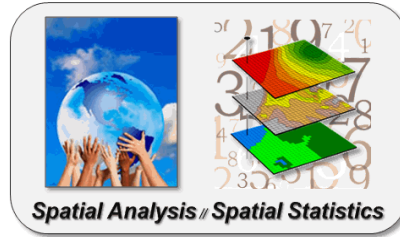


Spatial/STEM:

A Mathematical/Statistical Framework for Understanding and Communicating Map Analysis and Modeling



Premise: There is a “map-ematics” that extends traditional math/stat concepts and procedures for the quantitative analysis of map variables (spatial data)

This presentation provides a fresh perspective on interdisciplinary instruction at the college level by combining the philosophy and approach of *STEM* with the spatial reasoning and analytical power of grid-based Map Analysis and Modeling

This PowerPoint with notes and online links to further reading is posted at
www.innovativegis.com/basis/Workshops/NGA2015/

Presented by

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Adjunct Faculty in Natural Resources, Warner College of Natural Resources, Colorado State University
Principal, Berry & Associates // Spatial Information Systems

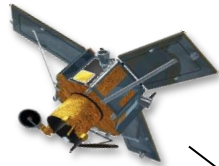
Email: jberry@innovativegis.com — Website: www.innovativegis.com/basis

(Nanotechnology)

Geospatial Technology

(Biotechnology)

Geospatial Technology is one of the three "mega technologies" for the 21st century and promises to forever change how we conceptualize, utilize and visualize spatial relationships in scientific research and commercial applications (U.S. Department of Labor)



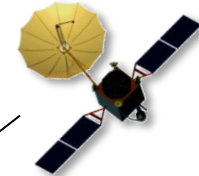
Remote Sensing
(Measure and Classify)

Geographic Information Systems
(Map and Analyze)



GPS/GIS/RS

The Spatial Triad



Global Positioning System
(Locate and Navigate)

Computer Mapping (70s)
Spatial Database Management (80s)

Technological Tool

Mapping
involves precise placement of physical features
(graphical inventory)



Where

Descriptive Mapping



What

Prescriptive Modeling



Map Analysis/Modeling (90s)
Web-based Mapping (00s)

Analytical Tool

Modeling
involves analysis of spatial patterns and relationships
(map analysis/modeling)

Why So What and What If

A Mathematical Structure for Map Analysis/Modeling

Technological Tool

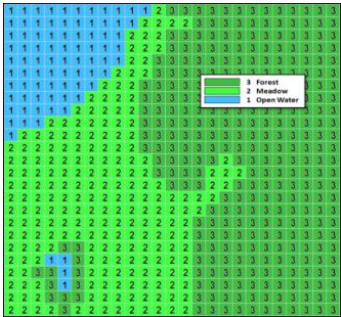
Geotechnology → RS — **GIS** — GPS

Analytical Tool

Mapping/Geo-Query (*Discrete, Spatial Objects*)

(*Continuous, Map Surfaces*)

Map Analysis/Modeling



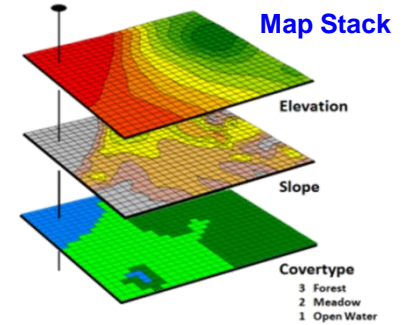
...organized set of numbers

Geo-registered
Analysis Frame → **Matrix**
of Numbers

"Map-ematics"

Maps as Data, not Pictures

Vector & *Raster* — Aggregated & *Disaggregated*
Qualitative & *Quantitative*



Spatial Analysis Operations

Mathematical Perspective

Classes of *Mathematical* operations

Basic GridMath & Map Algebra

(+ - * /)

Advanced GridMath

(Math, Trig, Logical Functions)

Map Calculus

(Spatial Derivative, Spatial Integral)

Map Geometry

(Euclidian Proximity, Effective Proximity, Narrowness)

Plane Geometry Connectivity

(Optimal Path, Optimal Path Density)

Solid Geometry Connectivity

(Viewshed, Visual Exposure)

Unique Map Analytics

(Contiguity, Size/Shape/Integrity, Masking, Profile)

Grid-based Map Analysis Toolbox



A Map-ematical Framework

Traditional math/stat procedures
can be extended to
geographic space to support

Quantitative Analysis of Mapped Data

"... *thinking analytically
with maps*"

Spatial Statistics Operations

Statistical Perspective

Classes of *Statistical* operations

Basic Descriptive Statistics

(Min, Max, Median, Mean, StDev, etc.)

