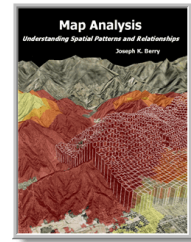


## Beyond Mapping III

# Topic 25: Calculating Effective Distance and Connectivity



[Map Analysis](#) book with companion CD-ROM for hands-on exercises and further reading

[Measuring Distance Is Neither Here nor There](#) — discusses the basic concepts of distance and proximity

[Use Cells and Rings to Calculate Simple Proximity](#) — describes how simple proximity is calculated

[Extend Simple Proximity to Effective Movement](#) — discusses the concept of effective distance responding to relative and absolute barriers

[Calculate and Compare to Find Effective Proximity](#) — describes how effective proximity is calculated

[Taking Distance to the Edge](#) — discusses advance distance operations

[Advancing the Concept of Effective Distance](#) — describes the algorithms used in implementing Starter value advanced techniques

[A Dynamic Tune-up for Distance Calculations](#) — describes the algorithms used in implementing dynamic effective distance procedures involving intervening conditions

[A Narrow-minded Approach](#) — describes how Narrowness maps are derived

[Narrowing-In on Absurd Gerrymanders](#) — discusses how a Narrowness Index can be applied to assess redistricting configurations

[Just How Crooked Are Things?](#) — discusses distance-related metrics for assessing crookedness

*Note: The processing and figures discussed in this topic were derived using MapCalc™ software. See [www.innovativegis.com](http://www.innovativegis.com) to download a free MapCalc Learner version with tutorial materials for classroom and self-learning map analysis concepts and procedures.*

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## Measuring Distance Is Neither Here nor There

(GeoWorld, April 2005, pg. 18-19)

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Measuring distance is one of the most basic map analysis techniques. Historically, *distance* is defined as *the shortest straight-line between two points*. While this three-part definition is both

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easily conceptualized and implemented with a ruler, it is frequently insufficient for decision-making. A straight-line route might indicate the distance “as the crow flies,” but offer little information for the walking crow or other flightless creature. It is equally important to most travelers to have the measurement of distance expressed in more relevant terms, such as time or cost.

The limitation of a map analysis approach is not so much in the concept of distance measurement, but in its implementation. Any measurement system requires two components—a standard *unit* and a *procedure* for measurement. Using a ruler, the “unit” is the smallest hatching along its edge and the “procedure” is the line implied by aligning the straightedge. In effect, the ruler represents just one row of a grid implied to cover the entire map. You just position the grid such that it aligns with the two points you want measured and count the squares (top portion of figure 1). To measure another distance you merely realign the implied grid and count again.

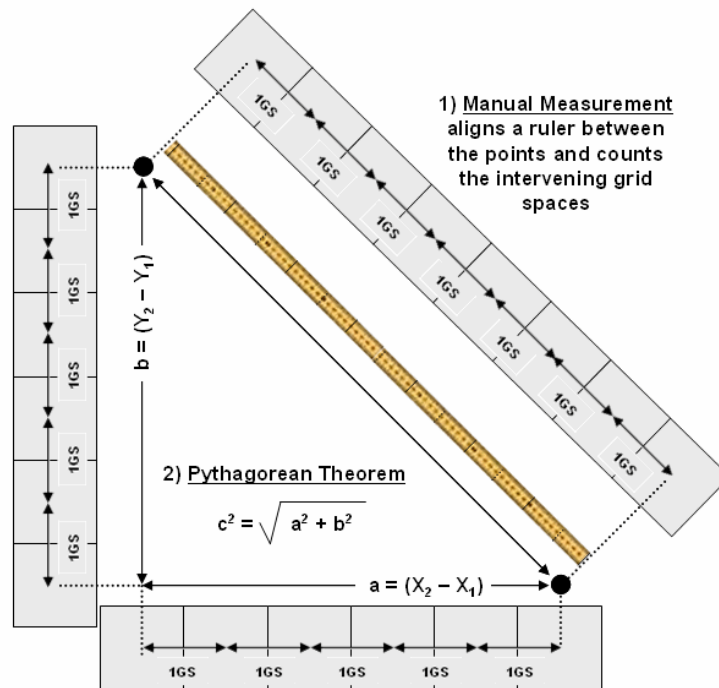


Figure 1. Both Manual Measurement and the Pythagorean Theorem use grid spaces as the fundamental units for determining the distance between two points.

