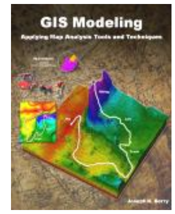
Beyond Mapping IV

***Topic 9** – Math/Stat Framework for Map Analysis (Further Reading)*



GIS Modeling book

<Click here> for a printer-friendly version of this topic (.pdf).

(Back to the Table of Contents)

Map-ematically Messing with Mapped Data

(GeoWorld, February 2012)

(return to top of Topic)

Earlier discussion introduced the idea of *spatial*STEM for teaching map analysis and modeling fundamentals within a mathematical context that resonates with science, technology, engineering and math/stat communities ("*SpatialSTEM Has Deep Mathematical Roots*," GeoWorld, January 2012). The discussion established a general framework and grid-based data structure needed for quantitative analysis of spatial patterns and relationships. This section focuses on the nature of mapped data, an example of a grid-math/algebra application and discussion of extended spatial analysis operations.

Figure 1 identifies the two primary perspectives of spatial data—1) *Numeric* that indicates how numbers are distributed in "number space" (*What* condition) and 2) *Geographic* that indicated how numbers are distributed in "geographic space" (*Where* condition). The numeric perspective can be grouped into categories of *Qualitative* numbers that deal with general descriptions based on perceived "quality" and *Quantitative* numbers that deal with measured characteristics or "quantity."

Further classification identifies the familiar numeric data types of Nominal, Ordinal, Interval, Ratio and Binary. It is generally well known that very few math/stat operations can be performed using qualitative data (Nominal, Ordinal), whereas a wealth of operations can be used with quantitative data (Interval, Ratio). Only a specialized few operations utilize Binary data.

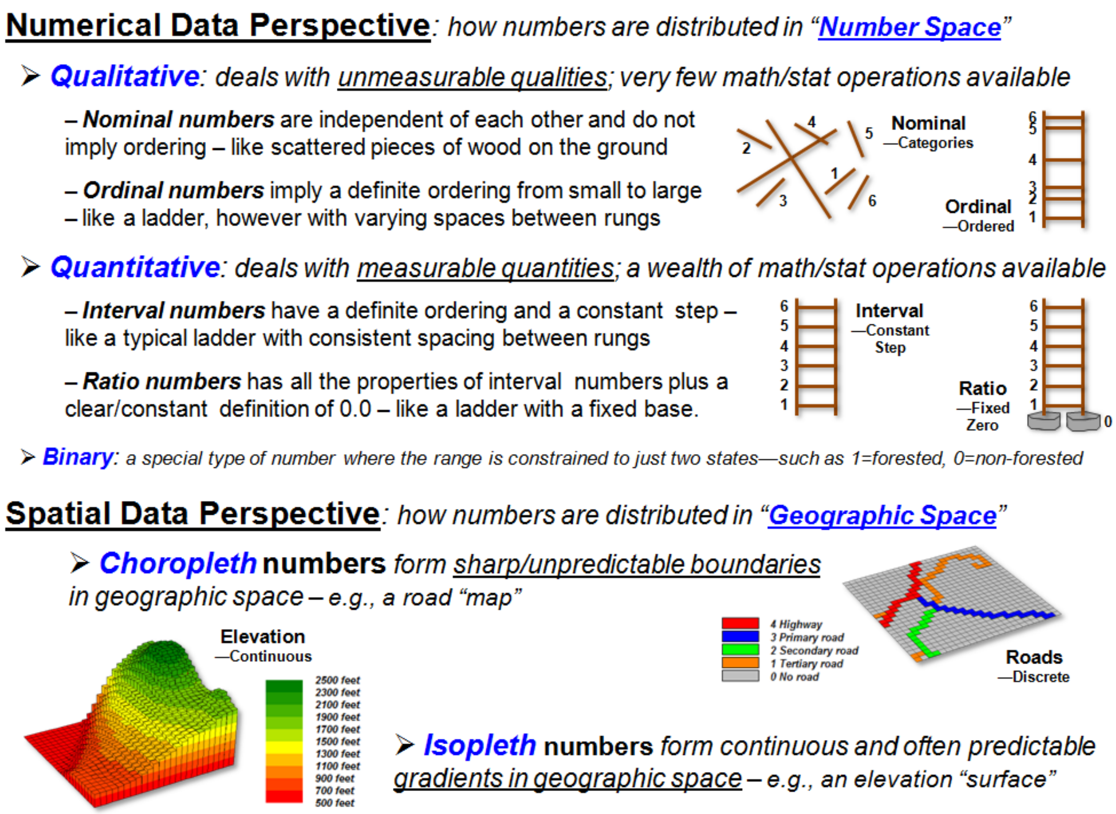


Figure 1. Spatial Data Perspectives—Where is What.

Less familiar are the two geographic data types. *Choropleth* numbers form sharp and unpredictable boundaries in space, such as the values assigned to the discrete map features on a road or cover type map. *Isopleth* numbers, on the other hand, form continuous and often predictable gradients in geographic space, such as the values on an elevation or temperature surface.

Putting the Where and What perspectives of spatial data together, *Discrete Maps* identify mapped data with spatially independent numbers (qualitative or quantitative) forming sharp abrupt boundaries (choropleth), such as a cover type map. Discrete maps generally provide limited footholds for quantitative map analysis. On the other hand, *Continuous Maps* contain a range of values (quantitative only) that form spatial gradients (isopleth), such as an elevation surface. They provide a wealth of analytics from basic grid math to map algebra, calculus and geometry.

Site-specific farming provides a good example of basic grid math and map algebra using continuous maps (figure 2). *Yield Mapping* involves simultaneously recording yield flow and

GPS position as a combine harvests a crop resulting in a grid map of thousands of geo-registered numbers that track crop yield throughout a field. *Grid Math* can be used to calculate the mathematical difference in yield at each location between two years by simply subtracting the respective yield maps. *Map Algebra* extends the processing by spatially evaluating the full algebraic percent change equation.

