An Analytical Framework for GIS Modeling

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Abstract:

The U.S. Department of Labor has identified Geotechnology as one of three mega technologies for the 21st century noting that it will forever change how we will conceptualize, utilize and visualize spatial information. Of the spatial triad comprising Geotechnology (GPS, GIS and RS), the spatial analysis and modeling capabilities of Geographic Information Systems provides the greatest untapped potential, but these analytical procedures are least understood. This paper develops a conceptual framework for understanding and relating various grid-based map analysis and modeling procedures, approaches and applications. Discussion topics include; 1) the nature of discrete versus continuous mapped data; 2) spatial analysis procedures for reclassifying and overlaying map layers; 3) establishing distance/connectivity and depicting neighborhoods; 4) spatial statistics procedures for surface modeling and spatial data mining; 5) procedures for communicating model logic/commands; and, 6) the impact of spatial reasoning/dialog on the future of Geotechnology.

Keywords: Geographic Information Systems, GIS modeling, grid-based map analysis, spatial analysis, spatial statistics, map algebra, map-ematics

This paper presents a conceptual framework used in organizing material presented in a graduate course on GIS Modeling presented at the University of Denver. For more information and materials see http://www.innovativegis.com/basis/Courses/GMcourse11/.

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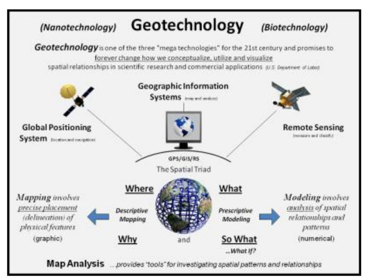
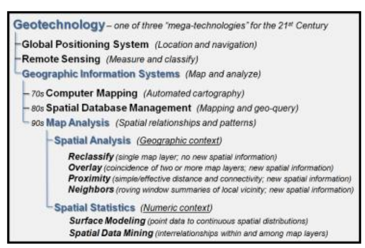
Table of Contents





1.0 **Introduction**

Historically information relating to the spatial characteristics of infrastructure, resources and activities has been difficult to incorporate into planning and management. Manual techniques of map analysis are both tedious and analytically limiting. The rapidly growing field of *Geotechnology* involving modern computer-based systems, on the other hand, holds promise in providing capabilities clearly needed for determining effective management actions.

Geotechnology refers to "any technological application that utilizes spatial location in visualizing, measuring, storing, retrieving, mapping and analyzing features or phenomena that occurs on, below or above the earth" (Berry, 2009). It is recognized by the U.S. Department of Labor as one of the "three mega-technologies for the 21st Century," along with Biotechnology and Nanotechnology (Gewin, 2004). As depicted in the left inset of figure 1 there are three primary mapping disciplines that enable Geotechnology— *GPS (Global Positioning System)* primarily used for location and navigation, *RS (Remote Sensing)* primarily used to measure and classify the earth's cover, and *GIS (Geographic Information Systems/Science/Solutions)* primarily used for mapping and analysis of spatial information.

The interpretation of the "S" in GIS varies from "Systems" with an emphasis on data management and the computing environment. A "Science" focus emphasizes the development of geographic theory, structures and processing capabilities. A "Solutions" perspective emphasizes application of the technology within a wide variety of disciplines and domain expertise.

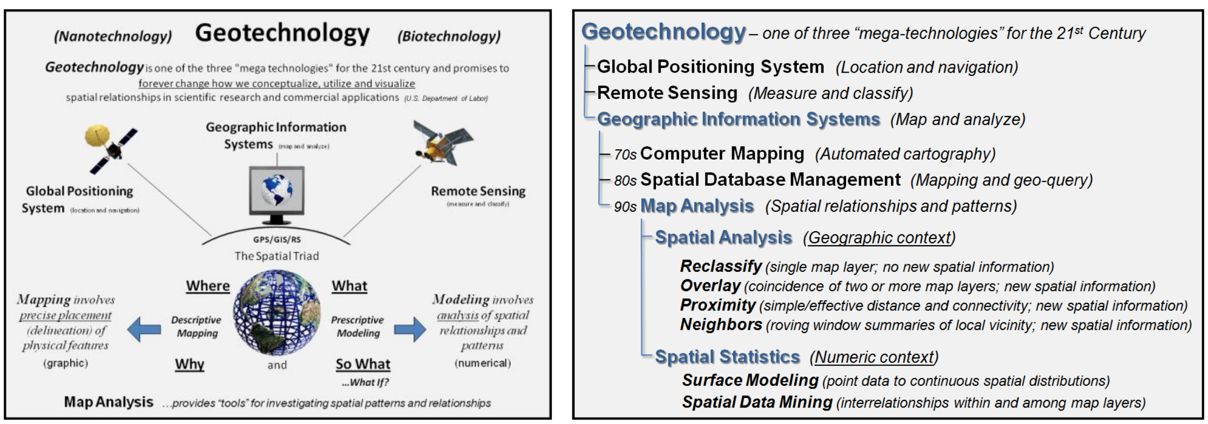


Figure 1. Overview organization of components, evolution and types of tools defining Map Analysis.

Since the 1960s the decision-making process has become increasingly quantitative, and mathematical models have become commonplace. Prior to the computerized map, most spatial analyses were severely limited by their manual processing procedures. Geographic information systems technology provides the means for both efficient handling of voluminous data and effective spatial analysis capabilities. From this perspective, GIS is rooted in the digital nature of the computerized map.

While today's emphasis in Geotechnology is on sophisticated *multimedia mapping* (e.g., Google Earth, internet mapping, web-based services, virtual reality, etc.), the early 1970s saw

computer mapping as a high-tech means to automate the map drafting process. The points, lines and areas defining geographic features on a map are represented as an organized set of X,Y coordinates. These data drive pen plotters that can rapidly redraw the connections at a variety of colors, scales, and projections. The map image, itself, is the focus of this automated cartography.

During the early 1980s, ***Spatial database management systems (SDBMS)*** were developed that linked computer mapping capabilities with traditional database management capabilities. In these systems, identification numbers are assigned to each geographic feature, such as a timber harvest unit or sales territory. For example, a user is able to point to any location on a map and instantly retrieve information about that location. Alternatively, a user can specify a set of conditions, such as a specific vegetation and soil combination, and all locations meeting the criteria of the geographic search are displayed as a map.

As Geotechnology continued its evolution, the 1990s emphasis turned from descriptive "geo-query" searches of existing databases to investigative ***Map Analysis***. Today, most GIS packages include processing capabilities that relate to the capture, encoding, storage, analysis and visualization of spatial data. This paper describes a conceptual framework and a series of techniques that relate specifically to the analysis of mapped data by identifying fundamental map analysis operations common to a broad range of applications. As depicted in the lower portion of the right inset in figure 1, the classes of map analysis operations form two basic groups involving *Spatial Analysis* and *Spatial Statistics*.

Spatial Analysis extends the basic set of discrete map features of points, lines and polygons to surfaces that represent continuous geographic space as a set of contiguous grid cells. The consistency of this grid-based structuring provides a wealth of new analytical tools for characterizing "contextual spatial relationships," such as effective distance, optimal paths, visual connectivity and micro-terrain analysis. Specific classes of spatial analysis operations that will be discussed include *Reclassify*, *Overlay*, *Proximity* and *Neighbors*.

In addition, it provides a mathematical/statistical framework by numerically representing geographic space. Whereas traditional statistics is inherently non-spatial as it seeks to represent a data set by its typical response regardless of spatial patterns, ***Spatial Statistics*** extends this perspective on two fronts. First, it seeks to map the variation in a data set to show where unusual responses occur, instead of focusing on a single typical response. Secondly, it can uncover "numerical spatial relationships" within and among mapped data layers, such as generating a prediction map identifying where likely customers are within a city based on existing sales and demographic information. Specific classes of spatial statistics operations that will be discussed include *Surface Modeling* and *Spatial Data Mining*.

By organizing primitive analytical operations in a logical manner, a generalized GIS modeling approach can be developed. This fundamental approach can be conceptualized as a "map algebra" or "map-ematics" in which entire maps are treated as variables (Tomlin and Berry, 1979; Berry, 1985; Tomlin, 1990). In this context, primitive map analysis operations can be seen as analogous to traditional mathematical operations. The sequencing of map operations is similar to the algebraic solution of equations to find unknowns. In this case however, the unknowns represent entire maps. This approach has proven to be particularly effective in presenting spatial analysis techniques to individuals with limited experience in geographic information processing.

2.0 **Nature of Discrete versus Continuous Mapped Data**

For thousands of years, points, lines and polygons have been used to depict map features. With the stroke of a pen a cartographer could outline a continent, delineate a highway or identify a specific building's location. With the advent of the computer and the digital map, manual drafting of spatial data has been replaced by the cold steel of the plotter.

In digital form, mapped data have been linked to attribute tables that describe characteristics and conditions of the map features. Desktop mapping exploits this linkage to provide tremendously useful database management procedures, such as address matching, geo-query and network routing. ***Vector-based*** data forms the foundation of these techniques and directly builds on our historical perspective of maps and map analysis.