Basic Classification

(Reclassify, Contouring, Normalization)

Map Comparison

(Joint Coincidence, Statistical Tests)

Unique Map Statistics

(Roving Window and Regional Summaries)

Surface Modeling

(Density Analysis, Spatial Interpolation)

Advanced Classification

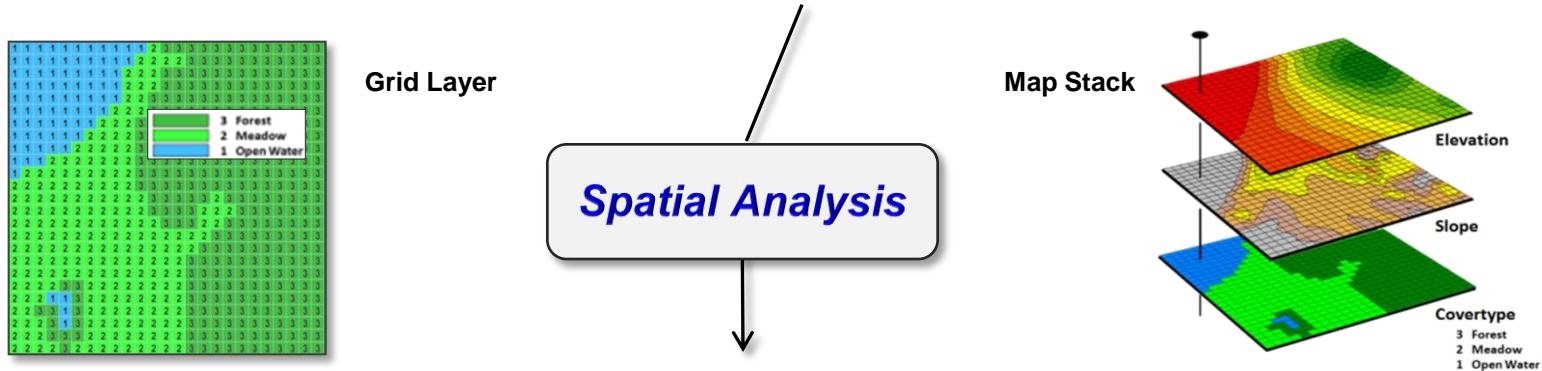
(Map Similarity, Maximum Likelihood, Clustering)

Predictive Statistics

(Map Correlation/Regression, Data Mining Engines)

Spatial Analysis Operations *(Geographic Context)*

GIS as “**Technical Tool**” (*Where is What*) vs. “**Analytical Tool**” (*Why, So What and What if*)



Spatial Analysis extends the basic set of discrete map features (points, lines and polygons) to map surfaces that represent continuous geographic space as a set of contiguous grid cells (matrix), thereby providing a Mathematical Framework for *map analysis* and *modeling* of the

Contextual Spatial Relationships within and among grid map layers

Map Analysis Toolbox



✓ Unique spatial operations

Mathematical Perspective:

Basic GridMath & Map Algebra (+ - * /)

Advanced GridMath (Math, Trig, Logical Functions)

Map Calculus (Spatial Derivative, Spatial Integral)

Map Geometry (Euclidian Proximity, Effective Proximity, Narrowness)

Plane Geometry Connectivity (Optimal Path, Optimal Path Density)

Solid Geometry Connectivity (Viewshed, Visual Exposure)

✓ **Unique Map Analytics** (Contiguity, Size/Shape/Integrity, Masking, Profile)