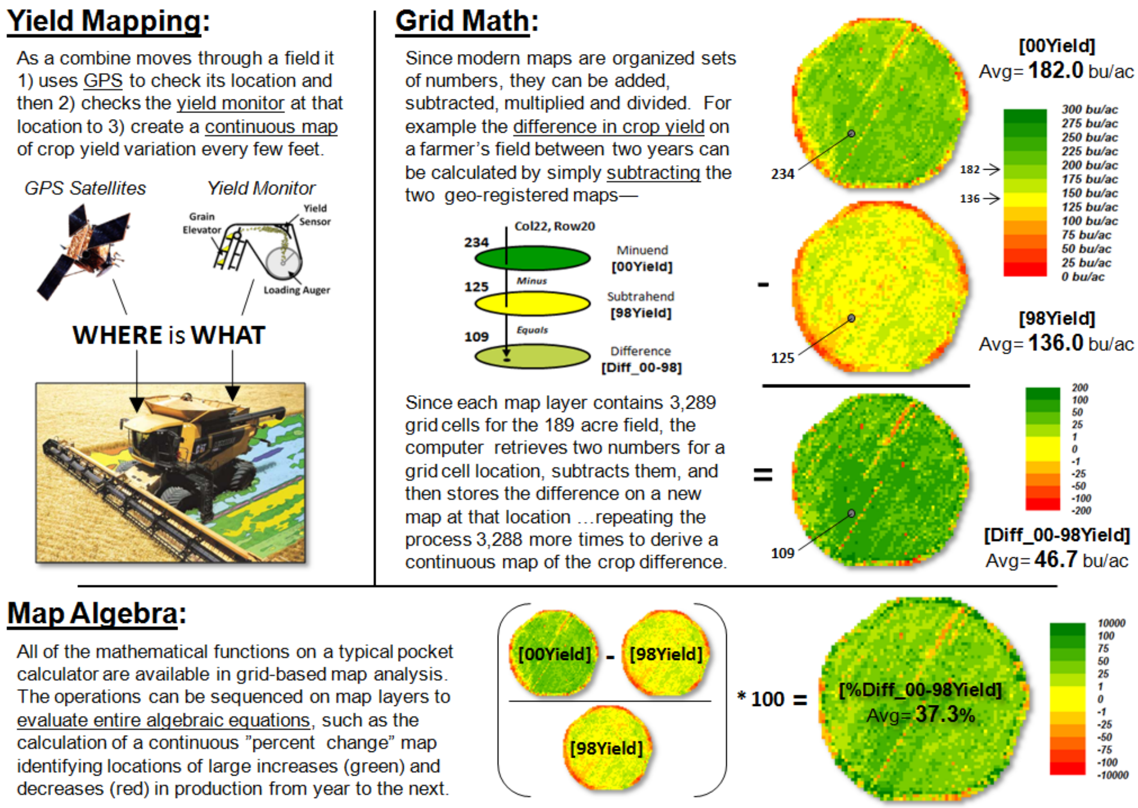


Figure 2. Basic Grid Math and Algebra example.

The paradigm shift in this map-*ematical* approach is that map variables, comprised of thousands of geo-registered numbers, are substituted for traditional variables defined by only a single value. Map algebra's continuous map solution shows localized variation, rather than a single "typical" value being calculated (i.e., 37.3% increase in the example) and assumed everywhere the same in non-spatial analysis.

Figure 3 expands basic Grid Math and Map Algebra into other mathematical arenas. *Advanced Grid Math* includes most of the buttons on a scientific calculator to include trigonometric functions. For example, taking the cosine of a slope map expressed in degrees and multiplying it times the planimetric surface area of a grid cell calculates the surface area of the "inclined plane" at each grid location. The difference between planimetric area represented by traditional maps and surface area based on terrain steepness can be dramatic and greatly affect the characterization of "catchment areas" in environmental and engineering models of surface runoff.

A *Map Calculus* expresses such functions as the derivative and integral within a spatial context. The derivative traditionally identifies a measure of how a mathematical function changes as its input changes by assessing the slope along a curve in 2-dimensional abstract space.

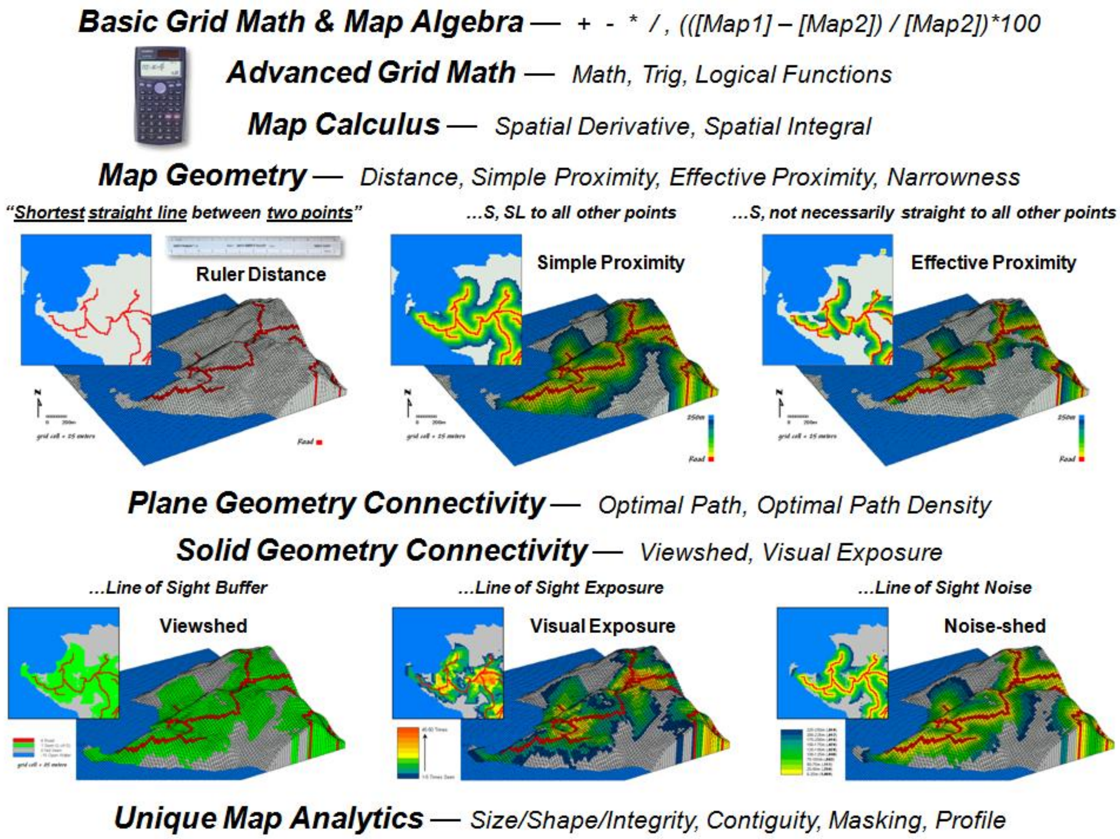


Figure 3. Spatial Analysis operations.

The spatial equivalent calculates a "slope map" depicting the rate of change in a continuous map variable in 3-dimensional geographic space. For an elevation surface, slope depicts the rate of change in elevation. For an accumulation cost surface, its slope map represents the rate of change in cost (i.e., a marginal cost map). For a travel-time accumulation surface, its slope map indicates the relative change in speed and its aspect map identifies the direction of optimal movement at each location. Also, the slope map of an existing topographic slope map (i.e., second derivative) will characterize surface roughness (i.e., areas where slope itself is changing).

Traditional calculus identifies an integral as the net signed area of a region along a curve expressing a mathematical function. In a somewhat analogous procedure, areas under portions of continuous map surfaces can be characterized. For example, the total area (planimetric or surface) within a series of watersheds can be calculated; or the total tax revenue for various neighborhoods; or the total carbon emissions along major highways; or the net difference in crop yield for various soil types in a field. In the spatial integral, the net sum of the numeric values for portions of a continuous map surface (3D) is calculated in a manner comparable to calculating the area under a curve (2D).

Traditional geometry defines Distance as "the shortest straight line between two points" and routinely measures it with a ruler or calculates it using the Pythagorean Theorem. *Map Geometry* extends the concept of distance to Simple Proximity by relaxing the requirement of just "two points" for distances to all locations surrounding a point or other map feature, such as a road.

A further extension involves Effective Proximity that relaxes "straight line" to consider absolute and relative barriers to movement. For example effective proximity might consider just uphill locations along a road or a complex set of variable hiking conditions that impede movement from a road as a function of slope, cover type and water barriers.

The result is that the "shortest but not necessarily straight distance" is assigned to each grid location. Because a straight line connection cannot be assumed, optimal path routines in *Plane Geometry Connectivity* (2D space) are needed to identify the actual shortest routes. *Solid Geometry Connectivity* (3D space) involves line-of-sight connections that identify visual exposure among locations. A final class of operations involves *Unique Map Analytics*, such as size, shape, intactness and contiguity of map features.