In a GIS (and grade school geometry) the grid is a fixed reference and distance is calculated as the hypotenuse of a right triangle formed by the grid's rows and columns (bottom portion of figure 1). Yet, this mathematical procedure is often too limited in both its computer implementation and information content.

**Proximity** establishes the distance to all locations surrounding a point—the set of *shortest*

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*straight-lines among groups of points.* Rather than sequentially computing the distance between pairs of locations, concentric equidistance zones are established around a location or set of locations (figure 2). This procedure is similar to the wave pattern generated when a rock is thrown into a still pond. Each ring indicates one “unit farther away”— increasing distance as the wave moves away. Another way to conceptualize the process is nailing one end of a ruler at a point and spinning it around. The result is a series of “data zones” emanating from a location and aligning with the ruler’s tic marks.

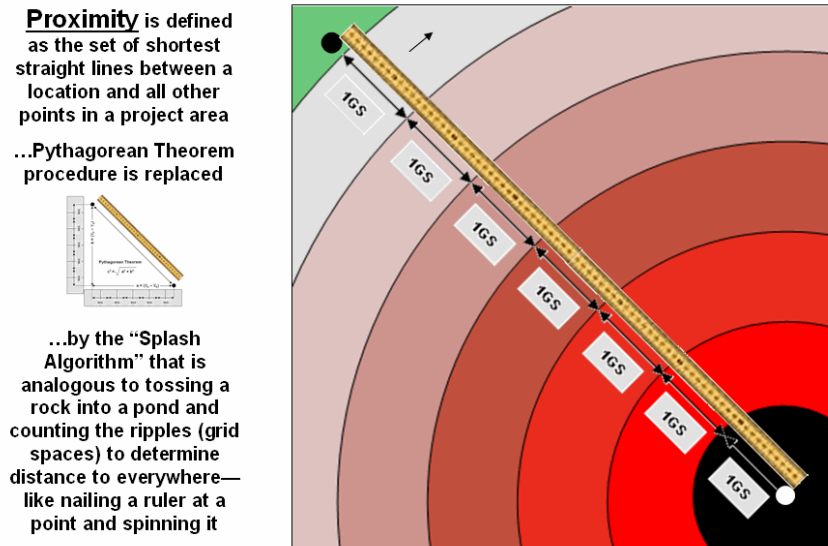


Figure 2. Proximity identifies the set of shortest straight-lines among groups of points (distance zones).

However, nothing says proximity must be measured from a single point. A more complex proximity map would be generated if, for example, all locations with houses (set of points) are simultaneously considered target locations (right side of figure 3).

In effect, the procedure is like throwing a handful of rocks into pond. Each set of concentric rings grows until the wave fronts meet; then they stop. The result is a map indicating the shortest straight-line distance to the nearest target location (house) for each non-target location. In the figure, the red tones indicate locations that are close to a house, while the green tones identify areas that are far from a house.

In a similar fashion, a proximity map to roads is generated by establishing data zones emanating from the road network—sort of like tossing a wire frame into a pond to generate a concentric pattern of ripples (middle portion of figure 3). The same result is generated for a set of areal features, such as sensitive habitat parcels (right side of figure 3).

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## Simple Proximity Surfaces