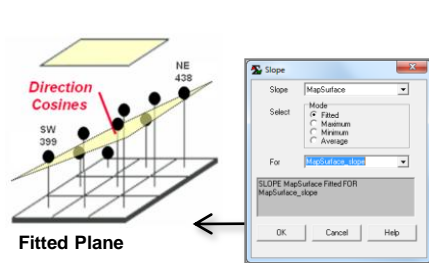
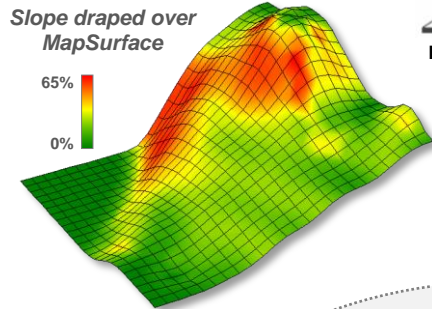
Spatial Analysis Operations (Math Examples)



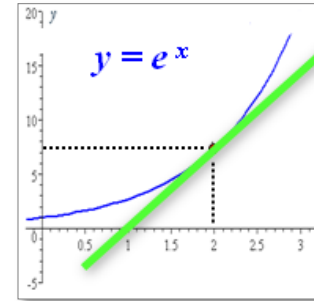
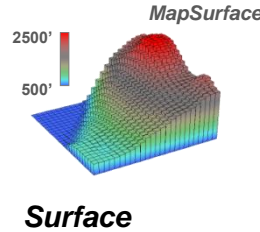
Advanced Grid Math — Math, Trig, Logical Functions
Map Calculus — Spatial Derivative, Spatial Integral

Spatial Derivative

...is equivalent to the slope of the **tangent plane** at a location



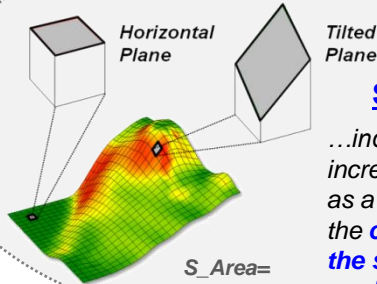
SLOPE MapSurface Fitted FOR MapSurface_slope



The **derivative** is the instantaneous "rate of change" of a function and is equivalent to the slope of the **tangent line** at a point

$D_{z,xy}$ Elevation

Advanced Grid Math



Surface Area

...increases with increasing inclination as a Trig function of the **cosine of the slope angle**

$$S_Area = Fn(Slope)$$

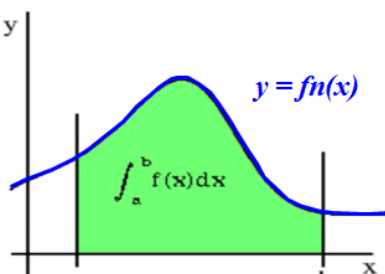
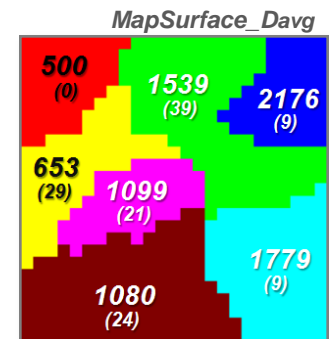
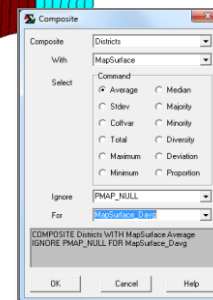
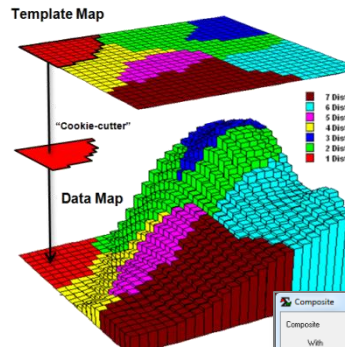
$$S_area = cellsize / \cos(D_{z,xy} \text{ Elevation})$$

\int Districts_Average Elevation

Spatial Integral

...summarizes the values on a surface for specified map areas (Total= **volume** under the surface)

COMPOSITE Districts WITH MapSurface Average FOR MapSurface_Davg



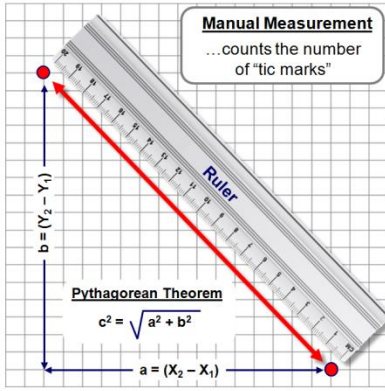
The **integral** calculates the **area** under the curve for any section of a function.