Grid-based data, on the other hand, is a less familiar way to describe geographic space and its relationships. At the heart of this procedure is a new map feature that extends traditional irregular discrete *Points*, *Lines* and *Polygons* (termed "spatial objects") to uniform continuous map *Surfaces*.

The top portion of figure 2 shows an elevation surface displayed as a traditional contour map, a superimposed analysis frame and a 2-D grid map. The highlighted location depicts the elevation value (500 feet) stored at one of the grid locations. The pop-up table at the lower-right shows the values stored on other map layers at the current location in the *Analysis Frame*. As the cursor is moved, the "drill-down" of values for different grid locations in the *Map Stack* are instantly updated.

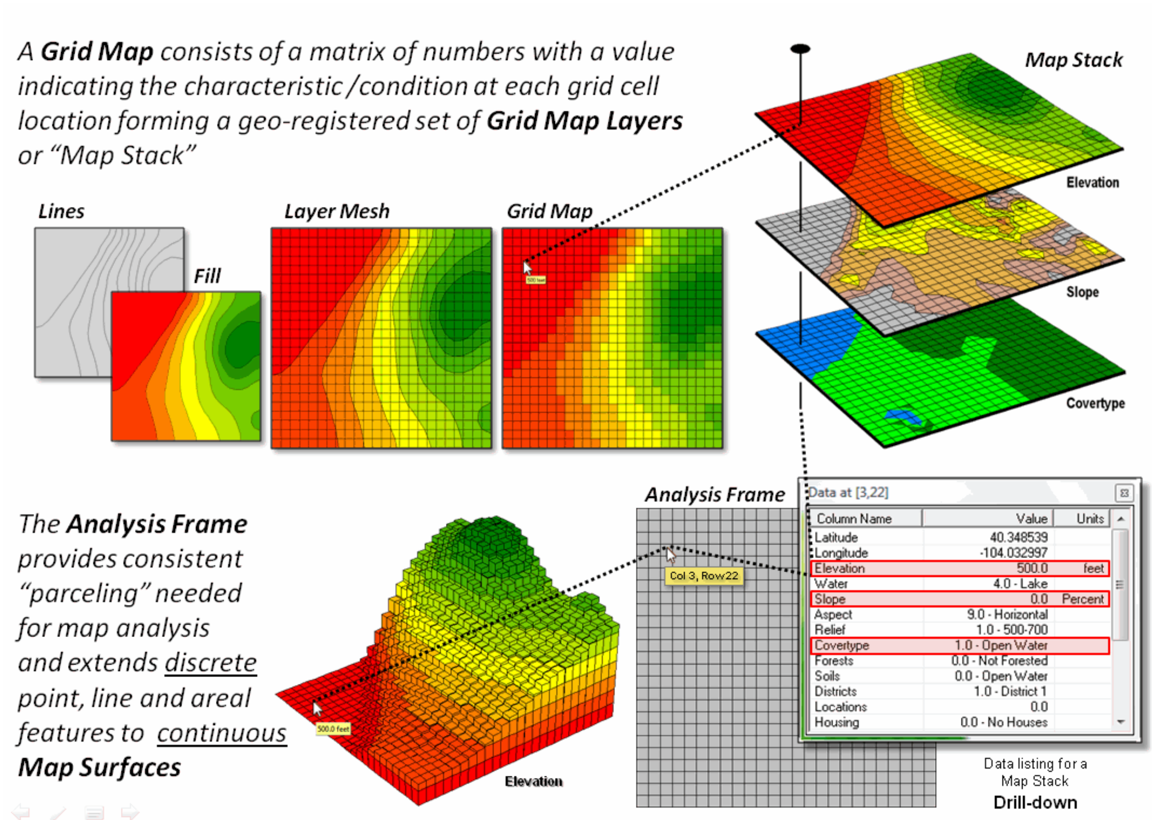


Figure 2. Grid-based map layers form a geo-registered stack of maps that are pre-conditioned for map analysis.

Extending the grid cells to the relative height implied by the map values at each location forms the 3-dimensional plot in the lower-left portion of the figure. The result is a "grid" plot that depicts the peaks and valleys of the spatial distribution of the mapped data forming the surface. The color zones identify contour intervals that are draped on the surface. In addition to providing a format for storing and displaying map surfaces, the analysis frame establishes the consistent structuring demanded for advanced grid-based map analysis operations. It is the consistent and continuous nature of the grid data structure that provides the geographic framework supporting the toolbox of grid-based analytic operations used in GIS modeling.

In terms of data structure, each map layer in the map stack is comprised of a title, certain descriptive parameters and a set of categories, technically referred to as *Regions*. Formally stated, a region is simply one of the thematic designations on a map used to characterize geographic locations. A map of water bodies entitled "Water," for example might include regions associated with dry land, lakes or ponds, streams and wetlands. Each region is represented by a name (i.e., a text label) and a numerical value. The structure described so far, however, does not account for geographic positioning and distribution. The handling of positional information is not only what most distinguishes geographic information processing from other types of computer processing; it is what most distinguishes one GIS system from another.

As mentioned earlier, there are two basic approaches in representing geographic positioning information: Vector, based on sets of discrete line segments, and Raster, based on continuous sets of grid cells. The vector approach stores information about the boundaries between regions, whereas raster stores information on interiors of regions. While this difference is significant in terms of implementation strategies and may vary considerably in terms of geographic precision, they need not affect the definition of a set of fundamental analytical techniques. In light of this conceptual simplicity, the grid-based data structure is best suited to the description of primitive map processing techniques and is used in this paper.

The grid structure is based on the condition that all spatial locations are defined with respect to a regular rectangular geographic grid of numbered rows and columns. As such, the smallest addressable unit of space corresponds to a square parcel of land, or what is formally termed a *Grid Cell*, or more generally referred to as a "point" or a "location." Spatial patterns are represented by assigning all of the grid cells within a particular region a unique *Thematic Value*. In this way, each point also can be addressed as part of a *Neighborhood* of surrounding values.

If primitive operations are to be flexibly combined, a processing structure must be used that accepts input and generates output in the same format. Using the data structure outlined above, this may be accomplished by requiring that each analytic operation Involve—

- <u>retrieval</u> of one or more map layers from the map stack,
- <u>manipulation</u> of that mapped data,
- <u>creation</u> of a new map layer whose categories are represented by new thematic values derived as a result of that manipulation, and
- <u>storage</u> of that new map layer back into the map stack for subsequent processing.

The cyclical nature of the retrieval-manipulation-creation-storage processing structure is analogous to the evaluation of "nested parentheticals" in traditional algebra. The logical sequencing of primitive map analysis operations on a set of map layers forms a spatial model of specified application. As with traditional algebra, fundamental techniques involving several primitive operations can be identified (e.g., a "travel-time map") that are applicable to numerous situations.

The use of these primitive map analysis operations in a generalized modeling context accommodates a variety of analyses in a common, flexible and intuitive manner. It also provides a framework for understanding the principles of map analysis that stimulates the development of new techniques, procedures and applications.

3.0 **Spatial Analysis Procedures for Reclassifying Maps**

The first and in many ways the most fundamental class of map analysis operations involves the reclassification of map categories. Each of the operations involves the creation of a new map by assigning thematic values to the categories/regions of an existing map. These values may be assigned as a function of the *initial value, position, size, shape* or *contiguity* of the spatial configuration associated with each map category (figure 3). All of the reclassification operations involve the simple repackaging of information on a single map layer and results in no new boundary delineations. Such operations can be thought of as the "purposeful re-coloring" of maps.

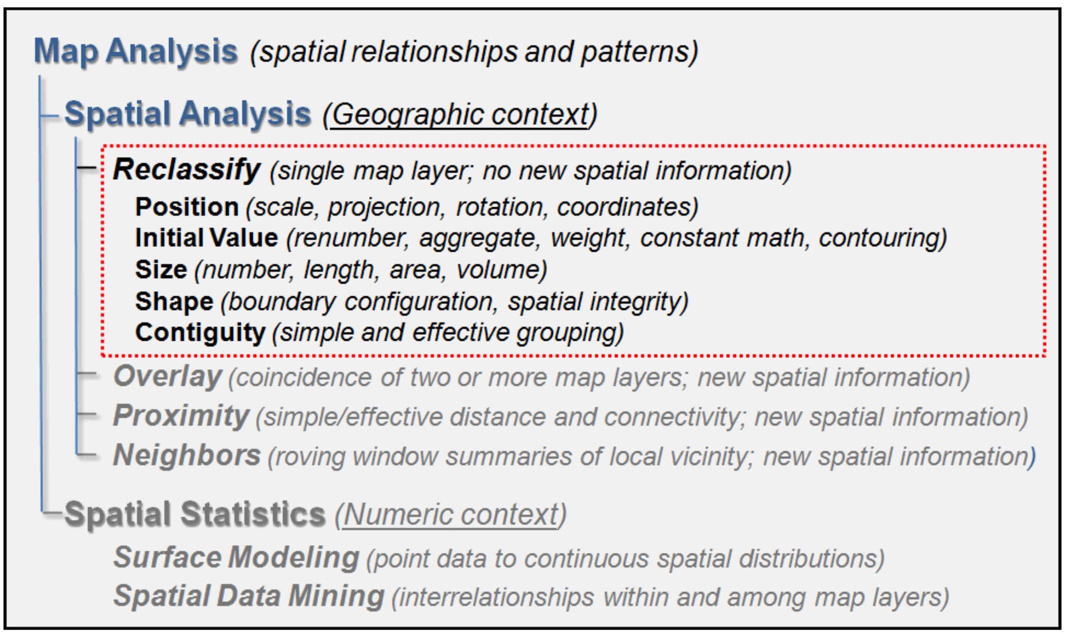


Figure 3. Reclassify operations involve reassigning map values to reflect new information about existing map features.