Grid-based map analysis takes us well beyond traditional mapping …as well as taking us well beyond traditional procedures and paradigms of mathematics. The next section considers extension of traditional statistics to spatial statistics within the *spatial*STEM framework.

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Author's Notes: *a table of URL links to further readings on the grid-based map analysis/modeling concepts, terminology, considerations and procedures described in this three-part series on spatialSTEM is posted at www.innovativegis.com/basis/MapAnalysis/Topic30/sSTEM/sSTEMreading.htm*.

Paint by Numbers Outside the Traditional Statistics Box

(GeoWorld, March 2012)

(return to top of Topic)

The previous section described a general framework and approach for teaching spatial analysis within a mathematical context that resonates with science, technology, engineering and math/stat communities (*spatial*STEM). The following discussion focuses on extending traditional statistics to a spatial statistics for understanding geographic-based patterns and relationships.

Whereas *Spatial analysis* focuses on "contextual relationships" in geographic space (such as effective proximity and visual exposure), *Spatial statistics* focuses on "numerical relationships" within and among mapped data (figure 1). From a spatial statistics perspective there are three primary analytical arenas— Summaries, Comparisons and Correlations.

Statistical summaries provide generalizations of the grid values comprising a single map layer (within), or set of map layers (among). Most common is a tabular summary included in a discrete map’s legend that identifies the area and proportion of occurrence for each map category, such as extremely steep terrain comprising 286 acres (19 percent) of a project area. Or for a continuous map surface of slope values, the generalization might identify the data range as from 0 to 65% and note that the average slope is 24.4 with a standard deviation of 16.7.

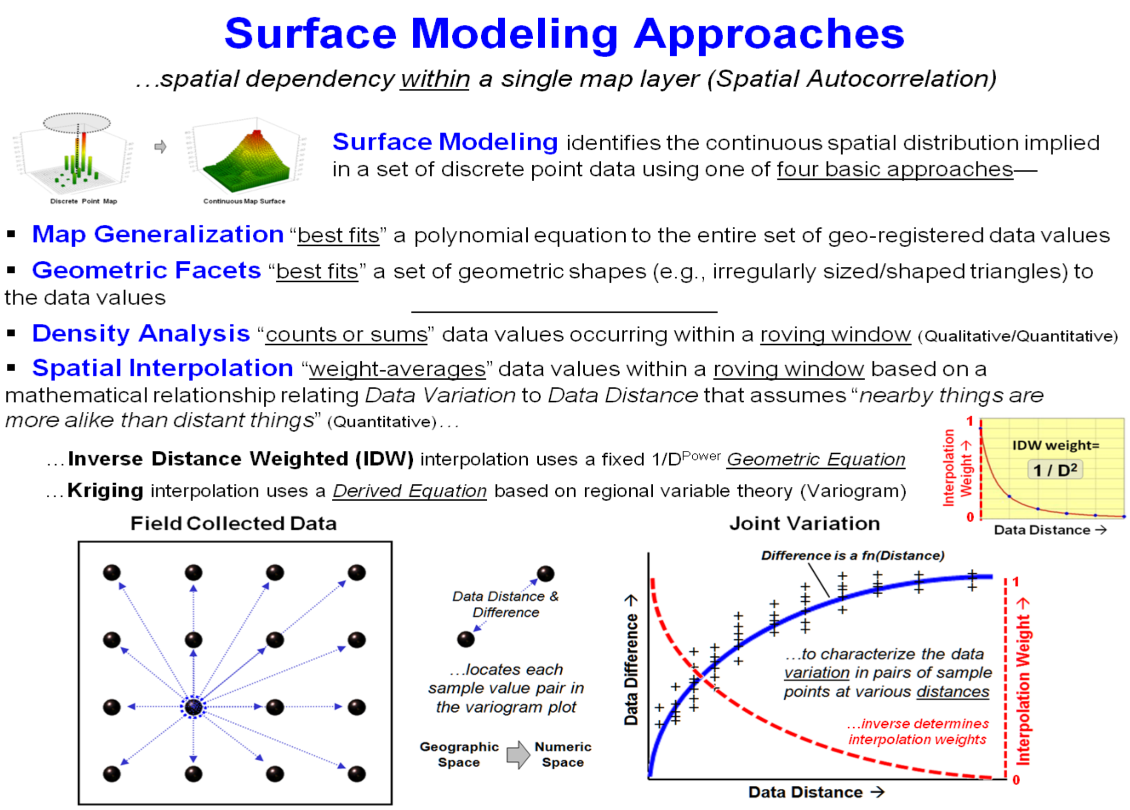


Figure 1. Spatial Statistics uses numerical analysis to uncover spatial relationships and patterns.

Summaries among two or more discrete maps generate cross-tabular tables that “count” the joint occurrence of all categorical combinations of the map layers. For example, the coincidence of steepness and cover maps might identify that there are 242 acres of forest cover on extremely steep slopes (16 percent), a particularly hazardous wildfire joint condition.

Map comparison and correlation techniques only apply to continuous mapped data. Comparisons within a single map surface involve normalization techniques. For example, a Standard Normal Variable (SNV) map can be generated to identify “how unusual” (above or below) each map location is compared to the typical value in a project area.

Direct comparisons among continuous map surfaces include appropriate statistical tests (e.g., F-test), difference maps and surface configuration differences based on variations in surface slope and orientation at each grid location.

Map correlations provide a foothold for advanced inferential spatial statistics. Spatial autocorrelation within a single map surface identifies the similarity among nearby values for each grid location. It is most often associated with surface modeling techniques that employ the assumption that "nearby things are more alike than distant things"—high spatial autocorrelation—for distance-based weight averaging of discrete point samples to derive a continuous map surface.

Spatial correlation, on the other hand, identifies the degree of geographic dependence among two or more map layers and is the foundation of spatial data mining. For example, a map surface of a bank's existing concentration of home equity loans within a city can be regressed against a map surface of home values. If a high level of spatial dependence exists, the derived regression equation can be used on home value data for another city. The resulting map surface of estimated loan concentration proves useful in locating branch offices.

In practice, many geo-business applications utilize numerous independent map layers including demographics, life style information and sales records from credit card swipes in developing spatially consistent multivariate models with very high R-squared values. Like most things from ecology to economics to environmental considerations, spatial expression of variable dependence echoes niche theory with grid-based spatial statistics serving as a powerful tool for understanding geographic patterns and relationships.

