*Spatial*STEM: Extending Traditional Mathematics and Statistics to Grid-based Map Analysis and Modeling

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Abstract

This paper describes a ***Spatial*STEM *approach* for teaching map analysis and modeling fundamentals within a mathematical/statistical context** that resonates with science, technology, engineering and math/stat communities. The premise is that "maps are numbers first, pictures later" and we do mathematical things to mapped data for insight and better understanding of spatial patterns and relationships within decision-making contexts …from *Where* is *What* graphical inventories to a *Why*, *So What* and *What If* problem solving environment— thinking with maps.

The map-*ematical* approach focuses on analytical tools used in spatial reasoning by non-GIS communities instead of traditional "GIS mechanics" of data acquisition, storage, retrieval, query and display of map features directed toward GIS specialists. The goal is to get the STEM communities to "think with maps" and infuse direct consideration of spatial patterns and relationships into their endeavors, as an alternative for spatially-aggregated math/stat procedures that assume uniform or random distribution in geographic space.

Keywords: GIS modeling, grid-based map analysis, spatial analysis, spatial statistics, map algebra, map-ematics

Topics

Author's Note: *this paper is a compilation of Beyond Mapping columns by the author in GeoWorld, January - March, 2012 and is posted at www.innovativegis.com/basis/Papers/Other/SpatialSTEM/Default.htm. Further Reading references provide URL links to additional Beyond Mapping columns posted at www.innovativegis.com.*

*Spatial*STEM Has Deep Mathematical Roots *(GW, January, 2012)*

Recently my interest has been captured by a new arena and expression for the contention that "maps are data"—*Spatial*STEM (or *s*STEM for short)—as a means for redirecting education in general, and GIS education in particular. I suspect you have heard of STEM (Science, Technology, Engineering and Mathematics) and the educational crisis that puts U.S. students well behind many other nations in these quantitatively-based disciplines.

While Googling around the globe makes for great homework in cultural geography, it doesn't advance quantitative proficiency, nor does it stimulate the spatial reasoning skills needed for problem solving. Lots of folks from Freed Zakaria to Bill Gates to President Obama are looking for ways that we can recapture our leadership in the quantitative fields. That's the premise of *Spatial*STEM– that "maps are numbers first, pictures later" and we do mathematical things to mapped data for insight and better understanding of spatial patterns and relationships within decision-making contexts.

This contention suggests that there is a "map-ematics" that can be employed to solve problems that go beyond mapping, geo-query, visualization and GPS navigation. This first of three sections discusses the quantitative nature of maps, data structure and organization to establish the underlying foundation. The next section asserts that grid-based spatial analysis operations are extensions of traditional mathematics by investigating map math, algebra, calculus, plane and solid geometry. The final section argues that grid-based spatial statistics operations are extensions of traditional statistics by considering map descriptive statistics, normalization, comparison, classification, surface modeling and predictive statistics.

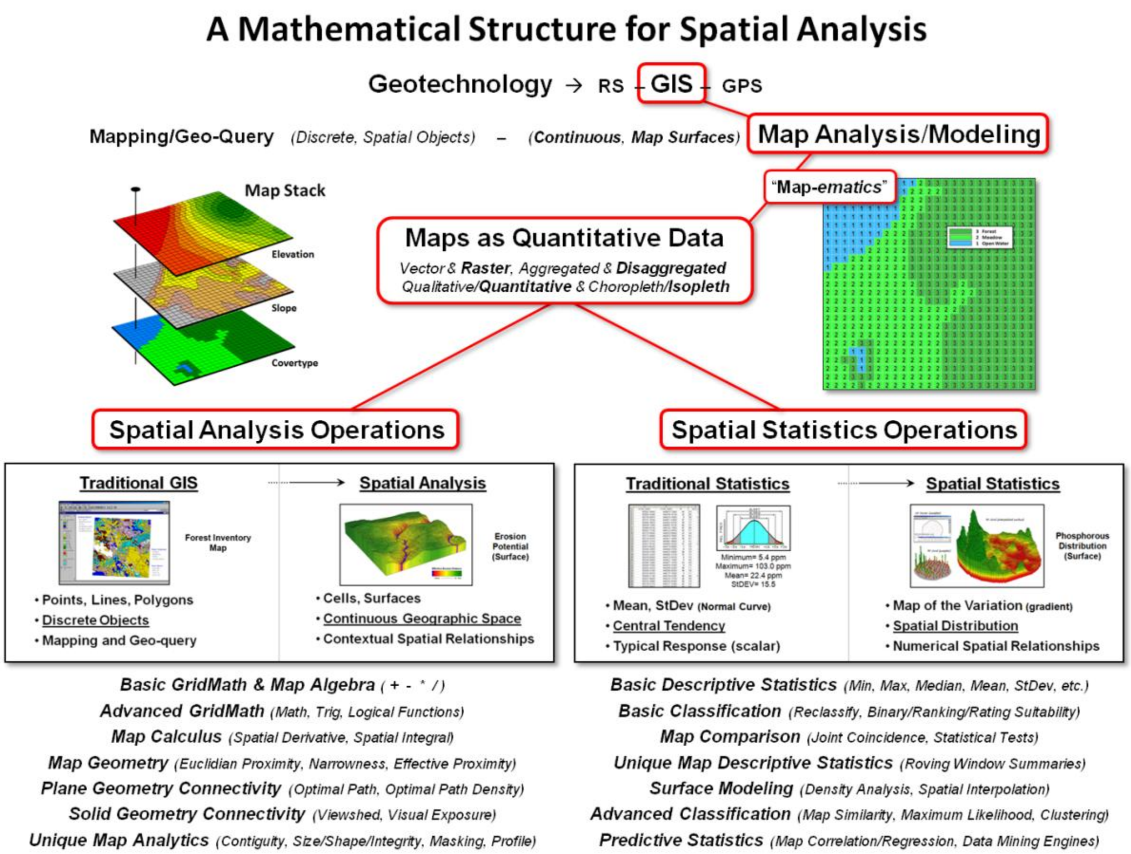

Figure 1. Conceptual overview of the SpatialSTEM framework.

