? What information would the summation surface provide?

Author’s Note: for “hands-on” experience in deriving and analyzing accumulation surfaces, see exercises TMAP2, TUTOR5, TUTOR6 and TU-RESP in the Tutorial Map Analysis Package (tMAP) software (Berry, 1993; GIS World Books).

Determining Optimal Path Corridors

(GeoWorld, December 1997, pg. 28)

[\(return to top of Topic\)](#)

The first section in this series described the construction and fundamental nature of accumulation surfaces. The second section discussed an analysis procedure for mapping relative geographic advantage that involved subtracting two surfaces. This time we will investigate the interpretation of slope and aspect of an accumulation surface and what information is derived if we add the surfaces.

Recall that as distance increases from a location(s) the values can be plotted as a 3-dimensional surface, like those shown in the left portion of figure 1. The lowest point on the surface identifies the starting location(s) as zero units away from itself. All other locations contain increasing distance values forming the characteristic “bowl-shape” of an accumulation surface.

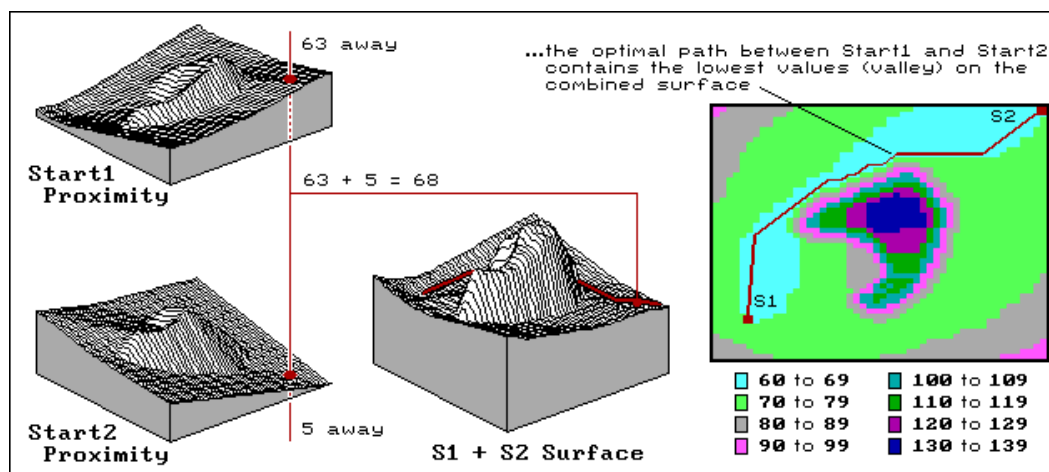


Figure 1. The sum of two proximity surfaces identifies the optimal path between two locations as the lowest values, while increasing values identify the “opportunity cost” of forcing a path through any location.

Effective distance (as opposed to “simple,” straight-line distance) can be derived by introducing absolute and relative barriers to movement. The “hill” in the Starter1 and Starter2 proximity surfaces reflect the increased impedance of a horseshoe-shaped barrier in the center of the map area. Each grid space on a friction map is coded with the “relative cost” of traversing that location. Note that the increased impedance is translated into the steeper slopes for the barrier area. Therefore a slope map of an accumulation surface unmasks the relative ease of optimal travel through each grid space.

The notion of “optimal movement” embedded in an accumulation surface is important. The “splash” algorithm used to build the surface considers movement from the eight surrounding cells to each location. The accumulated distance and the relative impedance

for each of the eight potential “steps” is evaluated. The least costly step, in terms of total movement, is assigned. Therefore an aspect map of an accumulation surface unmasks the direction of optimal movement through each grid space.

That’s a lot of spatially-specific information—the rate and direction of optimal movement throughout a map area. It allows us to relax the assumption that “everything moves in a straight line and with equal impedance.” In fact, things rarely move as simply as they respond to the complex patterns of absolute and relative barriers existing in the real world. Slope and aspect maps of an accumulation surface allow us to track the conditions of that complex movement at each map location.

As described in the first article, the optimal path from any location to the origin is identified as the steepest downhill route over the surface. In the second article, we found that subtracting two accumulation surfaces located the bisecting line between the origins as 0 (equidistant). The sign of the distance value on the difference map indicated which origin was closer, and its magnitude indicated how much closer.