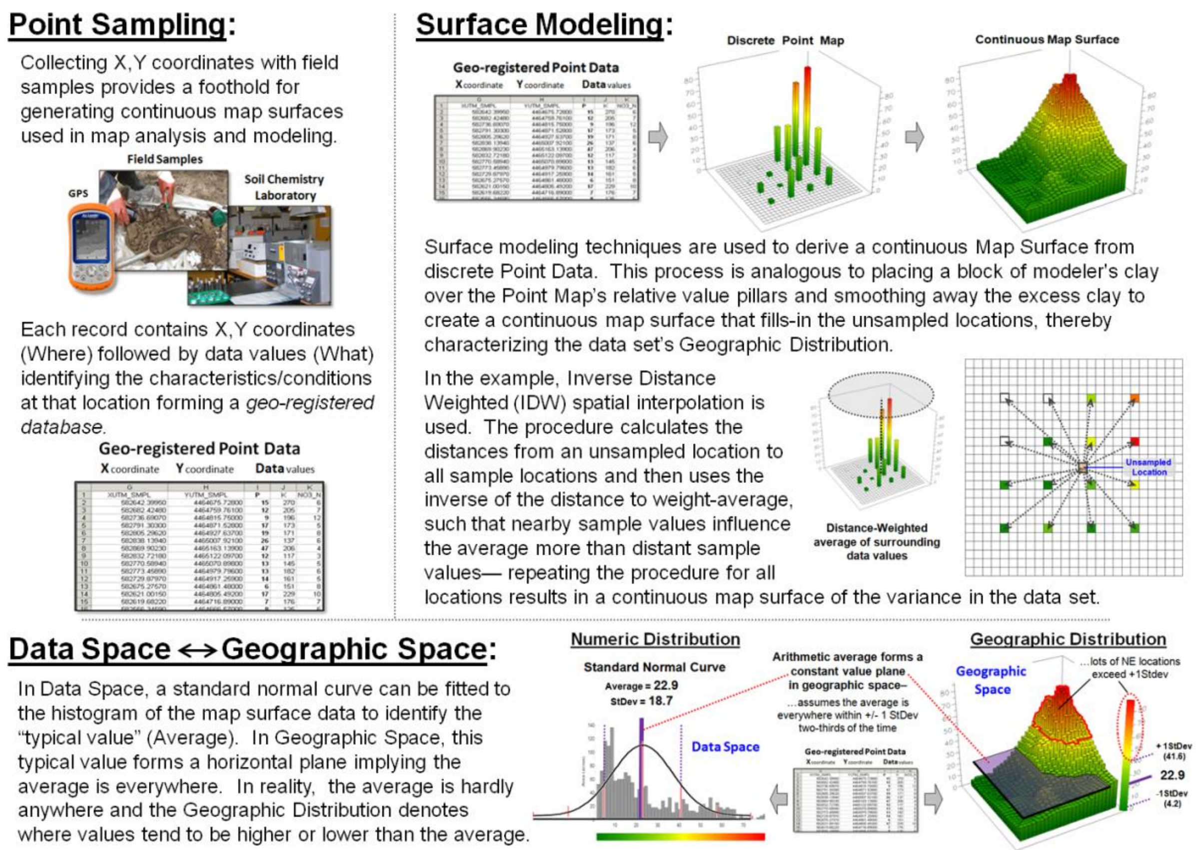


Figure 2. An example of Surface Modeling that derives a continuous map surface from set of discrete point data.

Figure 2 describes an example of basic surface modeling and the linkage between numeric space and geographic space representations using environmentally-oriented mapped data. Soil samples are collected and analyzed assuring that geographic coordinates accompany the field samples. The resulting discrete point map of the field soil chemistry data are spatially interpolated into a continuous map surface characterizing the data set's geographic distribution.

The bottom portion of figure 2 depicts the linkage between Data Space and Geographic Space representations of the mapped data. In data space, a standard normal curve is fitted to the data as means to characterize its overall "typical value" (Average= 22.9) and "typical dispersion" (StDev= 18.7) without regard for the data's spatial distribution.

In geographic space, the Average forms a flat plane implying that this value is assumed to be everywhere within +/- 1 Standard Deviation about two-thirds of the time and offering no information about where values are likely more or less than the typical value. The fitted continuous map surface, on the other hand, details the spatial variation inherent in the field collected samples.

Nonspatial statistics identifies the "central tendency" of the data, whereas surface modeling maps the "spatial variation" of the data. Like a Rochart ink blot, the histogram and the map surface provide two different perspectives. Clicking a histogram pillar identifies all of the grid cells within that range; clicking on a grid location identifies which histogram range contains it.

This direct linkage between the numerical and spatial characteristics of mapped data provides the foundation for the spatial statistics operations outlined in figure 3. The first four classes of operations are fairly self-explanatory with the exception "Roving Window" summaries. This technique first identifies the grid values surrounding a location, then mathematically/statistically summarizes the values, assigns the summary to that location and then moves to the next location and repeats the process.

Another specialized use of roving windows is for Surface Modeling. As described in figure 2, inverse-distance weighted spatial interpolation (IDW) is the weight-averaged of samples based on their relative distances from the focal location. For qualitative data, the total number of occurrences within a window reach can be summed for a density surface.

In figure 3 for example, a map identifying customer locations can be summed to identify the total number of customers within a roving window to generate a continuous map surface customer density. In turn, the average and standard deviation can be used to identify "pockets" of unusually high customer density.

Standard multivariate techniques using "data distance," such as Maximum Likelihood and Clustering, can be used to classify sets of map variables. Map Similarity, for example, can be used to compare each map location's pattern of values with a comparison location's pattern to create a continuous map surface of the relative degree of similarity at each map location.

Statistical techniques, such as Regression, can be used to develop mathematical functions between dependent and independent map variables. The difference between spatial and non-spatial approaches is that the map variables are spatially consistent and yield a prediction map that shows where high and low estimates are to be expected.

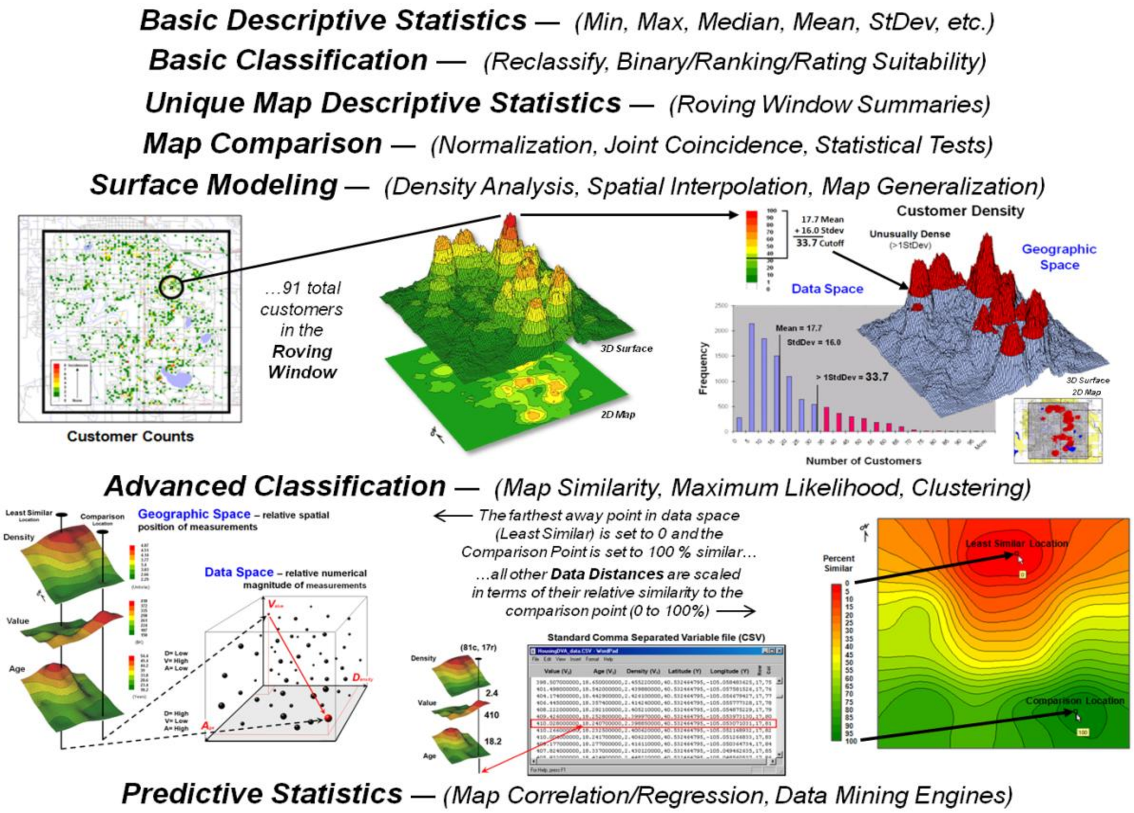


Figure 3. Classes of Spatial Statistics operations.