Figure 1 outlines the important components of map analysis and modeling within a mathematical structure that has been in play since the 1980s (see author's note). Of the three disciplines forming Geotechnology (Remote Sensing, Geographic Information Systems and Global Positioning System), GIS is at the heart of converting mapped data into spatial information. There are two primary approaches used in generating this information—*Mapping/Geo-query* and *Map Analysis/Modeling*.

The major differences between the two approaches lie in the structuring of mapped data and their intended use. Mapping and geo-query use a data structure akin to manual mapping in which discrete spatial objects (*points*, *lines* and *polygons*) form a collection of independent, irregular features to characterize geographic space. For example, a water map might contain categories of spring (points), stream (lines) and lake (polygons), with the features scattered throughout a landscape.

Map analysis and modeling procedures, however, operate on continuous map variables (i.e., map *surfaces*) composed of thousands of map values stored in geo-registered matrices. Within this context, a water map no longer contains separate and distinct features, but is a collection of adjoining grid cells with a map value indicating the characteristic at each location (e.g., spring= 1, stream= 2 and lake= 3).

Figure 2 illustrates two broad types of digital maps, formally termed *Vector* for storing discrete spatial objects and *Raster* for storing continuous map surfaces. In vector format, spatial data is stored as two linked data tables. A "spatial table" contains all of the X,Y coordinates defining a set of spatial objects that are grouped by object-identification numbers. For example, the location of the forest polygon identified on the left side of the figure is stored as ID#32, followed by an ordered series of X,Y coordinate pairs delineating its border (connect-the-dots).

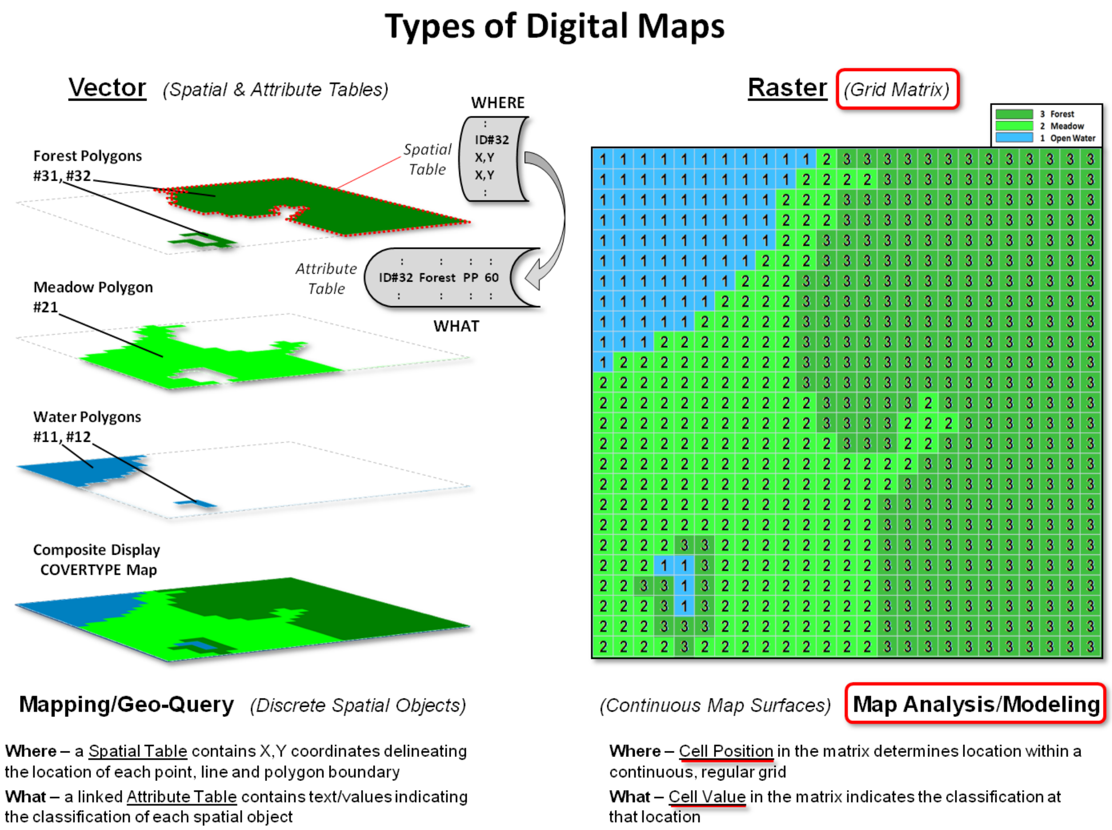

Figure 2. Basic data structure for Vector and Raster map types.

In a similar manner, the ID#s and X,Y coordinates defining the other cover-type polygons are sequentially listed in the table. The ID#s link the spatial table (where) to a corresponding "attribute table" (what) containing information about each spatial object as a separate record. For example, polygon ID#31 is characterized as a mature 60-year-old Ponderosa Pine (PP) forest stand.

The right side of 2 depicts raster storage of the same cover type information. Each grid space is assigned a number corresponding to the dominant cover type present— the "cell position" in the matrix determines the location (Where) and the "cell value" determines the characteristic/condition (What).

It's important to note that the raster representation stores information about the interior of polygons and "pre-conditions geographic space" for analysis by applying a consistent grid configuration to each grid map. Because each map's underlying data structure is the same, the computer simply "hits disk" to get information and doesn't have to calculate whether irregular sets of points, lines or polygons on different maps intersect.

Figure 3 depicts the fundamental concepts supporting raster data. As a comparison between vector and raster data structures, consider how the two approaches represent an elevation surface. In vector, contour lines are used to identify lines of constant elevation, and contour interval polygons are used to identify specified ranges of elevation. While contour lines are exacting, they fail to describe the intervening surface configuration.