For example, an ***initial value*** based reclassification operation may involve the ranking or weighting of qualitative map categories to generate a new map with quantitative values. A map of soil types might be assigned values that indicate the relative suitability of each soil type for residential development. Quantitative values also may be reclassified to yield new quantitative values. This might simply involve a specified reordering of map categories (e.g., given a map of soil moisture content, generate a map of suitability levels for plant growth). Or it could involve the application of a generalized reclassifying function, such as "level slicing," which splits a continuous range of map categories values into discrete intervals (e.g., derivation of a contour map from a map of terrain elevation values).

Other quantitative reclassification functions include a variety of arithmetic operations involving map category values and a specified computed constant. Among these operations are addition, subtraction, multiplication, division, exponentiation, maximization, minimization, normalization and other scalar mathematical and statistical operators. For example, a map of topographic elevation expressed in feet may be converted to meters by multiplying each map value by 3.28083 feet per meter.

Reclassification operations also can relate to location-based, as well as purely thematic attributes associated with a map. One such characteristic is ***position***. A map category represented by a single "point" location (grid cell), for example, might be reclassified according to its latitude and longitude. Similarly, a line segment or areal feature could be reassigned values indicating its center of gravity or orientation.

Another location-based characteristic is ***size***. In the case of map categories associated with linear features or point locations, overall length or number of points may be used as the basis for reclassifying those categories. Similarly, a map category associated with a planar area may be reclassified according to its total acreage or the length of its perimeter. For example, a map of surface water might be reassigned values to indicate the areal extent of individual lakes or length of stream channels. The same sort of technique also might be used to deal with volume. Given a map of depth to bottom for a group of lakes, for example, each lake might be assigned a value indicating total water volume based on the areal extent of each depth category.

In addition to value, position and size of features, ***shape*** characteristics also may be used as the basis for reclassifying map categories. Categories represented by point locations have measurable shapes insofar as the set of points imply linear or areal forms (e.g., just as stars imply constellations). Shape characteristics associated with linear forms identify the patterns formed by multiple line segments (e.g., dendritic stream pattern). The primary shape characteristics associated with areal forms include boundary convexity, nature of edge and topological genius.

Convexity and edge address the "boundary configuration" of areal features. Convexity is the measure of the extent to which an area is enclosed by its background relative to the extent to which the area encloses its background. The convexity index for a feature is computed by the ratio of its perimeter to its area. The most regular configuration is that of a circle which is convex everywhere along its boundary, and therefore, not enclosed by the background at any point. Comparison of a feature's computed convexity with that of a circle of the same area results in a standard measure of boundary regularity. The nature of the boundary at each edge location can be used for a detailed description of boundary along a features edge. At some locations the boundary might be an entirely concave intrusion, whereas other locations might have entirely convex protrusions. Depending on the degree of "edginess," each point can be assigned a value indicating the actual boundary configuration at that location.

Topological genius relates to the "spatial integrity" of an area. A category that is broken into numerous fragments and/or containing several interior holes indicates less spatial integrity than those without such violations. The topological genius can be summarized as the Euler number which is computed as the number of holes within a feature less one short of the number of fragments which make up the entire feature.   An Euler number of zero indicates features that are spatially balanced, whereas larger negative or positive numbers indicate less spatial integrity.

A related operation, termed "parceling," characterizes ***contiguity***. This procedure identifies individual clumps of one or more locations having the same numerical value and which are geographically contiguous (e.g., generation of a map identifying each lake as a unique value from a generalized water map representing all lakes as a single category).

This explicit use of shape/contiguity as analytic parameters is unfamiliar to most Geotechnology users. However, a non-quantitative consideration of landscape structure is implicit in any visual assessment of mapped data. Particularly promising is the potential for applying these techniques in areas of image classification and wildlife habitat modeling. A map of forest stands, for example, may be reclassified such that each stand is characterized according to the relative amount of forest edge with respect to total acreage and frequency of interior forest canopy gaps. Those stands with a large portion of edge and a high frequency of gaps will generally indicate better wildlife habitat for many species.

4.0 **Spatial Analysis Procedures for Overlaying Maps**

Operations for overlaying maps begin to relate to the spatial, as well as the thematic nature of mapped data. The general class of overlay operations can be characterized as "light-table gymnastics." The operations involves creation of a new map on which the value assigned to each location or set of locations is a function of the independent values associated with that location on two or more existing map layers. There are three basic approaches to overlaying maps—*Location-specific,* and *Region-wide* (figure 4).

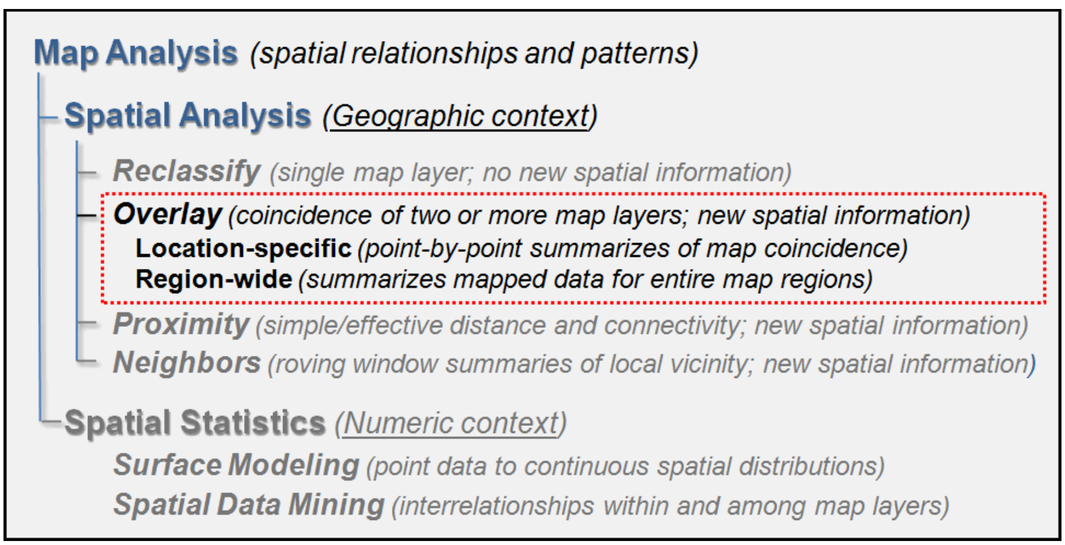


Figure 4. Overlay operations involve characterizing the spatial coincidence of mapped data.

In simple Location-specific overlaying, the value assigned is a function of the point-by-point aligned coincidence of existing maps. In Region-wide compositing, values are assigned to entire thematic regions as a function of the values associated with those regions contained on other map layers. Whereas the first overlay approach conceptually involves the vertical spearing of a set of map layers, the latter approach uses one layer to identify boundaries (termed the “template map”) for which information is extracted in a horizontal summary fashion from the other map layers (termed the “data maps”).

The most basic group of ***Location-specific*** overlay operations computes new map values from those of existing map layers according to the nature of the data being processed and the specific use of that data within a modeling context. Typical of many environmental analyses are those which involve the manipulation of quantitative values to generate new quantitative values. Among these are the basic arithmetic operations, such as addition, subtraction, multiplication, division, roots, and exponentiation. For example, given maps of assessed land values in 2000 and 2005, respectively, one might generate a map showing the change in land values over that period as follows: *(expressed in MapCalc™ software syntax)*

COMPUTE 2005_map MINUS 2000_map FOR Change_map
COMPUTE Change_map DIVIDEDBY 2005_map FOR Relative_Change_map
COMPUTE Relative_Change_map TIMES 100.0 FOR Percent_Change_map

Or as a single map algebra equation:

CALCULATE ((2005_map - 2000_map) / 2005_map) * 100.0 FOR Percent_Change_map

Functions that relate to simple statistical parameters such as maximum, minimum, median, mode, majority, standard deviation, average, or weighted average also may be applied in this manner. The type of data being manipulated dictates the appropriateness of the mathematical or statistical procedure used. For example, the addition of qualitative maps such as soils and land use would result in meaningless sums, as their numeric values have no mathematical relationship. Other map overlay techniques include several which may be used to process values that are either quantitative or qualitative and generate values that may take either form. Among these are masking, comparison, calculation of diversity and intersection.