Surface
Curve

Spatial Analysis Operations *(Distance Examples)*

Map Geometry — *(Distance, Proximity, Effective Movement, Narrowness)*
Plane Geometry Connectivity — *(Optimal Path, Optimal Path Density)*
Solid Geometry Connectivity — *(Viewshed, Visual Exposure)*

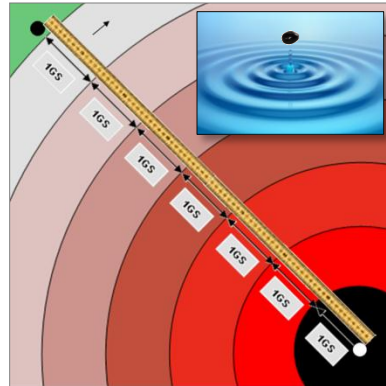
Distance



Mathematical Measurement
...solves the Pythagorean Theorem

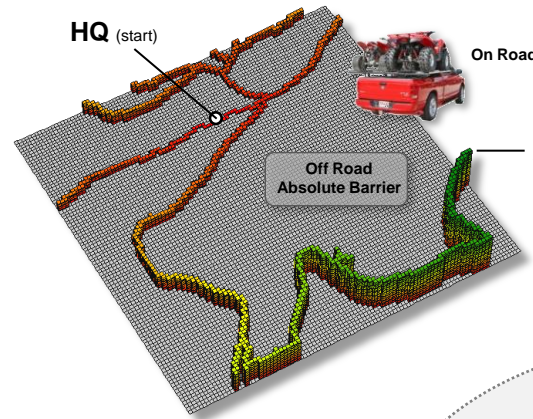
Shortest straight line between two points...

Proximity

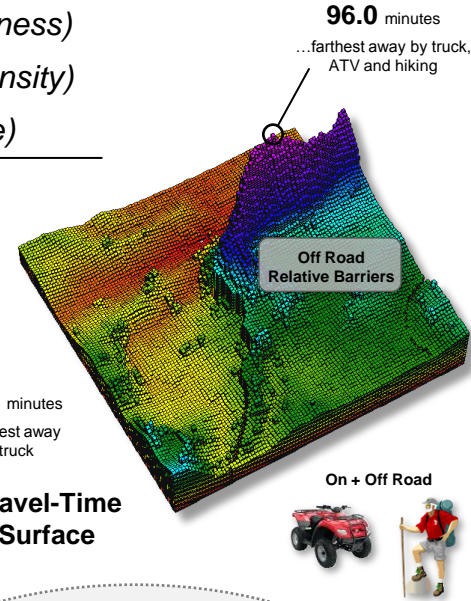


...from a point to everywhere...