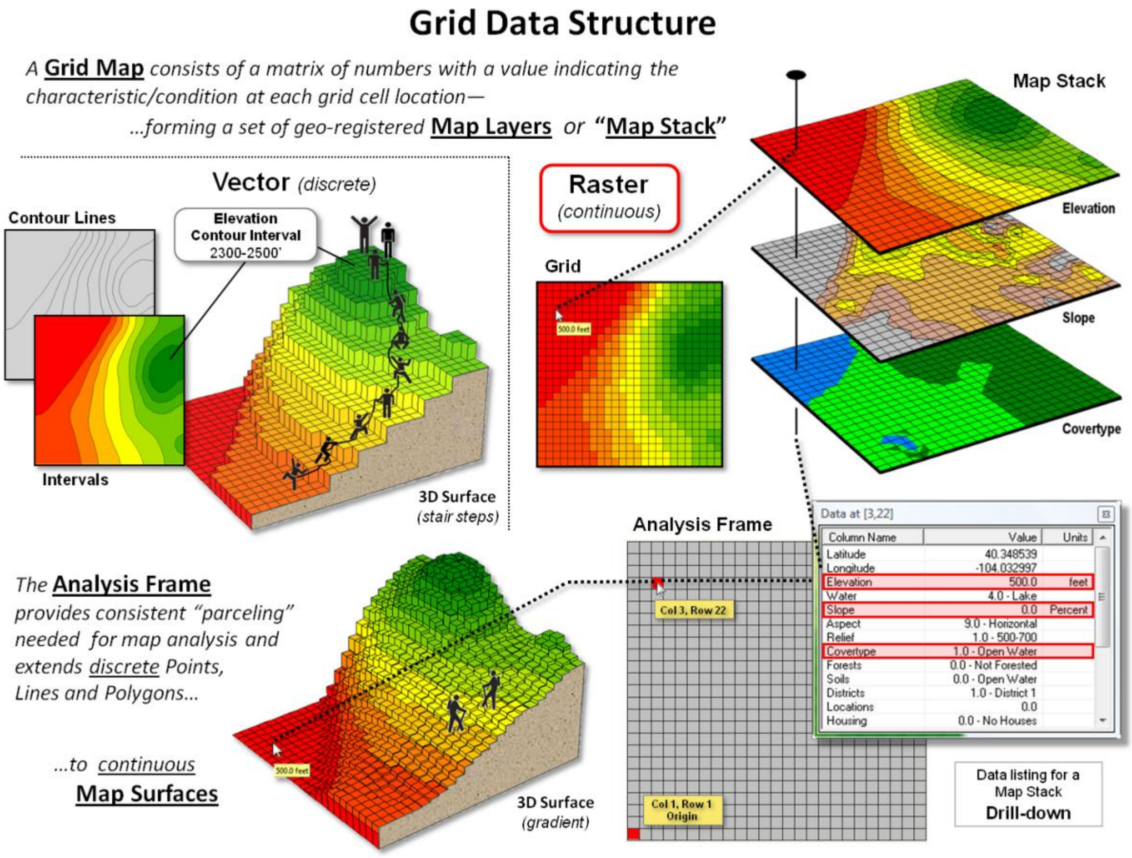

Figure 3. Organizational considerations and terminology for grid-based mapped data.

Contour intervals describe the interiors, but they overly generalize the actual "ups and downs" of the terrain into broad ranges that form an unrealistic stair-step configuration (center-left portion of Figure 3). As depicted in the figure, rock climbers would need to summit each of the contour interval "200-foot cliffs" rising from presumed flat mesas. Similarly, surface water flow presumably would cascade like waterfalls from each contour interval "lake" like a Spanish multi-tiered fountain.

The upshot is that within a mathematical context, vector maps are ineffective representations of real-world gradients and actual movements and flows over these surfaces. Although contour line/interval maps have formed colorful and comfortable visualizations for generations, the data structure is too limited for modern map analysis and modeling.

The remainder of Figure 3 depicts the basic raster/grid organizational structure. Each grid map is termed a *Map Layer* and a set of geo-registered layers constitutes a *Map Stack*. All of the map layers in a project conform to a common *Analysis Frame* with a fixed number of rows and columns at a specified cell size that can be positioned anywhere in geographic space.

As in the case of the elevation surface in the lower-left portion of Figure 3, a continuous gradient is formed with subtle elevation differences that allow hikers to step from cell to cell while considering relative steepness. In addition, surface water can be mapped to sequentially stream from a location to its steepest downhill neighbor, thereby identifying a flow-path.

The underlying concept of this data structure is that grid cells for all of the map layers precisely coincide, and by simply accessing map values at a row, column location, a computer can "drill down" through the map layers, noting their characteristics. Similarly, noting the map values of surrounding cells identifies the characteristics within a location's vicinity on a given map layer or set of map layers.

Keep in mind that although terrain elevation is the most common example of a map surface, it's by no means the only one. In natural systems, temperature, barometric pressure, air-pollution concentration, soil chemistry and water turbidity are but a few examples of continuous mapped data gradients.

In human systems, population density, income level, life style concentration, crime occurrence and disease-incidence rate all form continuous map surfaces. In economic systems, home values, sales activity and travel time to/from stores form map variables that that track spatial patterns.

In fact the preponderance of spatial data is easily and best represented as grid-based continuous map surfaces that are preconditioned for use in map analysis and modeling. The computer does the heavy-lifting of the computation, but what is needed is a new generation of creative minds that goes beyond mapping to "thinking with maps" within this less familiar, quantitative framework—a *Spatial*STEM environment.

Author's Notes: *My involvement in map analysis/modeling began in the 1970s with doctoral work in computer-assisted analysis of remotely sensed data a couple of years before we had civilian satellites. The extension from digital imagery classification using multivariate statistics and pattern recognition algorithms in the 1970s to a comprehensive grid-based mathematical structure for all forms of mapped data in the 1980s was a natural evolution. See www.innovativegis.com, select "Online Papers" for a link to a 1986 paper on "A Mathematical Structure for Analyzing Maps" that serves as an early introduction to a comprehensive framework for grid-based map analysis and modeling.*

Map-ematically Messing with Mapped Data *(GW, February, 2012)*

The last section introduced the idea of *Spatial*STEM for teaching map analysis and modeling fundamentals within a mathematical context that resonates with science, technology, engineering and math/stat communities. The discussion established a general framework and grid-based data structure needed for quantitative analysis of spatial patterns and relationships. This section focuses on the nature of mapped data, an example of a grid-math/algebra application and discussion of extended spatial analysis operations.