Many of the more complex statistical techniques comprising spatial statistics that will be discussed later involve overlays where the inherent interdependence among spatial data is accounted for in the continuous nature of the analysis frame. These spatial data mining approaches treat each map as a “variable,” each grid cell as a “case” and each value as an “observation” in an analogous manner as non-spatial math/stat models. A predictive statistical model, such as regression, can be evaluated for each map location, resulting in a spatially continuous surface of predicted values. The mapped predictions contain additional information over traditional non-spatial procedures, such as direct consideration of coincidence among regression variables and the ability to locate areas of a given prediction level.

An entirely different approach to overlaying maps involves ***Region-wide*** summarization of values. Rather than combining information on a point-by-point basis, this group of operations summarizes the spatial coincidence of entire categories of two or more maps. For example, the categories on a Cover_type map can be used to define areas over which the coincident values on a Slope map are averaged. The computed values of average slope are then used to renumber each of the Cover_type categories. This processing can be implemented as:
(expressed in MapCalc™ software syntax)

COMPOSITE Cover_type with Slope AVERAGE FOR Covertype_avgslope

Summary statistics that can be used in this way include the total, average, maximum, minimum, median, mode, or majority value; the standard deviation, variance, coefficient of variation or diversity of values; and the correlation, deviation or uniqueness of particular value combinations.

For example, a map indicating the proportion of undeveloped land within each of several districts could be generated by superimposing a map of district boundaries on a map of land use and computing the ratio of undeveloped land to the total area of each district.

As with location-specific overlay techniques, data types must be consistent with the summary procedure used. Also of concern is the order of data processing. Operations such as addition and multiplication are independent of the order of processing (termed "commutative operations"). However, other operations, such as subtraction and division, yield different results depending on the order in which a group of numbers is processed (termed "non-commutative operations"). This latter type of operations cannot be used for region-wide summaries.

5.0 **Spatial Analysis Procedures for Establishing Proximity and Connectivity**

Most geographic information systems contain analytic capabilities for reclassifying and overlaying maps. The operations address the majority of applications that parallel manual map analysis techniques. However to more fully integrate spatial considerations into decision-making, new more advanced techniques are available. The concept of ***distance*** has been historically associated with the "shortest straight line distance between two points." While this measure is both easily conceptualized and implemented with a ruler, it frequently is insufficient in a decision-making context.

A straight line route might indicate the distance "as the crow flies" but offers little information for a walking or hitchhiking crow or other flightless creature. Equally important to most travellers is to have the measurement of distance expressed in more relevant terms, such as time or cost. The group of operations concerned with distance, therefore, is best characterized as "rubber rulers."

The basis of any system for measurement of distance requires two components—a standard measurement unit and a measurement procedure. The *measurement unit* used in most computer-oriented systems is the "grid space" implied by the superimposing of an imaginary uniform grid over geographic space (e.g., latitude/longitude or a custom analysis grid). The distance from any location to another is computed as the number of intervening grid spaces. The *measurement procedure* always retains the requirement of "shortest" connection between two points; however, the "straight line" requirement may be relaxed.

A frequently employed measurement procedure involves expanding the concept of distance to one of ***proximity*** (figure 5). Rather than sequentially computing the distance between pairs of locations, concentric equidistant zones are established around a location or set of locations. In effect, a map of proximity to a "target" location is generated that indicates the shortest straight line distance to the nearest target grid cell for each non-target location.

Within many application contexts, the shortest route between two locations might not always be a straight line. And even if it is straight, the Euclidean length of that line may not always reflect a meaningful measure of distance. Rather, distance in these applications is defined in terms of ***movement*** expressed as travel-time, cost, or energy that may be consumed at rates that vary over time and space. Distance modifying effects may be expressed spatially as "barriers" located within the geographic space in which the distance is being measured. Note that this implies that distance is the result of some sort of movement over that space and through those barriers.

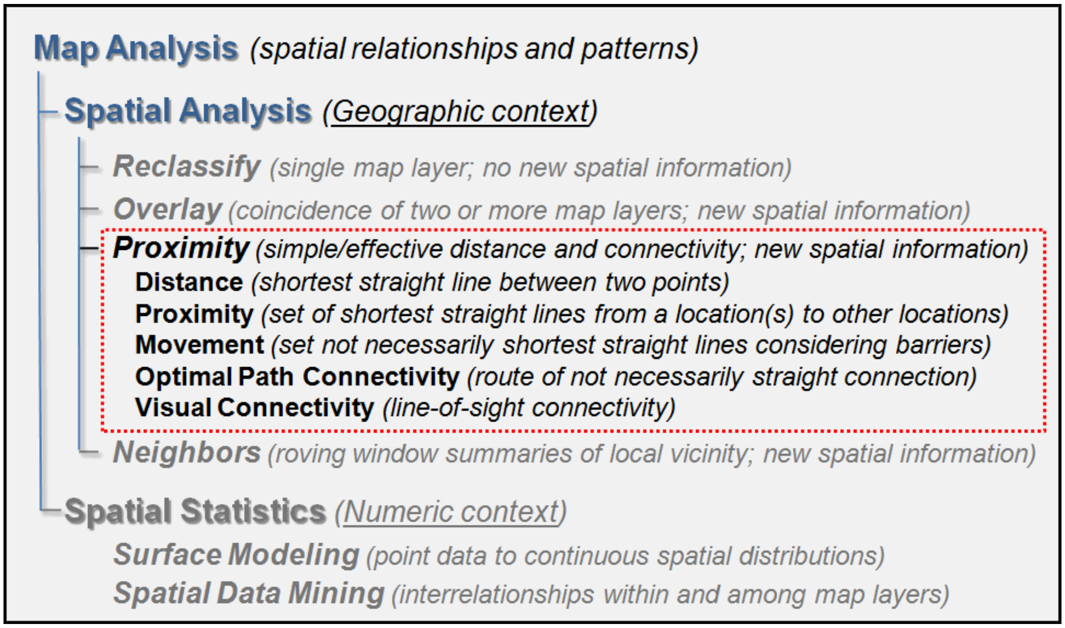


Figure 5. Proximity operations involve measuring distance and connectivity among map locations.

Two major types of barriers can be identified as to how they affect the implied movement. “Absolute barriers” are those that completely restrict movement and therefore imply an infinite distance between the points they separate, unless a path around the barrier is available. A flowing river might be regarded as an absolute barrier to a non-swimmer. To a swimmer or boater, however, the same river might be regarded as a relative rather than an absolute barrier. “Relative barriers” are ones that are passable but only at a cost/impedance which may be equated with an increase in effective distance.

For example, the hiking-time map from a location can be calculated as set of concentric zones that vary in shape by the relative influence various cover/slope categories for foot-travel responding to the two types of barriers. Open water such as lakes and large streams are treated as absolute barriers that completely restrict hiking. The land areas on the other hand, represent relative barriers to hiking which indicate varied impedance to movement for each grid cell as a function of the cover/slope conditions occurring at a location.

In a similar example, movement by automobile might be effectively constrained to a network of roads (absolute barriers) of varying speed limits (relative barriers) to generate a riding travel-time map. Or from an even less conventional perspective, effective-distance can be expressed in such terms as accumulated cost of electric transmission line construction from an existing trunk line to all other locations in a project area. The cost surface that is developed can be a function of a variety of social and engineering factors, such as visual exposure and adverse terrain, expressed as absolute and/or relative barriers.

The ability to move, whether physically or abstractly, may vary as a function of the implied movement, as well as the static conditions at a location. One aspect of movement that may affect the ability of a barrier to restrict that movement is direction. A topographic incline, for example, will generally impede hikers differently according to whether their movement is uphill, downhill or across slope. Another possible modifying factor is accumulation. After hiking for a certain distance, “mole-hills” tend to become disheartening mountains and movement becomes more restricted for a tired hiker. A third attribute of movement that may dynamically alter the effect of a barrier is momentum or speed. If an old car has to stop on steep hill, it might not be able to resume movement, whereas if it were allowed to maintain its momentum (e.g., green light) it could easily reach the top. Similarly, a highway construction zone which reduces traffic speeds from 75 to 55 mph, for example, would have little or no effect during rush hour when traffic already is moving at a much slower speed.

Another distance related class of operators is concerned with the nature of ***connectivity*** among locations. Fundamental to understanding these procedures is the conceptualization of an "accumulation surface," It the map value of a simple proximity map from a location is used to indicate the third dimension of a surface a uniform bowl would be depicted. The surface configuration for an effective proximity map would have a similar appearance; however, the bowl would be warped with numerous ridges. Also, the nature of the surface is such that it cannot contain saddle points (i.e., false bottoms). This "bowl-like topology" is characteristic of all accumulation surfaces and can be conceptualized as a warped football stadium with each successive ring of seats identifying concentric, equidistant halos. The target or starting location(s) is the lowest point and all other locations are assigned progressively larger values of the shortest, but not necessarily straight distance from the start. When viewed in perspective this surface resembles a topographic surface with familiar valleys and hills. However in this case, the highlands indicate areas that are effectively farther away from the starting location and there can be no false bottoms along the ever-increasing gradient.