For example, a wildfire response model might generate a proximity map for two fire stations considering both on and off-road movement. Subtracting the maps locates the effective dividing line between the two stations. In retail marketing, the halfway line is extended into a broad band indicating a “combat zone” for customers. Areas outside the band have distinct proximity advantages, while the real battles are waged in the combat zone where there the differences are marginal.

If subtracting accumulation surfaces creates useful information, what do you think happens when you add them? The surface in the center of the figure is a summation surface for the example data. The highlighted location is 63 from Start1 and 5 from Start2, therefore it is a total of 68 units away from both. In other words, the best path connecting the two origins which passes through that location has a total length of 68. In fact, the values on the summation surface identify the length of the “best” path forced through any given map location.

The optimal path between the two locations (identified by the line in both the 2-D and 3-D views) contains the set of locations having the lowest values (a valley connecting the origins). The saw-toothed appearance of the optimal path is an artifact of arithmetic rounding, the nature of the splash algorithm and minimal friction outside the barrier in the center. Values above the valley floor indicate the length of the best, but sub-optimal paths forced through any location.

The difference between the lowest value on the summation surface and the value at any other location identifies the “opportunity cost” of forcing a route through that location. The 2-D display shows fixed intervals of increasing opportunity cost—you would be crazy to force a route through the darker tones (a mountain of opportunity cost). Next time we will look at constructing a “stepped accumulation surface” which enables you to determine the optimal path connecting a series of predetermined stops along the way. As

a sneak preview, it involves minimizing successive accumulation surfaces... whew!

Analyzing Stepped Accumulation Surfaces

(GeoWorld, January 1998, pg. 30)

[\(return to top of Topic\)](#)

Hopefully you have survived the last three columns on accumulation surfaces. The discussion has covered the fundamental nature of accumulation surfaces (increasing distance waves), procedures for determining relative geographic advantage (subtract), ways to uncover direction and speed of optimal movement (slope and aspect), and a technique for determining the corridor of optimal movement (add). If that wasn't enough, now we get to extend the discussion to a "stepped accumulation surface" and "optimal path zones."

Suppose you know several places you would like to visit, but don't have a particular route in mind. If you know the order you would like to visit them, then a directed stepped accumulation surface is for you. Simply generate an effective proximity surface from the first location like those discussed in the previous articles—splash, one ripple, two ripple, three ripple and more radiate out from your starting point. The familiar bowl of proximity identifies the effective distance to all other locations. All you have to do is "stream" down the bowl from your second point of interest to identify the optimal path.

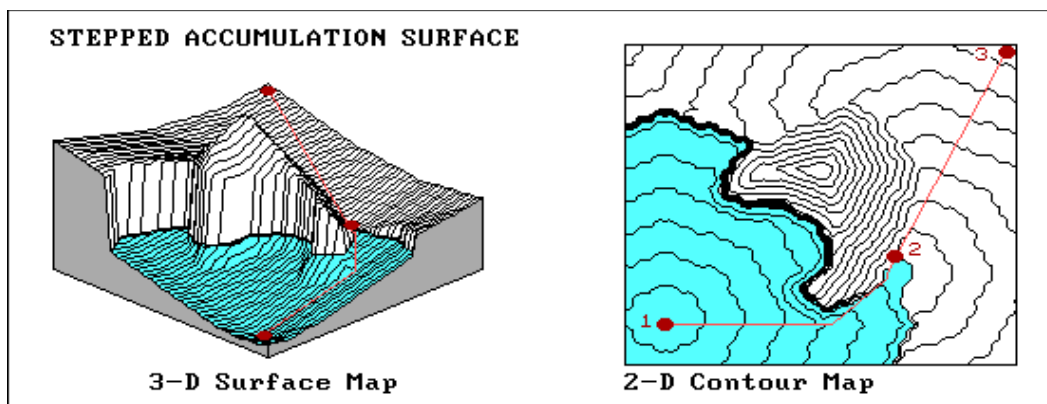


Figure 25.4. Stepped Accumulation Surface. Proximity from the first point is calculated (shaded) until the second point is reached, then proximity from that point is calculated (non-shaded) forming the next tier; the individual optimal paths along each of the stepped proximity surfaces forms the overall optimal route.