The bottom line in spatial statistics (as well as spatial analysis) is that the spatial character within and among map layers is taken into account. The grid-based representation of mapped data provides the consistent framework that needed for these analyses. Each database record contains geographic coordinates (X,Y= Where) and value fields identifying the characteristics/conditions at that location (V_i= What).

From this map-*ematical* view, traditional math/stat procedures can be extended into geographic space. The paradigm shift from our paper map legacy to "maps as data first, pictures later" propels us beyond mapping to map analysis and modeling. In addition, it defines a comprehensive and common *spatial*STEM educational environment that stimulates students with diverse backgrounds and interests to "think analytically with maps" in solving complex problems.

<u>Author's Notes</u>: *a table of URL links to further readings on the grid-based map analysis/modeling concepts, terminology, considerations and procedures described in this three-part series on spatialSTEM is posted at www.innovativegis.com/basis/MapAnalysis/Topic30/sSTEM/sSTEMreading.htm.*

The Spatial Key to Seeing the Big Picture

(GeoWorld, September 2013)

(return to top of Topic)

Earlier discussion described the standard Latitude/Longitude grid as a "Universal Spatial dB Key" that is comparable to the date/time tagging of records in most database systems ("*To Boldly Go Where No Map Has Gone Before,*" GeoWorld, October 2012). With general availability of GPS coordinates on most data collection devices, cameras, smartphones and tablets, earth position can be easily stamped with each data record. Couple that with geo-coding by street address and most data collected today has a triplet of numbers indicating location (where), as well as characteristic/condition (what)—XY and Value designating "where is what."

Data flowing from a "spatially aware database" can be thought of as a faucet spewing data that meets a query (figure 1). In turn, each value flows to the appropriate grid cell based on its Lat/Lon tag. The process can be conceptualized as the "what" attributes aligning within an analysis frame (matrix of numbers) that characterizes the spatial pattern/distribution inherent in a set of data.

While the long history of quantitative data analysis focused on the *numerical distribution* of data, quantitative analysis of the *spatial distribution* of geospatial data provides an new frontier for understanding spatial patterns and relationships influencing most physical, biological, environmental, economic, political and cultural systems. The recognition, development and application of this fresh math/stat paradigm (sort of a "map-ematics") promises to revolutionize how we extract and utilize information from field collected data (see Author's Note 1).

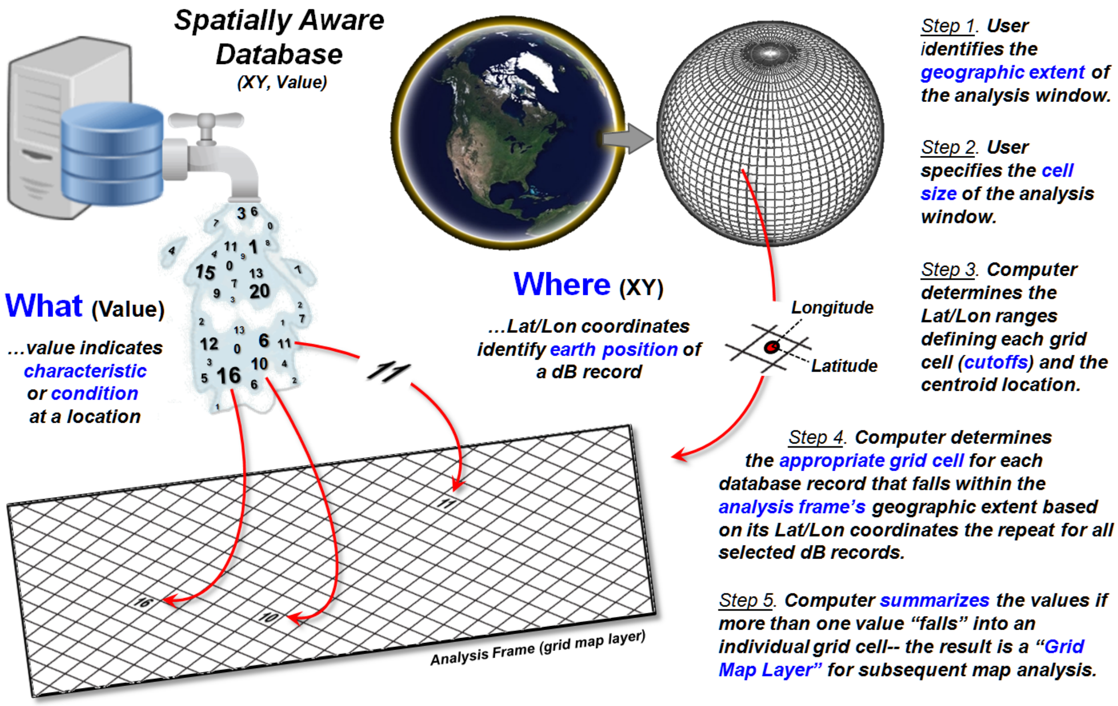


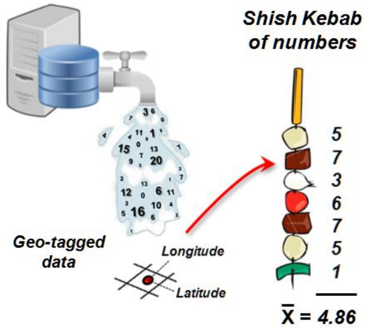
Figure 1. Steps in generating a grid map layer from spatially tagged data.

Converting spatially tagged data into grid maps is outlined on the right side of figure 1 as a five step process. The user first identifies the "geographic extent" of an area of interest by interactively dragging a box on a map or by entering Lat/Lon coordinates for the boundary (Step 1).

An appropriate "cell size" for analysis is then entered as length of a side of an individual grid cell (Step 2). The smaller the cell size the higher the spatial resolution affording greater detail in positioning but resulting in exponentially larger matrices for storage. User judgment is applied to balance the precision (correct placement), accuracy (correct characterization) and storage/performance demands (see Author's Note 2).

In Step 3, the computer divides the lengths of the NS and EW sides of the project area extent by the cell size to determine the number of rows and columns of a matrix (termed the *Analysis Frame*) used to store grid layer information (map variables). This establishes an algorithm for determining the Lat/Lon ranges defining each grid cell and its centroid position. Considerations and implications surrounding this technically tricky step (3D curved earth to 2D flat matrix) are reserved for later discussion.

Based on the positioning algorithm's calculations, each geo-tagged value flowing from the database can be placed in the appropriate row/column position in the analysis frame's matrix (Step 4). The processing is repeated for all of the selected dB records. If more than one value "falls" into a grid cell the values are summarized on-the-fly (Step 5).