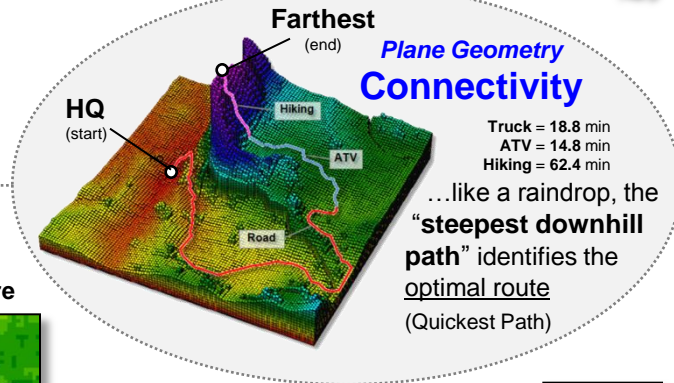
Movement *(absolute/relative barriers)*



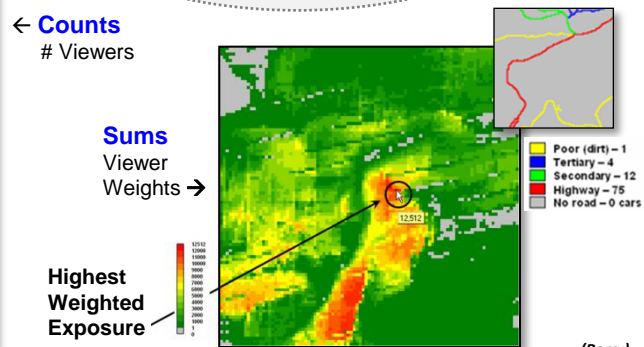
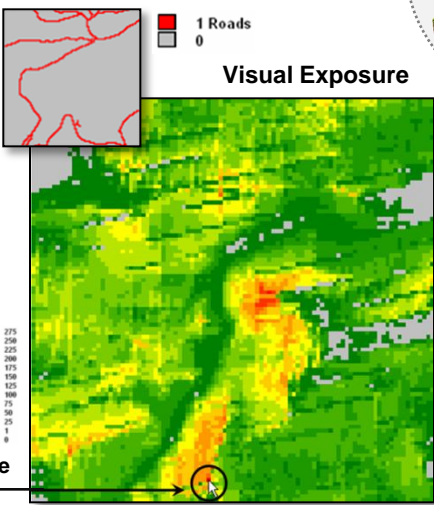
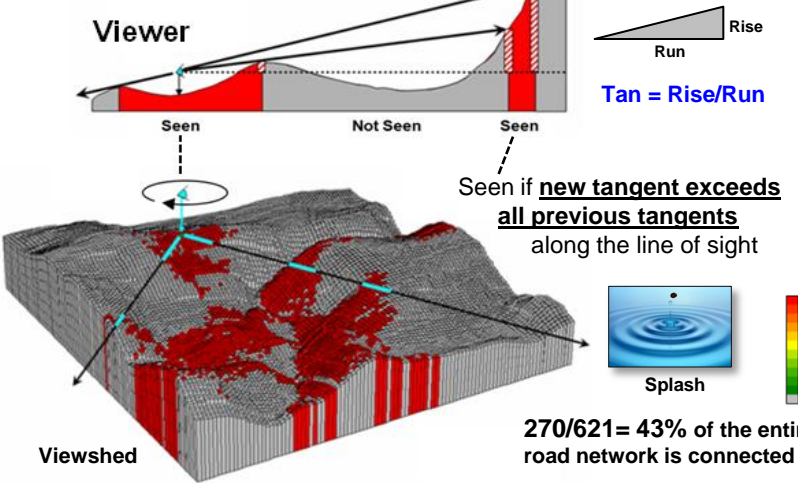
...not necessarily straight lines (movement)



Travel-Time Surface

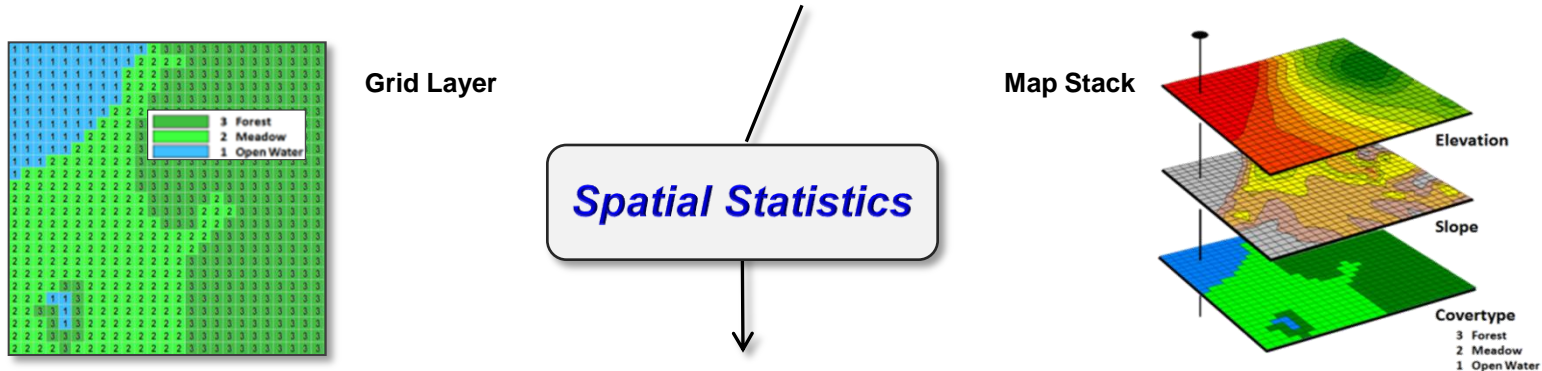


Solid Geometry Connectivity



Spatial Statistics Operations (Numeric Context)