In the case of simple distance, the delineation of paths, or "connectivity" lines, locate the shortest straight line between any point on the surface and the starting point. Similarly, the steepest downhill path along the accumulated surface of an effective distance respecting intervening absolute/relative barriers identifies the shortest but not necessarily straight path connecting any location to the starting point.

The procedure is analogous to the steepest downhill path of water over a topographic surface to indicate surface runoff. However the steepest path over a proximity accumulation surface always ends at the starting/target location(s) and effectively retraces the shortest route. If an accumulation cost surface is considered, such as the cost for transmission line construction, the minimum cost route will be located. If transmission line construction to a set of dispersed locations were simultaneously considered, an "optimal path density" map could be generated which identifies the number of individual optimal paths passing through each location from the dispersed termini to a truck line start/target. Such a map would be valuable in locating major feeder-lines (i.e., areas of high optimal path density) radiating from a central trunk line.

Another connectivity operation determines the narrowness of features. The narrowness at each point within a map feature is defined as the length of the shortest line segment that can be constructed through that point to diametrically opposing edges of the feature (termed a "cord" in plane geometry). The result of this processing is a continuous map of features with lower values indicating relative narrow locations within the feature. Panama, for example, is at the narrowest point in the continental western hemisphere. Or for a narrowness map of forest stands, low values indicate interior locations with easy access to edges.

The process of determining viewshed involves establishing inter-visibility among locations. Locations forming the viewshed of an area are connected by straight line-of-sight in three-dimensional space to the "viewer" location, or set of viewers. Topographic relief and surface objects form absolute barriers that preclude connectivity. Atmospheric haze forms a relative barrier and leaf on/off conditions can cause the vegetation canopy result in different levels of opacity. If multiple viewers are designated, locations within the viewshed may be assigned a value indicating the number or density of visual connections.

6.0 **Spatial Analysis Procedures for Depicting Neighborhoods**

The fourth and final group of spatial analysis operations includes procedures that create a new map in which the value assigned to a location is computed as a function of independent values within a specified distance and direction around that location (i.e., its geographic neighborhood). This general class of operations can be conceptualized as "roving windows" moving throughout a project area. The summary of information within these windows can be based on the *configuration of the surface* (e.g., slope and aspect) or the *statistical summary* of thematic values (figure 6).

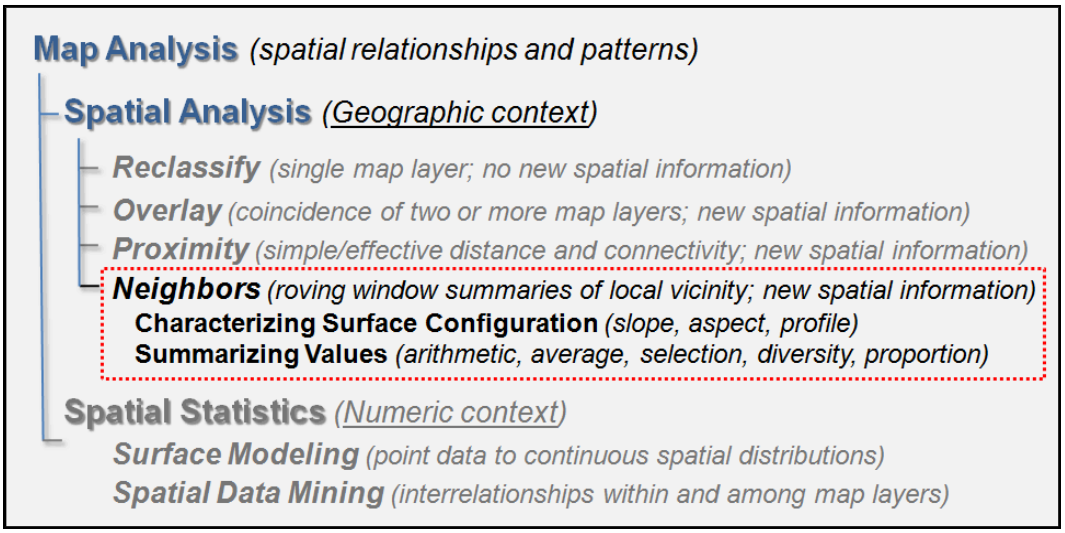


Figure 6. Neighborhood operations involve characterizing mapped data within the vicinity of map locations.

The initial step in characterizing geographic neighborhoods is the establishment of neighborhood membership. An instantaneous neighborhood (roving window) is uniquely defined for each target location as the set of points which lie within a specified distance and direction around that location. The roving window can assume different shapes based on geometric shape, direction and distance. In most applications the window has a uniform shape and orientation (e.g., a circle or square). However as noted in the previous section, the distance may not necessarily be Euclidean nor symmetrical, such as a neighborhood of "down-wind" locations within a quarter mile of a smelting plant. Similarly, a neighborhood of the "ten-minute drive" along a road network could be defined.

The summary of information within a neighborhood may be based on the relative spatial ***configuration*** of values that occur within the window. This is true of the operations that measure topographic characteristics, such as slope, aspect and profile from elevation values. One such approach involves the "least-squares fit" of a plane to adjacent elevation values. This process is similar to fitting a linear regression line to a series of points expressed in two-dimensional space. The inclination of the plane denotes terrain slope and its orientation characterizes the aspect/azimuth. The window is successively shifted over the entire elevation map to produce a continuous slope or aspect map.

Note that a “slope map” of any surface represents the first derivative of that surface. For an elevation surface, slope depicts the rate of change in elevation. For an accumulation cost surface, its slope map represents the rate of change in cost (i.e., a marginal cost map). For a travel-time accumulation surface, its slope map indicates the relative change in speed and its aspect map identifies the direction of optimal movement at each location. Also, the slope map of an existing topographic slope map (i.e., second derivative) will characterize surface roughness (i.e., areas where slope is changing).

The creation of a “profile map” uses a window defined as the three adjoining points along a straight line oriented in a particular direction. Each set of three values can be regarded as defining a cross-sectional profile of a small portion of a surface. Each line is successively evaluated for the set of windows along that line. This procedure may be conceptualized as slicing a loaf of bread, then removing each slice and characterizing its profile (as viewed from the side) in small segments along the upper edge. The center point of each three member neighborhood is assigned a value indicating the profile form at that location. The value assigned can identify a fundamental profile class (e.g., inverted “V” shape indicative of a ridge or peak) or it can identify the magnitude in degrees of the “skyward angle” formed by the intersection of the two line segment of the profile. The result of this operation is a continuous map of the surface’s profile as viewed from a specified direction. Depending on the resolution of an elevation map, its profile map could be used to identify gullies or valleys running east-west (i.e., “V” shape in multiple orthogonal profiles).

The other group of neighborhood operations are those that ***summarize*** thematic values. Among the simplest of these involve the calculation of summary statistics associated with the map values occurring within each neighborhood. These statistics may include the maximum income level, the minimum land value, the diversity of vegetation within a half-mile radius (or perhaps, a five-minute effective hiking distance) of each target point.

Note that none of the neighborhood characteristics described so far relate to the amount of area occupied by the map categories within each roving window neighborhood. Similar techniques might be applied, however, to characterize neighborhood values which are weighted according to spatial extent. One might compute, for example, the total land value within three miles of each target location on a per-acre basis. This consideration of the size of the neighborhood components also gives rise to several additional neighborhood statistics including the majority value (i.e., the value associated with the greatest proportion of the neighborhood area); the minority value (i.e., the value associated with the smallest proportion); and the uniqueness (i.e., the proportion of the neighborhood area associated the value occurring at the target point itself).

Another locational attribute which may be used to modify thematic summaries is the geographic distance from the target location. While distance has already been described as the basis for defining a neighborhood’s absolute limits, it also may be used to define the relative weights of values within a neighborhood. Noise level, for example, might be measured according to the inverse square of the distance from surrounding sources. The azimuthal relationship between neighborhood location and the target point also may be used to weight the value associated with that location. In conjunction with distance weighting, this gives rise to a variety of spatial sampling and interpolation techniques. For example “weighted nearest-neighbors” interpolation of lake-bottom temperature samples assigns a value to a non-sampled location as the distance-weighted average temperature of a set of sampled points within its vicinity.

7.0 **Spatial Statistics Procedures for Surface Modeling**

Whereas spatial analysis involves characterizing the "geographic context" of mapped data, spatial statistics involves characterizing the "numerical relationships" of mapped data. There are two basic classes of spatial statistics: *Surface Modeling* and *Spatial Data Mining*.

"Surface Modeling" converts discrete point-sampled data into continuous map surfaces through density analysis, spatial interpolation or map generalization techniques (figure 7). ***Density Analysis*** establishes a surface by summarizing the point values within a specified distance of each map location. For example, a "customer density surface" can be derived from geo-coded sales data to depict the peaks and valleys of customer concentrations throughout a city by simply counting the number of point values within a specified distance from each location.