Figure 1 identifies the two primary perspectives of spatial data—1) *Numeric* that indicates how numbers are distributed in "number space" (*"what"* condition) and 2) *Geographic* which indicates how numbers are distributed in "geographic space" (*"where"* condition). The numeric perspective can be grouped into categories of *Qualitative* numbers that deal with general descriptions based on perceived "quality" and *Quantitative* numbers that deal with measured characteristics or "quantity."

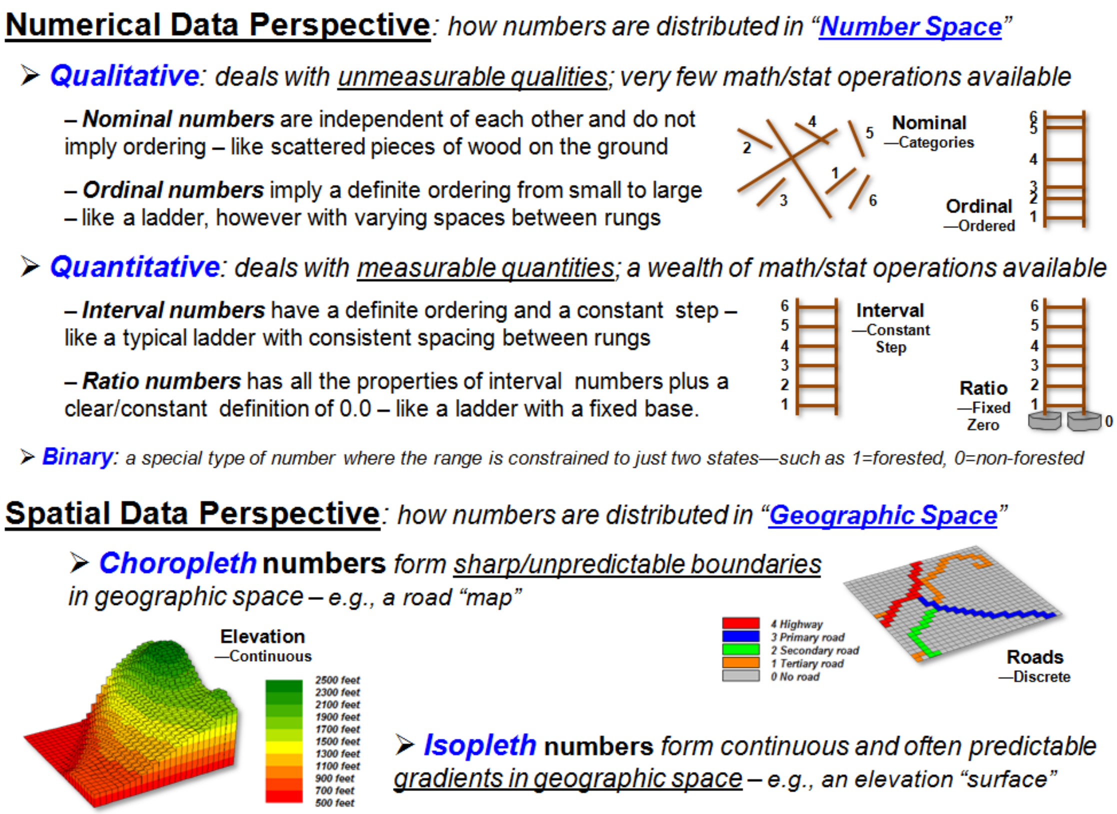

Figure 1. Spatial Data Perspectives—Where is What.

Further classification identifies the familiar numeric data types of nominal, ordinal, interval, ratio and binary. It is generally well known that very few math/stat operations can be performed using qualitative data (nominal, ordinal), whereas a wealth of operations can be used with quantitative data (interval, ratio). Only a specialized few operations utilize binary data.

Less familiar are the two geographic data types. *Choropleth* numbers form sharp and unpredictable boundaries in space, such as the values assigned to the discrete map features on a road or cover-type map. *Isopleth* numbers, on the other hand, form continuous and often predictable gradients in geographic space, such as the values on an elevation or temperature surface.

Putting the "where" and "what" perspectives of spatial data together, *Discrete Maps* identify mapped data with spatially independent numbers (qualitative or quantitative) forming sharp abrupt boundaries (choropleth), such as a cover-type map. Discrete maps generally provide limited footholds for quantitative map analysis. However, *Continuous Maps* contain a range of values (quantitative only) that form spatial gradients (isopleth), such as an elevation surface. They provide a wealth of analytics from basic grid math to map algebra, calculus and geometry.

Site-specific farming provides a good example of basic grid math and map algebra using continuous maps (see Figure 2). *Yield Mapping* involves simultaneously recording yield flow and GPS position as a combine harvests a crop resulting in a grid map of thousands of geo-registered numbers that track crop yield throughout a field. *Grid Math* can be used to calculate the mathematical difference in yield at each location between two years by simply subtracting the respective yield maps. *Map Algebra* extends the processing by spatially evaluating the full algebraic percent change equation.