GIS as “Technical Tool” (*Where is What*) vs. “**Analytical Tool**” (*Why, So What and What if*)



Spatial Statistics seeks to map the variation in a data set instead of focusing on a single typical response (central tendency), thereby providing a Statistical Framework for *map analysis* and *modeling* of the

Numerical **Spatial Relationships** within and among grid map layers

Statistical Perspective:

Basic Descriptive Statistics (*Min, Max, Median, Mean, StDev, etc.*)

Basic Classification (*Reclassify, Contouring, Normalization*)

Map Comparison (*Joint Coincidence, Statistical Tests*)

✓ **Unique Map Statistics** (*Roving Window and Regional Summaries*)

✓ **Surface Modeling** (*Density Analysis, Spatial Interpolation*)

Advanced Classification (*Map Similarity, Maximum Likelihood, Clustering*)

Predictive Statistics (*Map Correlation/Regression, Data Mining Engines*)

Map Analysis Toolbox

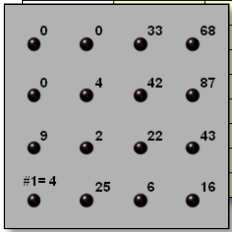


✓ **Unique spatial operations**

Spatial Statistics *(Linking Data Space with Geographic Space)*

Geo-registered Sample Data

Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
	1000	1000	10	6
	1500	17	22	
	2000	34	42	
	2500	22	33	
	1000	20	16	
	1500	28	43	
	2000	42	87	
	2500	34	68	



Non-Spatial Statistics

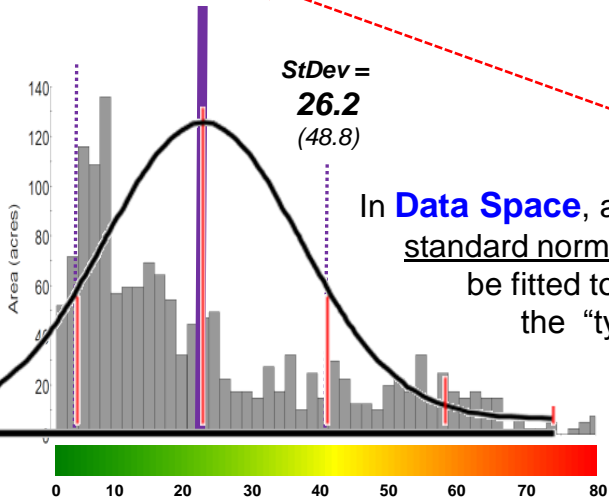
Standard Normal Curve

Average = 22.6

StDev = 26.2 (48.8)

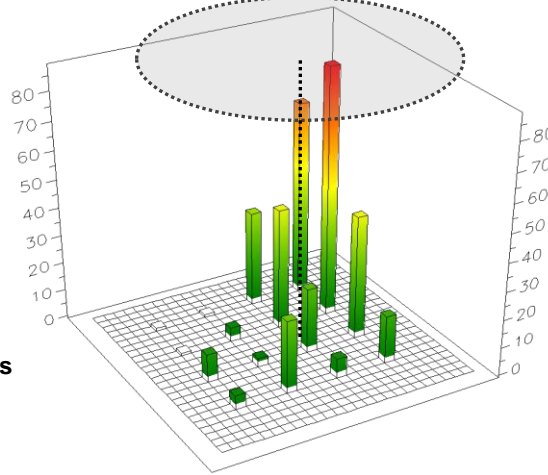
In **Data Space**, a standard normal curve can be fitted to the data to identify the "typical value" (average)

Histogram



Numeric Distribution

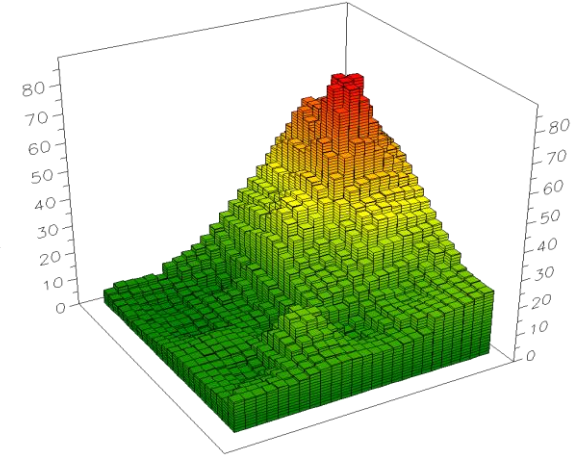
Roving Window (weighted average)