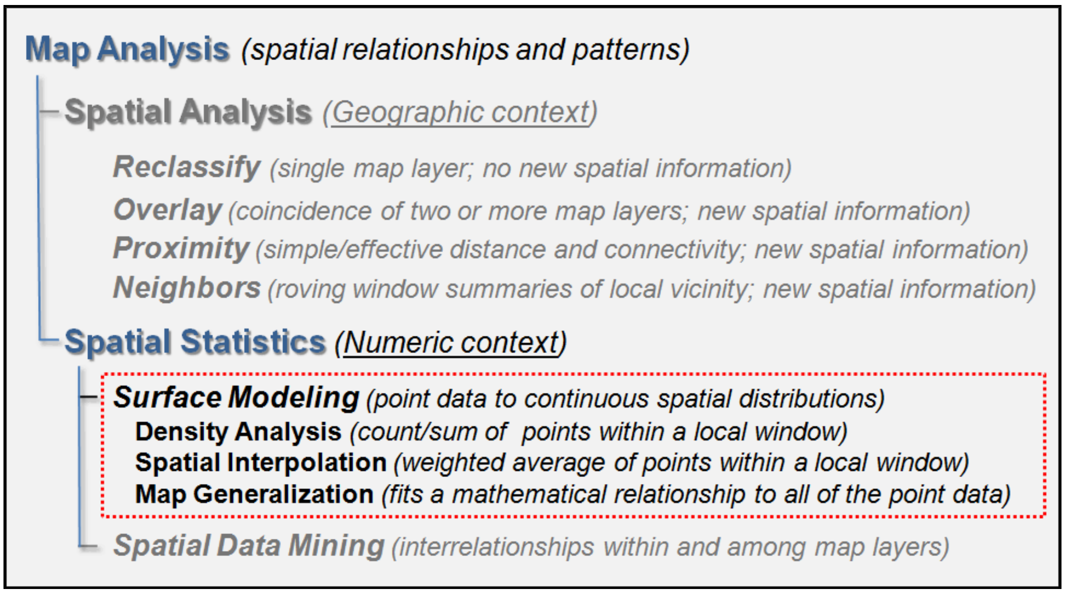


Figure 7. Surface Modelling operations involve creating continuous spatial distributions from point sampled data.

The procedure is analogous to poking pins on a map at each customer location and visually interpreting the pattern of customer clusters. However the result is a quantitative representation of customer data expressed over continuous geographic space. If the point values are summed a "total sales surface" is created (e.g. total sales per square mile). If the values are averaged, an "average sales surface" is created.

The subtle peaks (lots of customers/sales nearby) and valleys (few customers/sales nearby) form a continuous "spatial distribution" that is conceptually similar to a "numerical distribution" that serves as the foundation of traditional statistics. In this instance, three dimensions are used to characterize the data's dispersion—X and Y coordinates to position the data in geographic space and a Z coordinate to indicate the relative magnitude of the variable (i.e., number of customers or total/average sales). From this perspective, surface modeling is analogous to fitting a density function, such as a standard normal curve, in traditional statistics. It translates discrete point samples into a continuous three-dimensional surface that characterizes both the numeric and geographic distribution of the data.

Spatial Interpolation is similar to density analysis as it utilizes spatial patterns in a point sampled data set to generate a continuous map surface. However spatial interpolation seeks to estimate map values instead of simply summarizing them. Conceptually it “maps the variance” in a set of samples by using geographic position to help explain the differences in the sample values. Traditional statistics reduces a set of sample values to a single central tendency value (e.g., average) and a metric of data dispersion about the typical estimate (e.g., standard deviation). In geographic space this scalar estimate translates into a flat plane implying that the average is assumed everywhere, plus or minus the data dispersion. However, non-spatial statistics does not suggest where estimates might be larger or where they might be smaller than the typical estimate. The explicit spatial distribution, on the other hand, seeks to estimate values at every location based on the spatial pattern inherent in the sample set.

All spatial interpolation techniques establish a "roving window" that—

- moves to a grid location in a project area (analysis frame),
- calculates a weighted average of the point samples around it (roving window),
- assigns the weighted average to the center cell of the window, and then
- moves to the next grid location.

The extent of spatial interpolation’s roving window (both size and shape) affects the result, regardless of the summary technique. In general, a large window capturing a larger number of values tends to smooth the data. A smaller window tends to result in a rougher surface with more abrupt transitions.

Three factors affect the window's extent: its reach, the number of samples, and balancing. The reach, or search radius, sets a limit on how far the computer will go in collecting data values. The number of samples establishes how many data values should be used. If there is more than the specified number of values within a specified reach, the computer just uses the closest ones. If there are not enough values, it uses all that it can find within the reach. Balancing of the data attempts to eliminate directional bias by ensuring that the values are selected in all directions around window's center.

The weighted averaging procedure is what determines the different types of spatial interpolation techniques. The Inverse Distance Weighted (IDW) algorithm first uses the Pythagorean Theorem to calculate the Distance from a Grid Location to each of the data samples within the summary window. Then the distances are converted to weights that are inversely proportional to the distance (e.g., $1/D^2$), effectively making more distant locations less influential. The sample values are multiplied by their corresponding computed weights and the “sum of the products” is divided by the “sum of the weights” to calculate a weighted average estimate. The estimate is assigned to center cell location and the process is repeated for all map locations.

The inverse distance procedure used a fixed, geometric-based method to estimate map values at locations that were not sampled. An alternative algorithm, termed Krigging, an advanced spatial statistics technique that uses data trends in the sample data to determine the weights for averaging. Other roving window-based techniques, such as Natural Neighbor, Minimum Curvature, Radial Basis Function and Modified Shepard’s Method, use different techniques to weight the samples within the roving window.

Map Generalization, on the other hand, involves two additional surface modeling approaches that do not use a roving window: Polynomial Surface Fitting and Geometric Facets. Both approaches consider the entire set of sample points in deriving a continuous map surface. In

Polynomial Surface Fitting the degree of the polynomial equation determines the configuration of the final surface. A first degree polynomial fits a plane to the data such that the deviations from the plane to the data are minimized. The result is a tilted plane that indicates the general trend in the data. As higher degree polynomials are used the plane is allowed to bend and warp to better fit the data.

In Geometric Facets interpolation the procedure determines the optimal set of triangles connecting all the points. A Triangulated Irregular Network (TIN) forms a tessellated model based on triangles in which the vertices of the triangles form irregularly spaced nodes that allows dense information in areas where the implied surface is complex, and sparse information in simpler or more homogeneous areas.

While there numerous techniques for characterizing the spatial distribution inherent in a set of point-sampled data there are three critical conditions that govern the results: data type, sampling intensity and sampling pattern. The data must be continuous in both numeric space (interval or ratio) and geographic space (isopleth). Also, the number of samples must be large enough and well-dispersed across the project area to capture inherent geographic pattern in the data.

8.0 **Spatial Statistics Procedures for Spatial Data Mining**

Surface modeling establishes the continuous spatial distribution as map layers in map stack. “Spatial Data Mining” is the process of characterizing numerical interrelationships among the mapped data layers through three basic approaches: descriptive, predictive and prescriptive statistics (figure 8).

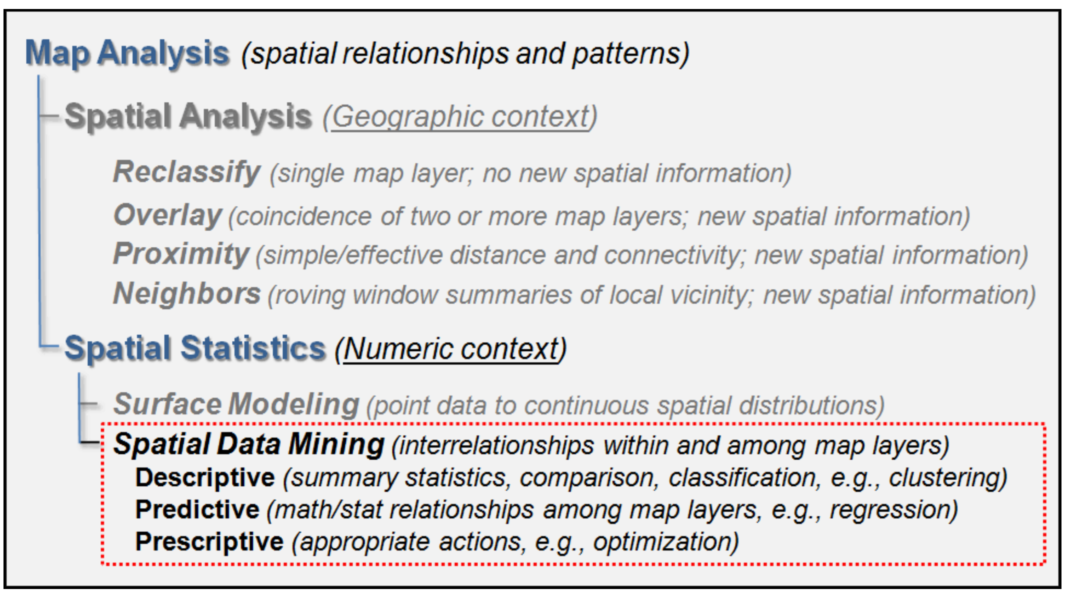


Figure 8. Spatial Data Mining operations involve characterizing numerical patterns and relationships among mapped data.