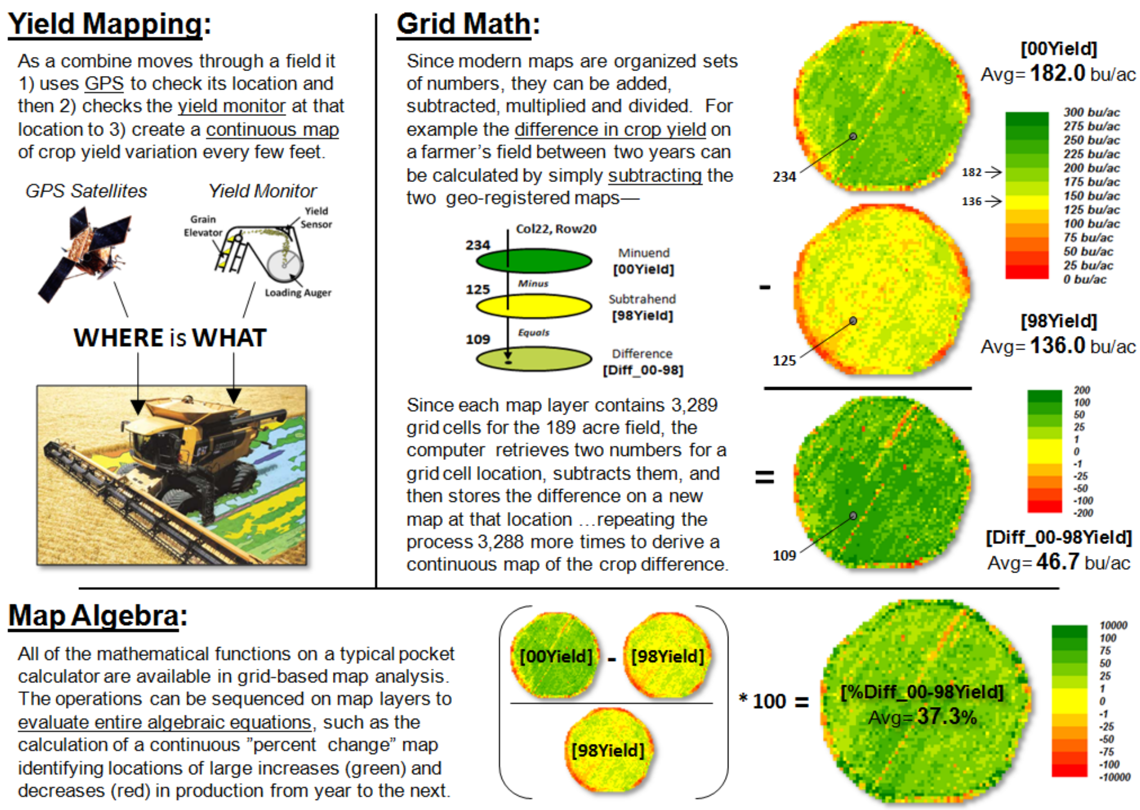

Figure 2. Basic Grid Math and Algebra example.

The paradigm shift in this map-ematical approach is that map variables, comprised of thousands of geo-registered numbers, are substituted for traditional variables defined by a single value. Map algebra's continuous map solution shows localized variation, rather than a single "typical" value being calculated (i.e., 37.3% increase in the example) and assumed everywhere the same in non-spatial analysis.

Figure 3 expands basic Grid Math and Map Algebra into other mathematical arenas. *Advanced Grid Math* includes most of the buttons on a scientific calculator to include trigonometric functions. For example, taking the cosine of a slope map expressed in degrees and multiplying it times the planimetric surface area of a grid cell calculates the surface area of the "inclined plane" at each grid location. The difference between planimetric area represented by traditional maps and surface area based on terrain steepness can be dramatic and greatly affect the characterization of "catchment areas" in environmental and engineering models of surface runoff.

Map Calculus expresses such functions as the derivative and integral within a spatial context. The derivative traditionally identifies a measure of how a mathematical function changes as its input changes by assessing the slope along a curve in 2-dimensional abstract space.

The spatial equivalent calculates a "slope map" depicting the rate of change in a continuous map variable in 3-dimensional geographic space. For an elevation surface, slope depicts the rate of change in elevation. For an accumulation cost surface, its slope map represents the rate of change in cost (i.e., a marginal cost map). For a travel-time accumulation surface, its slope map indicates the relative change in speed and its aspect map identifies the direction of optimal movement at each location. Also, the slope map of an existing topographic slope map (i.e., second derivative) will characterize surface roughness (i.e., areas where slope itself is changing).