Spatially Summarizing Data Values (Step 5)

…that "fall" into each grid cell

Quantitative Values	Qualitative Values	Binary Values (0,1)
Most frequently represented as the arithmetic average. Occasionally reported as mode, median, standard deviation, coefficient of variation, etc. to characterize the numerical distribution.	Most frequently represented as the maximum or minimum value. Occasionally reported as presence/absence of a particular number or string of numbers (selection).	Most frequently represented as "counts" of a condition that are summed to identify total occurrence. Occasionally reported as a binary value indicating at least one occurrence (presence/absence).

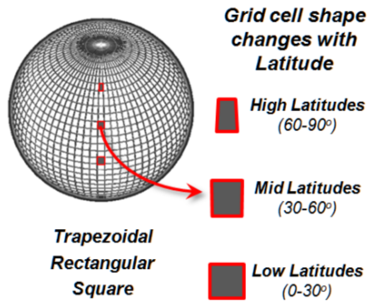
Figure 2. Summarizing multiple data values falling in a single grid cell.

Figure 2 depicts the considerations surrounding the summary of multiple data values sharing a single grid cell. The condition can be conceptualized as a "shish kebab of numbers" that needs to be reduced to an overall value that best typifies the actual characteristic/condition at that location.

The data type of the numbers determines the summary techniques available. Most often quantitative values are averaged as shown in the figure but other statistical metrics can be used depending on the application. Qualitative values are typically assigned the maximum or minimum value encountered in the string. Binary values, such as crime occurrence, are usually summed to identify total count of instances at each grid location.

The result of the five step procedure creates a grid map layer identifying the "*discrete*" spatial pattern of the data that is analogous to a histogram in non-spatial statistics. In most applications, spatial interpolation or density analysis techniques are used to derive a *continuous* grid map layer characterizing the spatial distribution of the data which is analogous to fitting a standard normal curve to a histogram (see Author's Note 3). Once in this generalized form, most traditional quantitative analysis techniques (plus some spatially unique techniques) can be applied to investigate the spatial distribution, as well as the numerical distribution of the data.

The muddling concerns in applying the Lat/Lon grid as a Universal Spatial dB Key is in representing curved 3D earth positions as flat 2D cells of a matrix. Figure 3 shows the reality of the grid cell shape that morphs from squares to stretched rectangles to elongated trapezoids with north/south movement away from the equator (see Author's Note 4).



Grid Cell Shape/Area …changes with Latitude		
Low Latitudes (0-30º)	**Mid Latitudes (30-60º)**	**High Latitudes (60-90º)**
1º Lat/Lon grid cells along the equator form squares of approximately 69 by 69 miles (~4761mi²). For most applications the assumption of a continuous set of "square" grid cells is valid.	Grid cells become somewhat stretched with increasing latitude (slight rectangular shape). For most applications the assumption of a continuous set of "square" grid cells is tolerable.	Grid cells become elongated with smaller top/bottom distances (trapezoidal shape). For most applications the assumption of "square" grid cells is unreliable and directional, proximal and areal calculations imprecise.
Latitude distances vary slightly from 68.71mi at the Equator → to 69.0mi at 40º → to 69.4 at the Poles. **Longitude distances** vary widely from 69.17mi at the Equator → to 53.0mi at 40º → to 34.5mi at 60º → to 27.6mi at the Arctic Circle → to 0.0mi at the Poles.		

Figure 3. The area and shape of Lat/Lon grid cells varies with increasing latitude.

Relatively small changes in the length of a degree of "latitude parallels" occur because of polar flattening— earth is an oblique spheroid instead of a perfect sphere due to centrifugal forces as the earth spins. However huge changes occur for "longitude meridians" as the lines converge at the poles— a degree of longitude is widest at the equator and gradually shrinks to zero at the poles.

The bottom line is that directly representing the Lat/Lon grid as a two-dimensional matrix can be unreliable for large project areas at the higher latitudes. However two caveats are in play. One is that projection algorithms can be applied on-the-fly to transform the curved 3D coordinates to a planar representation and then back to lat/Lon.

The other is that for many applications involving relatively small project areas at low or mid latitudes, the positional precision tolerable. The notion of "tolerable" precision is what most differentiates "mapping" from "map analysis." While neighbors and armies fight over inches in the placement of borders, most data analysts are more accommodating and satisfied knowing things are much higher (or lower) over there as compared to here—a few inches or feet (or even miles in some cases) misplacement doesn't obscure the big picture of the spatial distribution and relationships.

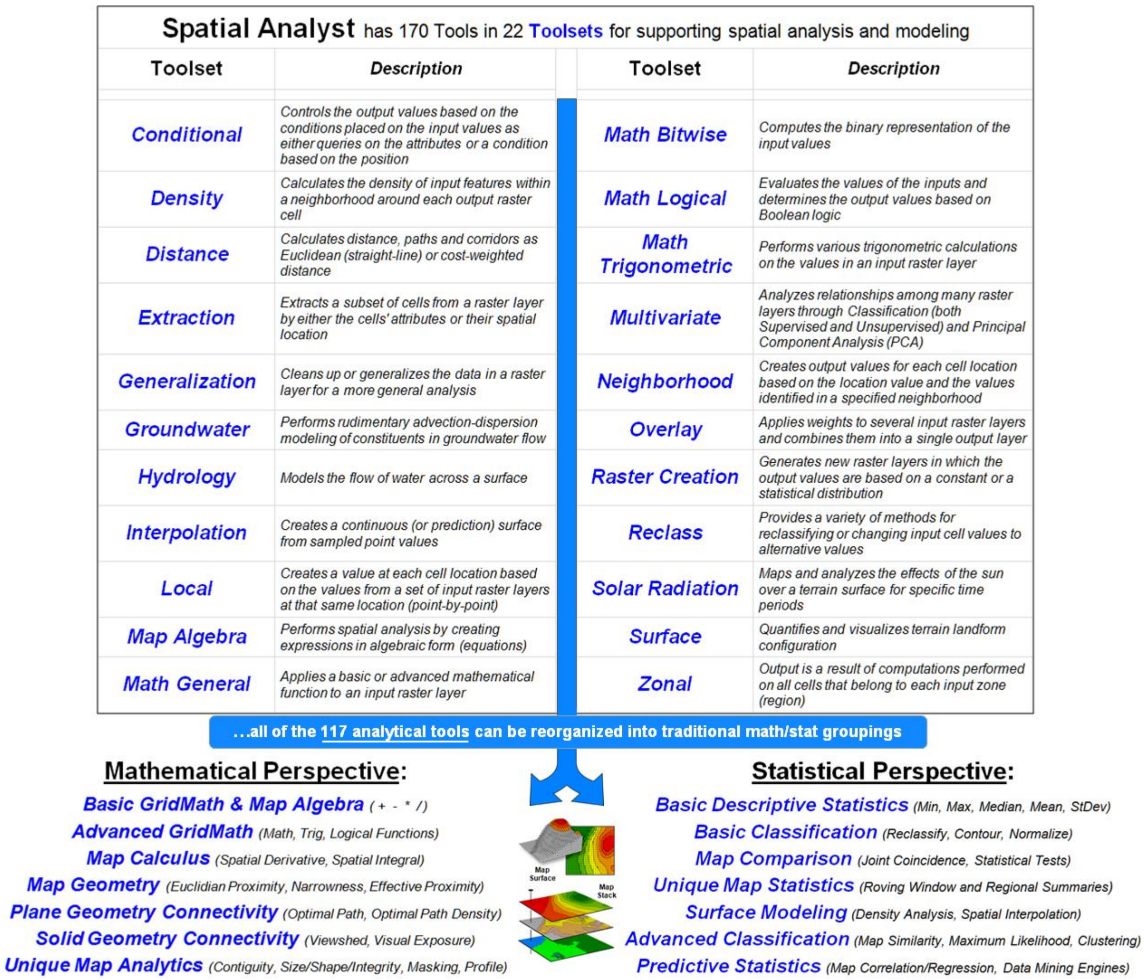
Author's Notes: *1) See, Topic 30, "A Math/Stat Framework for Grid-based Map Analysis and Modeling;" 2) see Introduction, section 2, "Determining Exactly Where Is What;" 3) see Topic 2, "Spatial Interpolation Procedures and Assessment" and Topic 7, "Linking Data Space and Geographic Space" in the online book Beyond Mapping III posted at www.innovativegis.com/basis/. 4) For a detailed discussion of latitude and longitude considerations see www.ncgia.ucsb.edu/giscc/units/u014/u014.html in the NCGIA Core Curriculum in Geographic Information Science, by Anthony P. Kirvan and edited by Kenneth Foote.*