Discrete Sample Map

Spatial Statistics

Spatial Distribution



Continuous Map Surface

Surface Modeling techniques are used to derive a continuous map surface from discrete point data— fits a Surface to the data (maps the variation).

In **Geographic Space**, the typical value forms a horizontal plane implying the average is everywhere

$\bar{X} = 22.6$

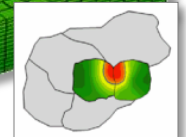
...lots of NE locations exceed Mean + 1Stdev

$$\begin{aligned} \bar{X} + 1\text{StDev} &= 22.6 + 26.2 \\ &= 48.8 \end{aligned}$$

Avg = 22.6

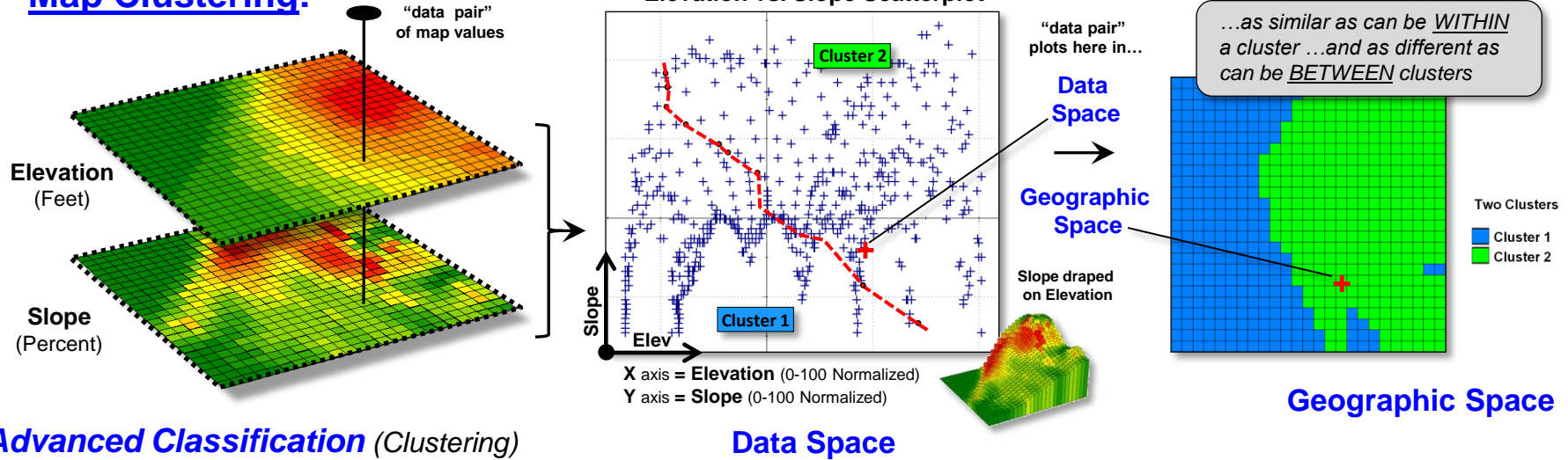
Unusually high values

+StDev
Average



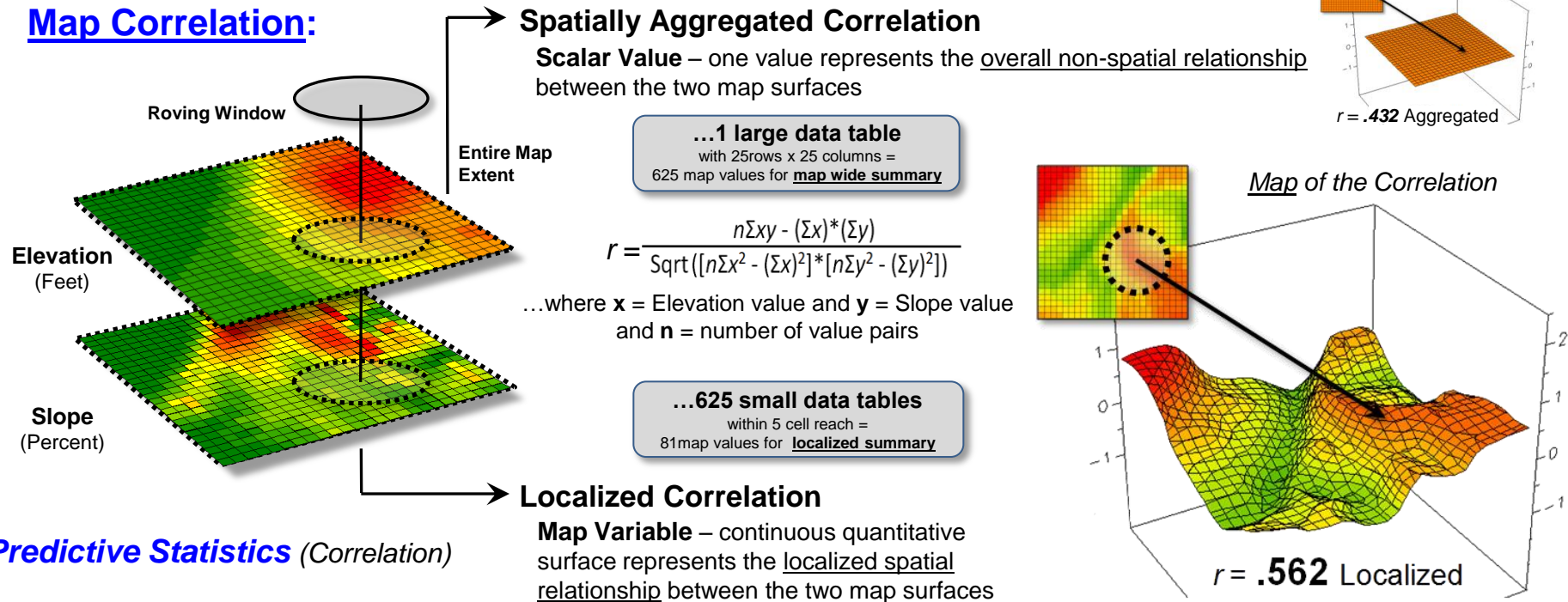
Spatial Statistics Operations (Data Mining Examples)

Map Clustering:



Advanced Classification (Clustering)

Map Correlation:



Predictive Statistics (Correlation)

Workshop Organization and Further Reading

Part 1) Overview

Part 2) Spatial Analysis
(Contextual relationships)

Part 3) Spatial Statistics
(Numerical relationships)

All of the PowerPoints with notes and online links to further reading are posted at ...
www.innovativegis.com/basis/Workshops/NGA2015/
Online book chapter...