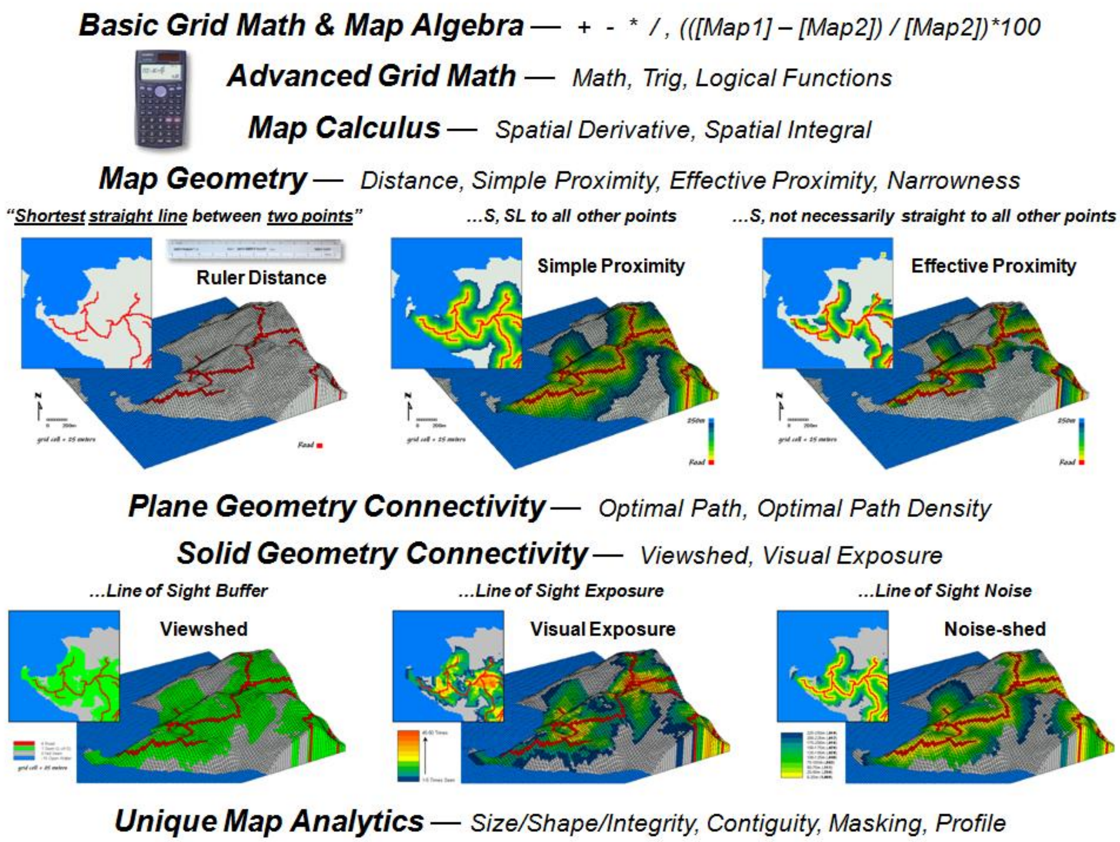

Figure 3. Spatial Analysis operations.

Traditional calculus identifies an integral as the net signed area of a region under a curve expressing a mathematical function. In a somewhat analogous procedure, areas under portions of continuous map surfaces can be characterized. For example, the total area (planimetric or surface) within a series of watersheds can be calculated; or the total tax revenue for various neighborhoods; or the total carbon emissions along major highways; or the net difference in crop yield for various soil types in a field. In the spatial integral, the net sum of the numeric values for portions of a continuous map surface (3D) is calculated in a manner comparable to calculating the area under a curve (2D).

Traditional geometry defines Distance as "the shortest straight line between two points" and routinely calculates it using the Pythagorean Theorem. *Map Geometry* extends the concept of distance to simple proximity by relaxing the requirement of just "two points" for distances to all locations surrounding a point or other map feature, such as a road.

A further extension involves effective proximity, which relaxes "straight line" to consider absolute and relative barriers to movement. For example effective proximity might consider just uphill locations along a road or a complex set of variable hiking conditions that impede movement from a road as a function of slope, cover type and water barriers.

The result is that the "shortest but not necessarily straight distance" is assigned to each grid location. Because a straight line connection cannot be assumed, optimal path routines in *Plane Geometry Connectivity* (2D space) are needed to identify the actual shortest routes. *Solid Geometry Connectivity* (3D space) involves line-of-sight connections that identify visual exposure among locations. A final class of operations involves *Unique Map Analytics*, such as size, shape, intactness and contiguity of map features.

Grid-based map analysis takes us well beyond traditional mapping …as well as taking us well beyond traditional procedures and paradigms of mathematics. The next installment of *Spatial*STEM discussion considers the extension of traditional statistics to spatial statistics.

Paint by Numbers Outside the Traditional Statistics Box *(GW, March, 2012)*

The two previous sections described a general framework and approach for teaching spatial analysis within a mathematical context that resonates with science, technology, engineering and math/stat communities (*Spatial*STEM). The following discussion focuses on extending traditional statistics to spatial statistics for understanding geographic-based patterns and relationships.

Whereas *spatial analysis* focuses on "contextual relationships" in geographic space (such as effective proximity and visual exposure), *spatial statistics* focuses on "numerical relationships" within and among mapped data (see Figure 1). From a spatial statistics perspective there are three primary analytical arenas— Summaries, Comparisons and Correlations.

Statistical summaries provide generalizations of the grid values comprising a single map layer (within), or set of map layers (among). Most common is a tabular summary included in a discrete map's legend that identifies the area and proportion of occurrence for each map category, such as extremely steep terrain comprising 286 acres (19 percent) of a project area. Or for a continuous map surface of slope values, the generalization might identify the data range as from 0-65 percent and note that the average slope is 24.4 with a standard deviation of 16.7.

Summaries among two or more discrete maps generate cross-tabular tables that "count" the joint occurrence of all categorical combinations of the map layers. For example, the coincidence of steepness and cover maps might identify that there are 242 acres of forest cover on extremely steep slopes (16 percent), a particularly hazardous wildfire joint condition.

Map comparison and correlation techniques only apply to continuous mapped data. Comparisons within a single map surface involve normalization techniques. For example, a Standard Normal Variable (SNV) map can be generated to identify "how unusual" (above or below) each map location is compared to the typical value in a project area.

Direct comparisons among continuous map surfaces include appropriate statistical tests (e.g., F-test), difference maps and surface-configuration differences based on variations in surface slope and orientation at each grid location.

Map correlations provide a foothold for advanced inferential spatial statistics. Spatial autocorrelation within a single map surface identifies the similarity among nearby values for each grid location. It is most often associated with surface modeling techniques that employ the assumption that “nearby things are more alike than distant things”—high spatial autocorrelation—for distance-based weight averaging of discrete point samples to derive a continuous map surface.

Spatial correlation, on the other hand, identifies the degree of geographic dependence among two or more map layers and is the foundation of spatial data mining. For example, a map surface of a bank’s existing concentration of home equity loans within a city can be regressed against a map surface of home values. If a high level of spatial dependence exists, the derived regression equation can be used on home value data for another city. The resulting map surface of estimated loan concentration proves useful in locating branch offices.

In practice, many geo-business applications utilize numerous independent map layers (e.g., demographics, life style information and sales records from credit card swipes) in developing spatially consistent multivariate models with very high R-squared values. Like most things from ecology to economics to environmental considerations, spatial expression of variable dependence echoes niche theory with grid-based spatial statistics serving as a powerful tool for understanding geographic patterns and relationships.