Now, construct a proximity surface from the second point to everywhere, and stream your third stop down it for the second leg of your journey. The left side of Figure 1 shows a stepped accumulation surface for the first two segments of a "directed hike" through the demonstration area. The shaded area shows the portion of the proximity

surface from the start until the concentric ripples encountered the second point. The non-shaded area picks up the count by adding the proximity from there to all other locations. The result is a two-tiered surface similar to a spiral staircase. If you stream down the stepped accumulation surface from the third stop, it first flows down the top tier to the second stop, then continues down to the first point (right side of Figure 1).

Additional stops are considered by repeating the successive construction of “proximity bowls” and streaming down them for sequential optimal path segments. An undirected procedure allows the computer to determine a spatially efficient ordering of the points. You simply identify a starting location, the points you need to visit and the computer calculates the optimal route connecting them. The solution spreads out from the first point until it encounters the closest visitation point, then streams down the truncated proximity surface for the first leg. The next tier spreads out until it encounters its closest point and streams down for the next leg. The process continues until all of the visitation points have been evaluated. If you intend to return to the starting point, the home leg considers the starting point and is evaluated last.

The undirected, stepped accumulation surface technique (whew ...quite a mouthful) is similar to the classic “traveling salesman” problem in network analysis. However, it provides a solution in continuous space, respecting the complex reality of absolute and relative barriers. This is important if the traveling salesman doesn’t have a car, or if the mover isn’t constrained to a bunch of lines, like a herd of elk, or shoppers in a store.

In a recent project (see Author’s Note), we encoded the floor plan for a retail superstore (98,000 1-foot grids), translating the fixtures into absolute barriers and congested areas into relative barriers. Shelving locations were identified on each fixture and linked to product codes. The checkout records for each market basket were used to “place” each item purchased on the appropriate shelf. These visitation points were evaluated for the “plausible” path used to collect the items between the door to the cash register.

Granted, a shopper could do “a random walk” to collect the items, but a shopper with a mission who knows the store, would be foolish to veer off the calculated route. Also, the consideration of several thousands of paths over a period of time converges on a map of relative in-store shopper activity. The analysis was summarized in hourly time-steps, displayed as normalized thematic maps, and animated to show the ebb and flow of shopper activity throughout the day.

The right side of Figure 2 shows the zones of influence for each leg of the optimal route. The locations within each zone are optimally connected to a particular segment. So, how might one use such information? In the shopper example, the distance from a shopper’s path can be interpreted as geographic impedance that must be overcome to veer off stride. Influence zones locate areas along a common portion of a shopper’s route. Overlaying these data with in-store departments, sales density surfaces and item categories produces a lot of information about shelving for retail marketing types. How might you use

accumulation surfaces? ...it's up to your innovative mind.

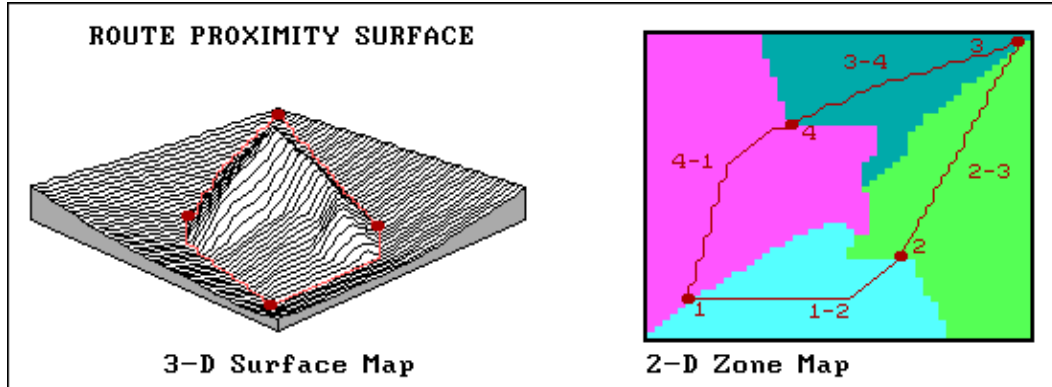


Figure 2. Route Proximity Surface. Proximity from the optimal route identifies the distance to the closest segment of the route for every location in a project area; the influence zone of each segment along the route identifies which segment is the closest.

Author's Note: see www.innovativegis.com/basis/mapanalysis/, Topic 6, "Analyzing In-Store Shopping Patterns."

[\(return to top of Topic\)](#)

[\(Back to the Table of Contents\)](#)