 A series of Beyond Mapping columns in GeoWorld compiled into **Topic 30, "SpatialSTEM: A Math/Stat Framework for Grid-based Map Analysis and Modeling"** in the online book Beyond Mapping III posted at ...
www.innovativegis.com/basis/MapAnalysis/Topic30/Topic30.htm/

Further Reading

SpatialSTEM: A New Perspective and Conceptual Framework for Grid-based Map Analysis and Modeling — white paper overview of SpatialSTEM approach, framework and considerations
http://www.innovativegis.com/basis/Papers/Other/sSTEM_overview/SpatialSTEM_overview.pdf

University Seminar Series on Geotechnology, three part university-wide seminar series sponsored by CSU Geospatial Centroid on “**Future Directions of Map Analysis and GIS Modeling**,” September 19 (Link to [Handout](#); [PowerPoint](#), 13MB; [Video](#) of presentation); “**GIS in Natural Resources and Agriculture**,” October 17 (Link to [Handout](#); [PowerPoint](#), 15MB; [Video](#) of presentation); “**Eye-Witness to GIS’s 40 year Evolution/Revolution**,” November 14, 2014 (Link to [Handout](#); [PowerPoint](#), 20MB; [Video](#) of presentation), Colorado State University, Fort Collins, Colorado. University seminar series.

<http://www.innovativegis.com/basis/Present/CentroidCSU2014/>

Part 4) Future Directions