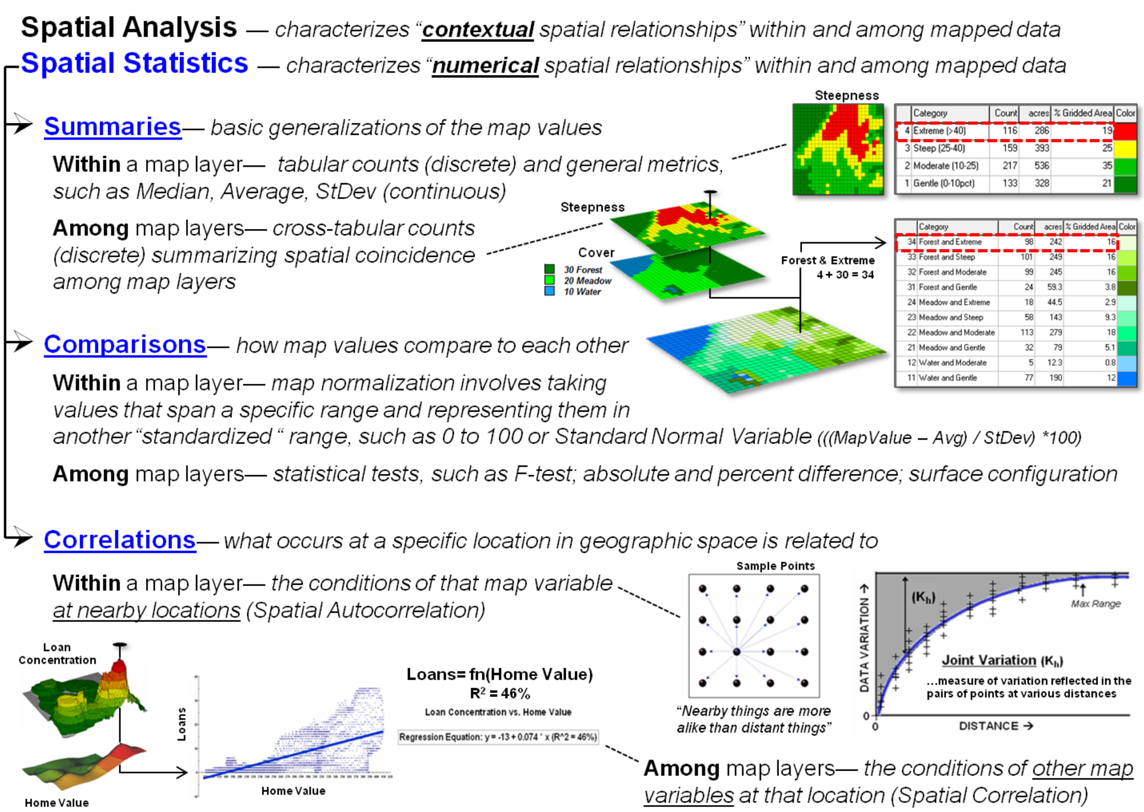

Figure 1. Spatial Statistics uses numerical analysis to uncover spatial relationships and patterns.

Figure 2 describes an example of basic surface modeling and the linkage between numeric space and geographic space representations using environmentally oriented mapped data. Soil samples are collected and analyzed, ensuring that geographic coordinates accompany the field samples. The resulting discrete point map of the field soil chemistry data are spatially interpolated into a continuous map surface characterizing the data set's geographic distribution.

The bottom portion of Figure 2 depicts the linkage between data space and geographic space representations of mapped data. In data space, a standard normal curve is fitted to the data as means to characterize its overall "typical value" (average= 22.9) and "typical dispersion" (StDev= 18.7), without regard for the data's spatial distribution.

In geographic space, the average forms a flat plane implying that this value is assumed to be everywhere within +/- 1 standard deviation about two-thirds of the time and offering no information about where values are likely more or less than the typical. The fitted continuous map surface, however, details the spatial variation inherent in the field collected samples.

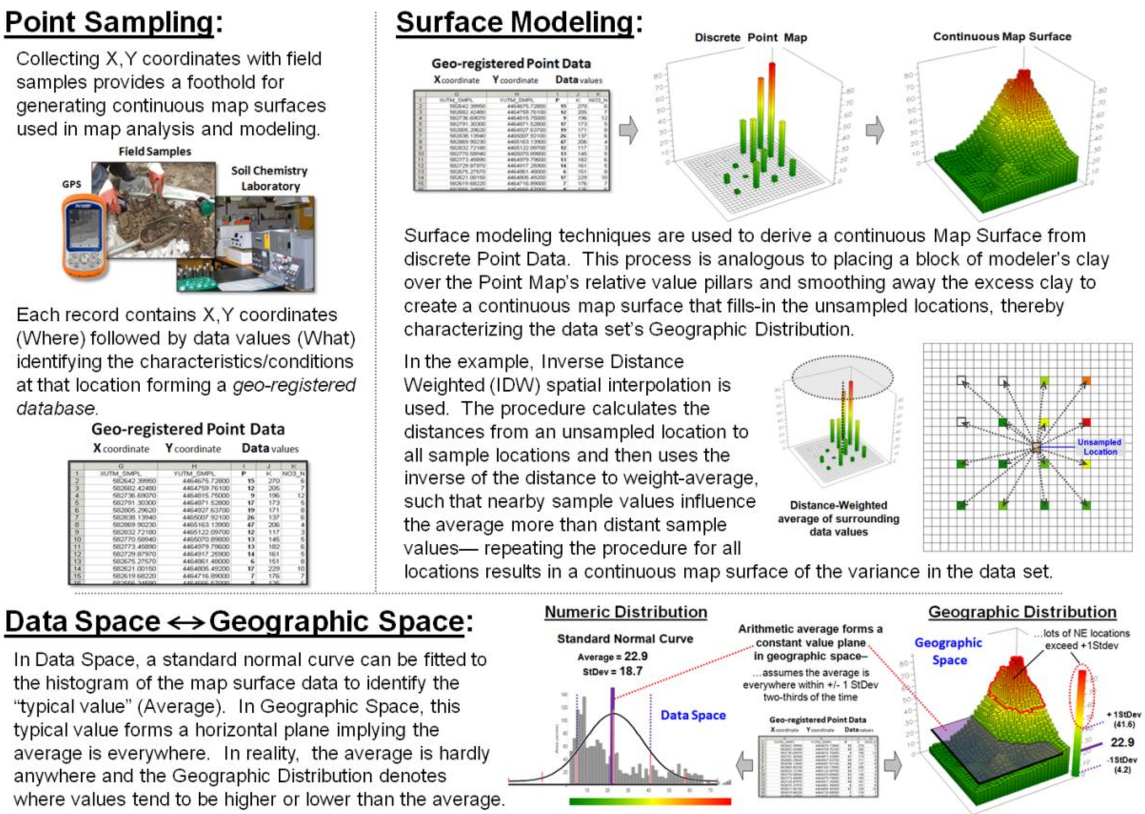

Figure 2. An example of Surface Modeling that derives a continuous map surface from set of discrete point data.