Recasting Map Analysis Operations for General Consumption

(GeoWorld, February 2013)

(return to top of Topic)

Earlier discussions have suggested that there is **"***a fundamental mathematical structure underlying grid-based map analysis and modeling that aligns with traditional non-spatial quantitative data analysis*" (see Author's Note 1). This conceptual framework provides a common foothold for understanding, communicating and teaching basic concepts, procedures and considerations in spatial reasoning and analysis resonating with both GIS and non-GIS communities—a **SpatialSTEM** schema—that can be applied to any grid-based map analysis system (see Author's Note 2).



Spatial Analyst has 170 Tools in 22 **Toolsets** for supporting spatial analysis and modeling

Toolset	Description	Toolset	Description
Conditional	Controls the output values based on the conditions placed on the input values as either queries on the attributes or a condition based on the position	Math Bitwise	Computes the binary representation of the input values
Density	Calculates the density of input features within a neighborhood around each output raster cell	Math Logical	Evaluates the values of the inputs and determines the output values based on Boolean logic
Distance	Calculates distance, paths and corridors as Euclidean (straight-line) or cost-weighted distance	Math Trigonometric	Performs various trigonometric calculations on the values in an input raster layer
Extraction	Extracts a subset of cells from a raster layer by either the cells' attributes or their spatial location	Multivariate	Analyzes relationships among many raster layers through Classification (both Supervised and Unsupervised) and Principal Component Analysis (PCA)
Generalization	Cleans up or generalizes the data in a raster layer for a more general analysis	Neighborhood	Creates output values for each cell location based on the location value and the values identified in a specified neighborhood
Groundwater	Performs rudimentary advection-dispersion modeling of constituents in groundwater flow	Overlay	Applies weights to several input raster layers and combines them into a single output layer
Hydrology	Models the flow of water across a surface	Raster Creation	Generates new raster layers in which the output values are based on a constant or a statistical distribution
Interpolation	Creates a continuous (or prediction) surface from sampled point values	Reclass	Provides a variety of methods for reclassifying or changing input cell values to alternative values
Local	Creates a value at each cell location based on the values from a set of input raster layers at that same location (point-by-point)	Solar Radiation	Maps and analyzes the effects of the sun over a terrain surface for specific time periods
Map Algebra	Performs spatial analysis by creating expressions in algebraic form (equations)	Surface	Quantifies and visualizes terrain landform configuration
Math General	Applies a basic or advanced mathematical function to an input raster layer	Zonal	Output is a result of computations performed on all cells that belong to each input zone (region)

Figure 1. Grid-based map analysis operations in any GIS system, such as Spatial Analyst, can be reorganized into commonly understood classes of traditional quantitative data analysis.

For example, the top portion of figure 1 identifies the 22 map analysis "toolsets" containing over 170 individual "tools" in the Spatial Analyst module (ArcGIS by Esri). The organization of the classes of operations involves a mixture of—

- Traditional math/stat procedures (*Conditional, Map Algebra, Math General, Math Bitwise, Math Logical, Math Trigonometric, Multivariate, Reclass*);
- Extensions of traditional math/stat procedures (*Distance, Interpolation, Surface*);

- Unique map analysis procedures (*Density, Local, Neighborhood, Overlay, Zonal*);
- Application-specific procedures (*Groundwater, Hydrology, Solar Radiation*); and
- Housekeeping tasks (*Extraction, Generalization, Raster Creation*).

In large part, this toolset structuring is the result of the module's development over-time responding to "business case" demands by clients instead of a comprehensive conceptual organization. In contrast, Tomlin's "Local, Focal, Zonal and Global" classes characterize the analytical operations on how the input data is obtained for processing, while my earlier groupings of "Reclassify, Overlay, Distance, Neighbors and Statistical" reflect the characteristics of the mapped data generated by the processing.

However, all three of these GIS-based schemas are foreign and confusing to the vast majority of potential map analysis users (all STEM disciplines) as they do not align with their traditional quantitative data analysis experiences. This conceptual disconnect keeps GIS on the sidelines of the much larger quantitative analysis communities and reinforces the idea that GIS is a "technical tool" (mapping and geoquery) not a full-fledged "analytical tool" (spatial analysis and statistics).

The bottom portion of figure 1 identifies the two broad categories of traditional data analysis—Mathematics and Statistics—broken into seven major groupings that resonate with non-GIS communities. All of Spatial Analysts' 117 analytical operations (the other 53 are "reporting/housekeeping") can be reorganized into the commonly recognized quantitative analysis categories.

Figures 2 and 3 at the end of this section show my initial attempts at the reorganization (see Author's Note 3).

The bottom line is that the SpatialSTEM framework recasts map analysis concepts and procedures into a more generally understood organization. Within this general schema, map analysis is recognized as a set of natural extensions to familiar non-spatial math/stat operations. For example—

- A high school **math teacher** might follow a discussion of the Pythagorean Theorem with "…but what if there is an impassible barrier between the two points? The distance is no longer a straight line but some sort of a 'bendy-twisty' route around the barrier. How would you calculate the not-necessarily-straight distance? The 'Splash Algorithm' does that by…" (you know the rest of the story).
- Or a **statistics instructor** might follow a lecture on the derivation of the Standard Normal Curve for characterizing the 'numerical distribution' of a data set with "…but what about the 'spatial distribution' of the data? Is data always uniform or randomly distributed in geographic space? How could you characterize/visualize the spatial distribution? 'Spatial Interpolation' does that by…" (you know the rest of the story).
- Or an **environmental science teacher** might follow a lecture on the use of riparian buffers with "…but are all 'buffer-feet the same'? What about the slope of the surrounding terrain? …and the type of soil? …and the density of vegetation? Wouldn't an area along a stream that is steep with an unstable soil and minimal vegetation require a much larger setback than an area that is flat with stable soils and dense vegetation? How could you create a variable-width buffer around

streams that considers the intervening erosion conditions? A simple 'sediment loading model does that by…" (you know the rest of the story).