Basic ***descriptive statistics*** involve calculating the mode, median, mean, range, variance, standard deviation, coefficient of variation and other scalar metrics for all or part of individual maps or a map stack. For example, region-wide overlay may be used to calculate the average parts per million by administrative districts for a spatially interpolated map surface of lead concentrations based on soil samples. A computed coefficient of variation may be included to establish the amount of data dispersion within each district. Or the mean of a weekly time series of airborne lead concentration over a city could be averaged to identify seasonal averages (location-specific overlay).

In a similar manner, two of the weekly maps could be compared by simply subtracting them for the absolute difference or computing the percent change. Another comparison technique could be a Paired *t*-test of the two maps to determine if there is an overall significant difference in the two sets of map values.

Map classification can be simply generalizing a continuous range of map values through contouring or employing sophisticated techniques like the supervised (e.g., maximum likelihood classifier) and unsupervised (e.g., clustering classifier) approaches used in remote sensing to distinguish land cover groups from map stacks of satellite spectral data. These approaches utilize standard multivariate statistics to assess the "data space distance" among individual data patterns within the map stack. If the data distance is small the map value patterns are similar and can be grouped; if the data distance is large separate thematic classes are indicated.

Predictive statistics establish a spatial relationship among a mapped variable and its driving variables, such as sales and demographic data or crop yield and soil nutrient maps. For example, linear regression can be used to derive an equation that relates crop yield (termed the "dependent" mapped variable) to phosphorous, potassium, nitrogen, pH, organic matter and other plant growth factors (termed the "independent" mapped variables).

Once a statistical relationship is established ***prescriptive statistics*** it can be used to generate a "prescription map." In the crop yield example, the equation can be used to predict yield given existing soil nutrient levels and then analyzed to generate a prescription map that identifies the optimal fertilizer application needed at each map location considering the current cost of fertilizer and the estimated market price of the increased yield expected. Another application might derive a regression relationship among product sales (dependent) and demographics (independent) through a test marketing project and then spatially apply the relationship to demographic data in another city to predict sales. The prediction map could be used to derive prescription maps of locations requiring infrastructure upgrade or marketing investment prior to product introduction.

For the most part, spatial data mining employs traditional data analysis techniques. What discriminates it is its data structure, organization and ability to infuse spatial considerations into data analysis. The keystone concept is the analysis frame of grid cells that provides a quantitative representation of the continuous spatial distributions of mapped variables. Also this structure serves as the primary key for linking spatial and non-spatial data sets, such as customer records, or crime incidence, or disease outbreak. With spatial data mining, geographic patterns and relationships within and among map layers are explicitly characterized and utilized in the numerical analyses.

9.0 **Procedures for Communicating Model Logic**

The preceding sections have developed a typology of fundamental map analysis procedures and described a set of spatial analysis (reclassify, overlay distance and neighbours) and spatial statistics (surface modeling and spatial data mining) operations common to broad range of techniques for analyzing mapped data. By systematically organizing these primitive operations, the basis for a generalized GIS modeling approach is identified. This approach accommodates a variety of analytical procedures in a common, flexible and intuitive manner that is analogous to the mathematical of conventional algebra.

As an example of some of the ways in which fundamental map analysis operations may be combined to perform more complex analyses, consider the GIS model outlined in figure 9. Note the format used in the schematic in which "boxes" represent encoded and derived maps and "lines" represent primitive map analysis operations. The flowchart structure indicates the logical sequencing of operations on the mapped data that progresses to the final map. The simplified model depicts the location of an optimal corridor for a highway considering only two criteria: an engineering concern to avoid steep slopes, and a social concern to avoid visual exposure.

It is important to note that the model flowchart is organized such that the "rows" identify the processing of the criteria, while the "columns" identify increasing levels of abstraction from physical base maps to conceptual suitability maps representing relative "goodness," and then to a map of the optimized best route.

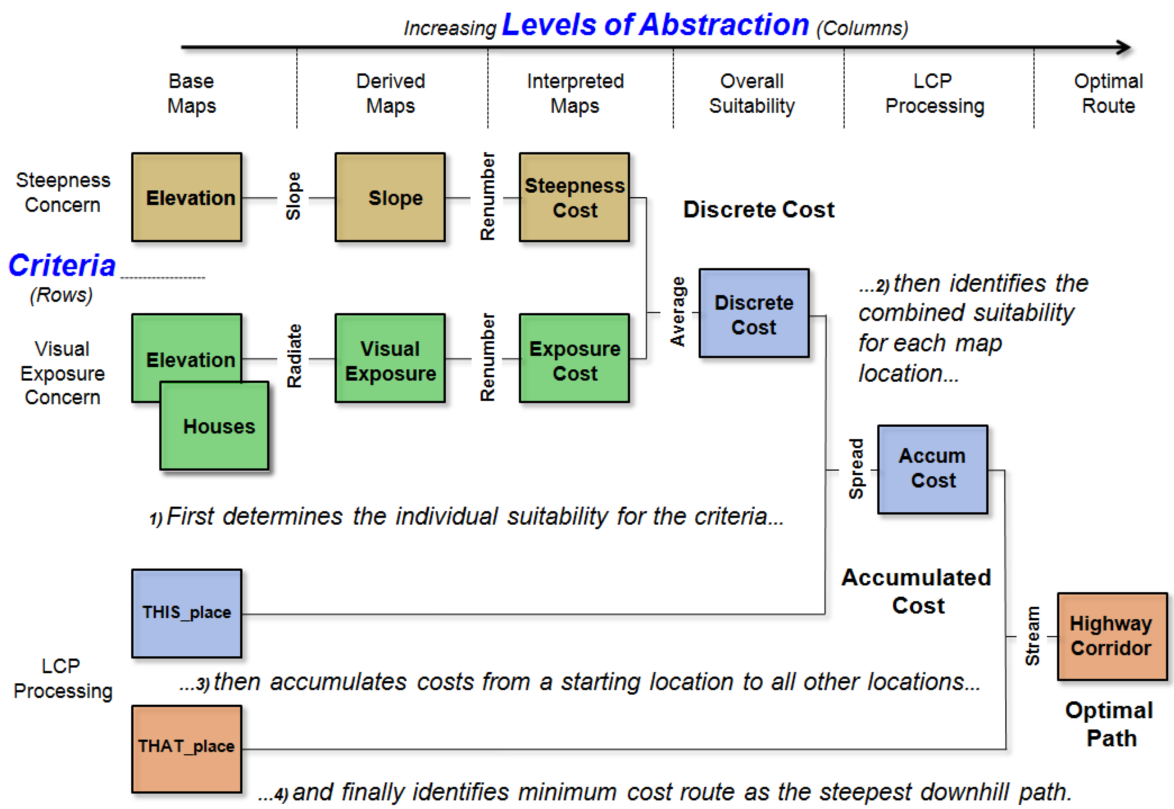


Figure 9. GIS modeling logic can be expressed as a flowchart of processing with "boxes" representing existing/derived maps and "lines" representing map analysis operations. (MapCalc™ software commands indicated)

Given a grid map layer of topographic Elevation values and a map of Houses, the model allocates a least-cost highway alignment between two predetermined locations. Cost is not measured in dollars, but in terms of relative suitability. The top-left portion of the model develops a "discrete cost surface" (identified as the Discrete Cost map) in which each location is assigned a relative cost based on the particular steepness/exposure combination occurring at that location. For example, those areas that are flat and not visible from houses would be assigned low cost values; whereas areas on steep slopes and visually exposed would be assigned high values. This discrete cost surface is used as a map of relative barriers for establishing an accumulated cost surface (Accum Cost) from one of the two termini to all other locations (THISplace). The final step locates the other terminus (THATplace) on the accumulated cost surface and identifies the minimum cost route as the steepest downhill path along the surface from that point to the bottom (i.e., the other end point).

The highway corridor application is representative of a “suitability model” where geographic considerations are translated into preferences for the location of an activity or facility. A “physical model,” on the other hand, characterizes a spatial process, such as overland water movement and subsequent sediment loading estimates to a stream channel. A “statistical model” establishes numerical relationships, such as crop yield throughout a farmer’s field based on the geographic distribution of soil nutrients like phosphorous, potassium and nitrogen. All three types of models use a common set of primitive map analysis operations in a cyclical manner to express solutions to complex spatial problems in a logical framework that is easily communicated.

10.0 **Impact of Spatial Reasoning and Dialog on the Future of Geotechnology**

In addition to the benefits of efficient data management and automated mapping engrained in GIS technology, the map analysis modeling structure has several additional advantages. Foremost among these is the capacity of “dynamic simulation” (i.e., spatial “what if” analysis). For example, the highway corridor model described in the previous section could be executed for several different interpretations of engineering and social criteria. What if the terrain steepness is more important, or what if the visual exposure is twice as important? Where does the least cost route change? Or just as important, where does it not change? From this perspective the model “replies” to user inquiries, rather than “answering” them with a single static map rendering—the processing provides information for decision-making, rather than tacit decisions.