Nonspatial statistics identifies the "central tendency" of the data, whereas surface modeling maps the "spatial variation." Like a Rorschacht ink blot, a map layer's histogram and surface plot provide two different perspectives. Clicking a histogram pillar identifies the grid cells within that range; clicking on a grid location identifies which histogram range contains it.

This direct link between the numerical and spatial characteristics of mapped data provides the foundation for the spatial statistics operations outlined in Figure 3. The first four classes of operations are fairly self-explanatory, with the exception of "Roving Window" summaries. This technique first identifies the grid values surrounding a location, then mathematically/statistically summarizes the values, assigns the summary to that location, and then moves to the next location and repeats the process.

Another specialized use of roving windows is for surface modeling. As described in Figure 2, inverse-distance weighted (IDW) spatial interpolation is the weight-averaged of samples based on their relative distances from the focal location. For qualitative data, the total number of occurrences within a window reach can be summed for a density surface.

In Figure 3, for example, a map identifying customer locations can be summed to identify the total number of customers within a roving window to generate a continuous map surface of customer density. In turn, the average and standard deviation can be used to identify “pockets” of unusually high customer density.

Standard multivariate techniques using “data distance,” such as maximum likelihood and clustering, can be used to classify sets of map variables. Map Similarity, for example, can be used to compare each map location’s pattern of values with a comparison location’s pattern to create a continuous map surface of the relative degree of similarity at each map location.

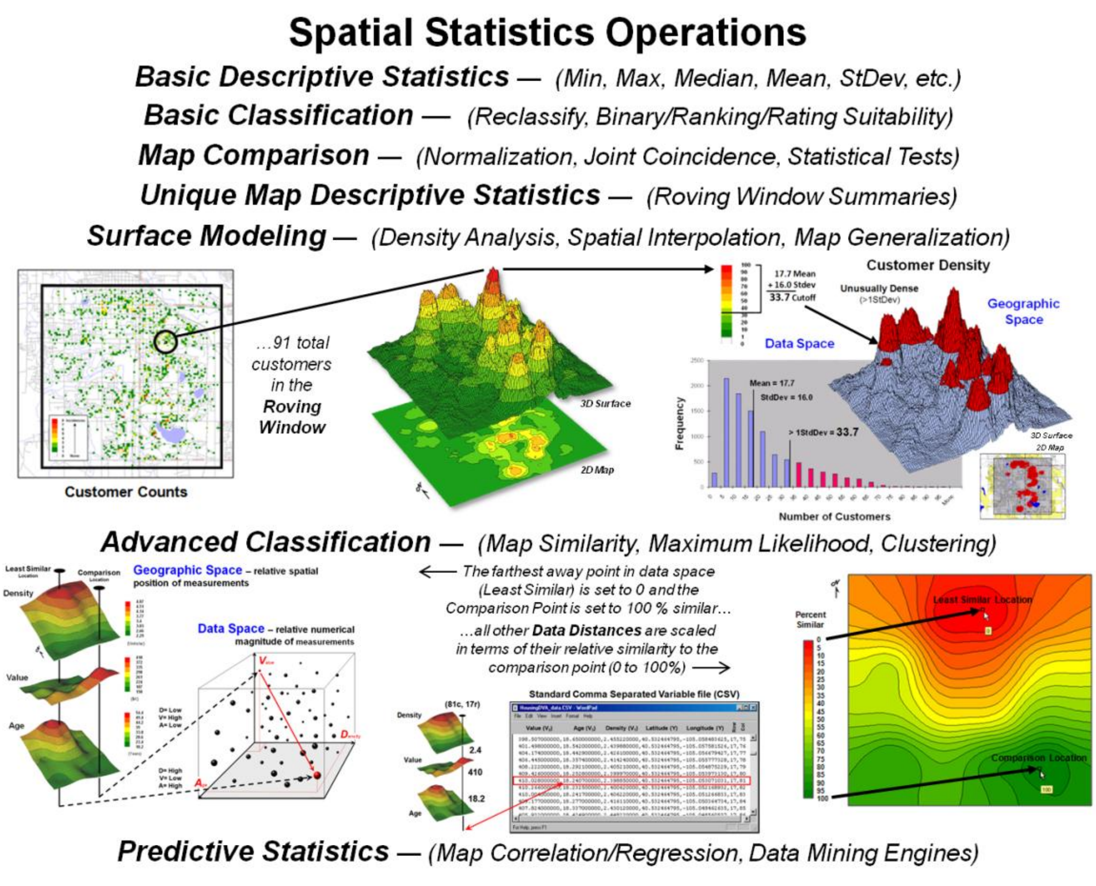

Figure 3. Classes of Spatial Statistics operations.

Statistical techniques, such as regression, can be used to develop mathematical functions between dependent and independent map variables. The difference between spatial and non-spatial approaches is that the map variables are spatially consistent and yield a prediction map that shows where high and low estimates are to be expected.

The bottom line in spatial statistics (as well as spatial analysis) is that the geographic character within and among map layers is taken into account. The grid-based representation of mapped data provides the consistent framework needed for such analyses. Each database record contains geographic coordinates (X,Y= Where) and value fields identifying the characteristics/conditions at that location (V_i= What).

From this "map-ematical" view, traditional math/stat procedures can be extended into geographic space. The paradigm shift from our paper-map legacy to "maps as data first, pictures later" propels us beyond mapping to map analysis and modeling. In addition, it defines a comprehensive and common *Spatial*STEM educational environment that stimulates students with diverse backgrounds and interests to "think analytically with maps" in solving complex problems.

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