- Or a **crop scientist** who historically calculated the increase (decrease) in yield over a previous year for a new genetic variety as the percent change in the total "weigh-wagon" records for an entire trial field. But with GPS-enabled yield maps that automatically collect on-the-fly yield measurements as a harvester moves through a field, a detailed map of the percent change can be generated by spatially evaluating the standard algebraic equation by… (you know the rest of the story).
- Or a **sales manager** can use 'address geo-coding' to sprinkle sales data onto a grid map and then compute 'roving window' totals to generate a sales density surface showing where sales are high (or low) throughout each of several sales territories. The map analysis can be extended to calculate areas of unusually high (or low) sales by identifying locations that are more than one standard deviation above (or below) the average sales density… (you know the rest of the story).

Dovetailing map analysis with traditional quantitative analysis thinking moves GIS from a "specialty discipline down the hall and to the right" for mapping and geoquery, to an integrated and active role in the spatial reasoning needed by tomorrow's scientists, technologists, decision-makers and other professionals in solving increasing complex and knurly real-world problems. From this perspective, "thinking with maps" becomes a true fabric of society thus fulfilling GIS's mega-technology promise.

…the following two listings cross-reference Spatial Analysis tools in ArcGIS software by Esri to commonly recognized quantitative math/stat analysis categories—

Spatial Analysis Operations — Mathematical Perspective

Raster-based Map Analysis and Modeling Operations

…for Esri ***Spatial Analyst*** *Software*

Mathematical Concepts	Spatial Analyst Toolsets and Tools
Basic GridMath & Map Algebra *(+ - * /):*	
	General Math Toolset, *Basic Arithmetic tools:* Plus, Minus, Times, Divide
	General Math Toolset, *Power tools:* Square, Square Root, Power
	Map Algebra Toolset: Raster Calculator
Advanced GridMath *(Math, Trig, Logical Functions):*	
	General Math Toolset, *Conversion tools:* Abs, Negate, Float, Int, Round Down, Round Up, Mod
	General Math Toolset, *Exponential and Logarithmic tools:* Exp, Exp2, Exp10, Ln, Log2, Log10
	Trigonometric Math Toolset: Cos, Sin, Tan, ACos, ASin, ATan, ATan2, CosH, SinH, TanH, ACosH, ASinH, ATanH
	Logical Math Toolset, *Relational tools:* Equal To, Not Equal, Greater Than, Greater Than Equal, Less Than, Less Than Equal
	Logical Math Toolset, *Boolean tools:* Boolean And, Boolean Or, Boolean Xor, Boolean Not
	Logical Math Toolset, *Combinatorial tools:* Combinatorial And, Combinatorial Or, Combinatorial XOr
	Logical Math Toolset, *Logical tools:* Diff, InList, Is Null, Over, Test
	Conditional Toolset: Con
	Bitwise Toolset: Bitwise XOr, And, Or, Bitwise Not
Map Calculus *(Spatial Derivative, Spatial Integral):*	
	Surface Toolset, *Surface Configuration tools:* Slope, Aspect, Curvature
	Zonal Toolset, *Zonal Statistics tools:* Zonal Statistics
Map Geometry *(Euclidian Proximity, Effective Proximity):*	
	Distance Toolset, *Euclidean Distance tools:* Euclidean Distance, Euclidean Direction, Euclidean Allocation
	Distance Toolset, *Effective Distance tools:* Cost Distance, Cost Allocation, Cost Back Link
Plane Geometry Connectivity *(Optimal Path, Optimal Path Density, Surface Configuration):*	
	Distance Toolset, *Effective Distance tools:* Cost Path, Path Distance, Corridor, Path Distance, Path Distance Allocation, Back Link
	Hydrology Toolset, *Flow Density tools:* Flow Accumulation
	Hydrology Toolset, *Surface Configuration tools:* Flow Length, Flow Direction, Sink, Fill, Watershed, Basin, Focal Flow
Solid Geometry Connectivity *(Visual Exposure):*	
	Surface Toolset, *Visual Connectivity tools:* Viewshed, Observer Points
Unique Map Analytics *(Reclassify, Contiguity, Shape):*	
	Reclass Toolset, *Reclassification tools:* Reclass, Slice
	Local Toolset, *Combinatorial tool:* Combine
	Generalization Toolset, *Contiguity tool:* Region Group, Nibble, Majority Filter
	Surface Toolset, *Surface Configuration tool:* Cut Fill
	Zonal Toolset, *Zonal Geometry:* Zonal Geometry

Figure 2. Reorganization of Spatial Analyst's analytical "tools" into traditional mathematical categories.

Spatial Statistics Operations — Statistical Perspective

Raster-based Map Analysis and Modeling Operations

…for Esri ***Spatial Analyst Software***

Statistical Concepts	Spatial Analyst Toolsets and Tools
Basic Descriptive Statistics *(Min, Max, Median, Mean, StDev, etc.)*	
Local Toolset, *Cell Statistics tools:* Cell Statistics	
Local Toolset, *Frequency tools:* Equal To Frequency, Greater Than Frequency, Less than Frequency	
Local Toolset, *Ranking tools:* Rank, Lowest Position, Highest Position, Popularity	
Overlay Toolset: Weighted Overlay, Weighted Sum	
Basic Classification *(Reclassify, Contour, Normalization):*	
Reclass Toolset, *Reclassification tools:* Reclass, Slice	
General Math Toolset, *Basic Arithmetic tools:* Plus, Minus, Times, Divide	
Map Comparison *(Joint Coincidence, Difference):*	
Local Toolset, *Combinatorial tool:* Combine	
General Math Toolset, *Basic Arithmetic tools:* Plus, Minus, Times, Divide	
Unique Map Statistics *(Zonal, Roving Window, Block Summaries):*	
Zonal Toolset, *Zonal Statistics tools:* Zonal Statistics	
Neighborhood Toolset, *Focal (roving window) tools:* Focal Statistics, Filter	
Neighborhood Toolset, *Block tool:* Block Statistics	
Raster Creation Toolset: Create Constant Raster, Create Normal Raster, Create Random Raster	
Surface Modeling *(Density Analysis, Spatial Interpolation, Trend):*	
Neighborhood Toolset, *Focal (roving window) tool:* Focal Statistics (sum)	
Interpolation Toolset: IDW, Kriging, Spline, Spline with Barriers, Natural Neighbor, Trend	
Advanced Classification *(Maximum Likelihood, Clustering):*	
Multivariate Toolset, *Classification tools:* Maximum Likelihood Classification, Iso Cluster Unsupervised Classification	
Correlation and Regression: *no direct tools* *in Spatial Analyst (dropped from AML Grid module; in Spatial Statistics toolbox)*	
Multivariate Toolset, *Classification tools:* Maximum Likelihood Classification, Iso Cluster Unsupervised Classification	
Correlation and Regression: *no direct tools* *(dropped from earlier AML Grid module)*	

Figure 3. Reorganization of Spatial Analyst's analytical "tools" into traditional statistical categories.

***Author's Note**: 1) see the Chronological Listing of Beyond Mapping columns posted at www.innovativegis.com/basis/MapAnalysis/ChronList/ChronologicalListing.htm; 2) for numerous links to papers, PowerPoint slide sets and other materials describing the SpatialSTEM framework, see www.innovativegis.com/Basis/Courses/SpatialSTEM/; 3) at the same SpatialSTEM posting, see the white paper entitled "Math/Stat Classification of Spatial Analysis and Spatial Statistics Tools (Spatial Analyst by Esri)" more detailed description of the recasting of Spatial Analyst's operations by traditional non-spatial mathematics and statistics categories.*

(return to top of Topic)

(Back to the Table of Contents)