Another advantage to map analysis and modeling is its flexibility. New considerations can be added easily and existing ones refined. For example, the non-avoidance of open water bodies in the highway model is a major engineering oversight. In its current form the model favors construction on lakes, as they are flat and frequently hidden from view. This new requirement can be incorporated readily by identifying open water bodies as absolute barriers (i.e., infinite cost) when constructing the accumulation cost surface. The result will be routing of the minimal cost path around these areas of extreme cost.

GIS modeling also provides an effective structure for communicating both specific application considerations and fundamental principles. The flowchart structure provides a succinct format for communicating the processing considerations of specific applications. Model logic, assumptions and relationships are readily apparent in the box and line schematic of processing flow.

Management of land always has required spatial information as its cornerstone process. Physical description of a management unit allows the conceptualization of its potential usefulness and constrains the list of possible management practices. Once a decision has been made and a plan implemented, additional spatial information is needed to evaluate its effectiveness. This strong allegiance between spatial information and effective management policy will become increasing important due to the exploding complexity of issues our world’s communities. The spatial reasoning ingrained in GIS modeling provides the ability “to truly think with maps” and is poised to take Geotechnology to the next level.

Acknowledgments

The basic framework described in this paper was developed through collaborative efforts with C. Dana Tomlin in the early 1980's in the development of a workbook and numerous workshop presentations on "Geographic Information Analysis" by J. K. Berry and C. D. Tomlin, Yale School of Forestry and Environmental Studies, New Haven, Connecticut, 206 pages.

References

- Berry, J.K. 1985. Computer-assisted map analysis: fundamental techniques. National Association of Computer Graphics, Computer Graphics '85 Conference Proceedings, pgs. 112-130.
- Berry, J.K. 2009. What's in a name? *GeoWorld*, Beyond Mapping column, March, Vol 22, No. 3, pp 12-13.
- Gewin, V. 2004. Mapping opportunities. *Nature* 427, 376-377 (22 January 2004).
- Tomlin, C.D. and J.K. Berry, 1979. A mathematical structure for cartographic modeling in environmental analysis. American Congress on Surveying and Mapping, 39th Symposium Proceedings, pgs. 269-283.
- Tomlin, C.D., 1990. Geographic Information Systems and Cartographic Modeling, Prentice Hall publishers, Englewood Cliffs, New Jersey.

Further Reading

Two primary resources support further study of the map analysis operations and GIS modeling framework described in this paper: the text book Map Analysis and the online book Beyond Mapping III. Table 1 below serves as a cross-listing of the sections in this paper to topics in the referenced resources containing extended discussion and hands-on experience with the concepts, procedures and techniques presented.

In addition, Chapter 29, "GIS Modeling and Analysis," by J.K. Berry in the Manual of Geographic Information Systems, American Society for Photogrammetry & Remote Sensing (ASPRS, in press) contains nearly 100 pages and over 50 illustrations expanding on the framework and material presented in this paper.

Table 1. **Cross-listing to Further Reading Resources**

The **Map Analysis** text is a collection of selected works from of Joseph K. Berry's popular "Beyond Mapping" columns published in GeoWorld magazine from 1996 through 2006. In this compilation Berry develops a structured view of the important concepts, considerations and procedures involved in grid-based map analysis and modeling. The companion CD contains further readings and MapCalc™ software for hands-on experience with the material.

Map Analysis: Understanding Spatial Patterns and Relationships by Joseph K. Berry, 2007, GeoTec Media Publishers, 224 pages, 105 figures; US$45 plus shipping and handling. For more information and ordering, see **www.geoplace.com/books/mapanalysis/**.

The **Beyond Mapping III** online book is posted at **www.innovativegis.com/basis/MapAnalysis/** and contains 28 topics on map analysis and GIS modeling. Over 350 pages and nearly 200 illustrations with hyperlinks for viewing figures in high resolution and permission granted for free use of the materials for educational purposes.

Section in this Paper	Map Analysis	Beyond Mapping III
1.0 **Introduction**	**Foreword, Preface, Introduction**	**Introduction**
2.0 **Nature of Discrete versus**	**Topic 1** Data Structure Implications	**Topic 1** Object-Oriented Technology

Continuous Mapped Data	**Topic 2** Fundamental Map Analysis Approaches	**Topic 7** Linking Data Space and Geographic Space **Topic 18** Understanding Grid-based Data
3.0 **Spatial Analysis Procedures for Reclassifying Maps**	**Topic 3** Basic Techniques in Spatial Analysis	**Topic 22** Reclassifying and Overlaying Maps
4.0 **Spatial Analysis Procedures for Overlaying Maps**	**Topic 3** Basic Techniques in Spatial Analysis	**Topic 22** Reclassifying and Overlaying Maps
5.0 **Spatial Analysis Procedures for Establishing Proximity and Connectivity**	**Topic 4** Calculating Effective Distance **Topic 5** Calculating Visual Exposure	**Topic 5** Analyzing Accumulation Surfaces **Topic 6** Analyzing In-Store Shopping Patterns **Topic 13** Creating Variable-Width Buffers **Topic 14** Deriving and Using Travel-Time Maps **Topic 15** Deriving and Using Visual Exposure Maps **Topic 17** Applying Surface Analysis **Topic 19** Routing and Optimal Paths **Topic 20** Surface Flow Modeling **Topic 25** Calculating Effective Distance
6.0 **Spatial Analysis Procedures for Depicting Neighborhoods**	**Topic 6** Summarizing Neighbors	**Topic 9** Analyzing Landscape Patterns **Topic 11** Characterizing Micro Terrain Features **Topic 26** Assessing Spatially-Defined Neighborhoods
7.0 **Spatial Statistics Procedures for Surface Modeling**	**Topic 9** Basic Techniques in Spatial Statistics	**Topic 2** Spatial Interpolation Procedures and Assessment **Topic 3** Considerations in Sampling Design **Topic 8** Investigating Spatial Dependency **Topic 24** Overview of Spatial Analysis and Statistics
8.0 **Spatial Statistics Procedures for Spatial Data Mining**	**Topic 10** Spatial Data Mining	**Topic 10** Analyzing Map Similarity and Zoning **Topic 16** Characterizing Patterns and Relationships **Topic 28** Spatial Data Mining in Geo-business
9.0 **Procedures for Communicating Model Logic**	**Topic 7** Basic Spatial Modeling Approaches **Topic 8** Spatial Modeling Example	**Topic 23** Suitability Modeling
10.0 **Impact of Spatial Reasoning and Dialog on the Future of Geotechnology**	**Epilogue**	**Topic 4** Where Is GIS Education? **Topic 12** Landscape Visualization **Topic 21** Human Dimensions of GIS **Topic 27** GIS Evolution and Future Trends **Epilogue**

In addition, a book chapter expanding on the framework described in this paper is posted online. *"GIS Modeling and Analysis"* by Joseph K. Berry in the Manual of Geographic Information Systems edited by Marguerite Madden, 2009, American Society for Photogrammetry, Bethesda, Maryland, USA (Section 5, Chapter 29, pages 527-585; ISBN 1-57083-086-X); **www.innovativegis.com/basis/Papers/Other/ASPRSchapter/**.

Finally, the Beyond Mapping Compilation Series by Joseph K. Berry is a collection of Beyond Mapping columns appearing in GeoWorld (formally GIS World) magazine from **March 1989** through **December 2013** that contains nearly 1000 pages and more than 750 figures providing a comprehensive and longitudinal perspective of the underlying concepts, considerations, issues and evolutionary development of modern geotechnology is posted at **www.innovativegis.com/basis/BeyondMappingSeries**.

Selected Additional Resources on grid-based map analysis include:

- DeMers, M. N. 2001. GIS Modeling in Raster, John Wiley & Sons, Ltd. Publisher, West Sussex, England.
- de Smith, M. J., M.F. Goodchild, and P.A Longley 2007. Geospatial Analysis: A Comprehensive Guide to Principles, Techniques and Software Tools, Troubador Publishing Ltd., Leicester, UK.
- Fotheringham, S. and P. Rogerson (editors) 1994. Spatial Analysis and GIS (Technical Issues in Geographic Information Systems) CRC Press, Florida.
- Fotheringham, S. and P. Rogerson (editors) 2009. Handbook of Spatial Analysis, Sage Publications Ltd. Publisher.
- Longley, P. A. and M. Batty (editors) 1997. Spatial Analysis: Modelling in a GIS Environment, John Wiley & Sons, Ltd. Publisher, West Sussex, England.
- Longley, P. A., M. F. Goodchild, D.J Maguire and D.W. Rhind 2001. Geographic Information Systems and Science, John Wiley & Sons, Ltd. Publisher, West Sussex, England.
- Longley, P. A. and M. Batty (editors) 2003. Advanced Spatial Analysis: The CASA Book of GIS, ESRI Press, Redlands, California.
- Maguire D. and M. Batty (editors) 2005. GIS, Spatial Analysis and Modeling, ESRI Press, Redlands, California.
- Mitchell, A. 1999. The ESRI Guide to GIS Analysis Volume 1: Geographic Patterns & Relationships, ESRI Press, Redlands, California.
- Mitchell, A. 2005. The ESRI Guide to GIS Analysis: Volume 2: Spatial Measurements and Statistics, ESRI Press, Redlands, California.