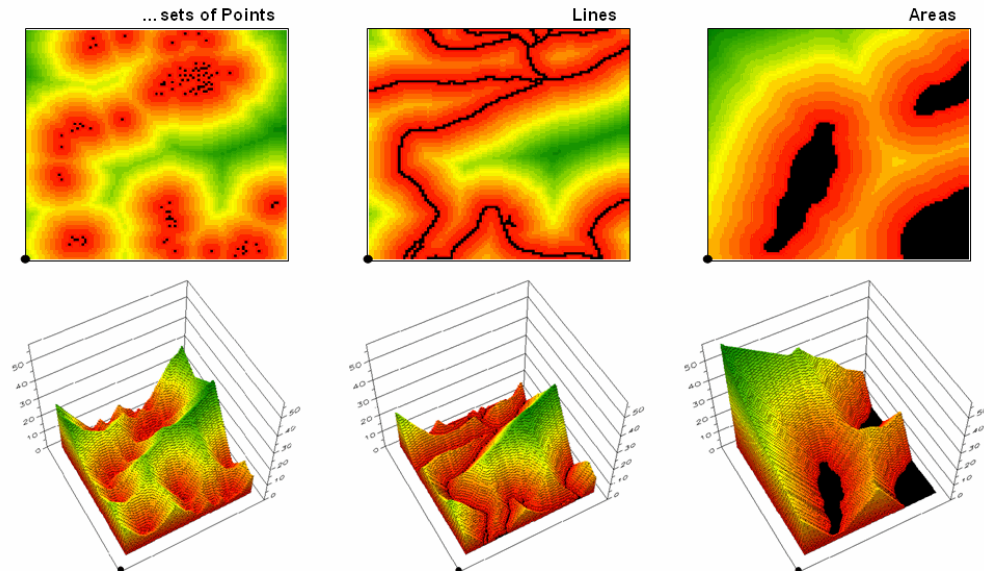


Figure 3. Proximity surfaces can be generated for groups of points, lines or polygons identifying the shortest distance from all location to the closest occurrence.

It is important to note that proximity is not the same as a buffer. A buffer is a discrete spatial object that identifies areas that are within a specified distance of map feature; all locations within a buffer are considered the same. Proximity is a continuous surface that identifies the distance to a map feature(s) for every location in a project area. It forms a gradient of distances away composed of many map values; not a single spatial object with one characteristic distance away.

The 3D plots of the proximity surfaces in figure 3 show detailed gradient data and are termed *accumulated surfaces*. They contain increasing distance values from the target point, line or area locations displayed as colors from red (close) to green (far). The starting features are the lowest locations (black= 0) with hillsides of increasing distance and forming ridges that are equidistant from starting locations. Next month will focus on how proximity is calculated—conceptually easy but way too much bookkeeping for even the most ardent accountant.

## Use Cells and Rings to Calculate Simple Proximity

(GeoWorld, May 2005, pg. 18-19)

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The last section established that proximity is measured by a series of propagating rings emanating from a starting location—splash algorithm. Since the reference grid is a set of square

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grid cells, the rings are formed by concentric sets of cells. In figure 1, the first “ring” is formed by the three cells adjoining the starting cell in the lower-right corner. The top and side cells represent orthogonal movement while upper-left one is diagonal. The assigned distance of the steps reflect the type of movement—orthogonal equals 1.000 and diagonal equals 1.414.

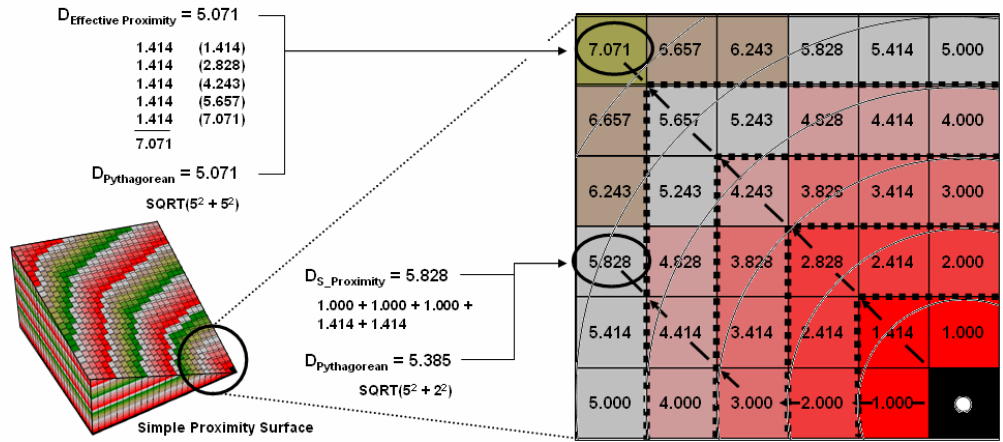


Figure 1. Simple proximity is generated by summing a series of orthogonal and diagonal steps emanating from a starting location.

As the rings progress, 1.000 and 1.414 are added to the previous accumulated distances resulting in a matrix of proximity values. The value 7.01 in the extreme upper-left corner is derived by adding 1.414 for five successive rings (all diagonal steps). The other two corners are derived by adding 1.000 five times (all orthogonal steps). In these cases, the effective proximity procedure results in the same distance as calculated by the Pythagorean Theorem.

Reaching other locations involve combinations of orthogonal and diagonal steps. For example, the other location in the figure uses three orthogonal and then two diagonal steps to establish an accumulated distance value of 5.828. The Pythagorean calculation for same location is 5.385. The difference ( $5.828 - 5.385 = .443 / 5.385 = 8\%$ ) is due to the relatively chunky reference grid and the restriction to grid cell movements.

Grid-based proximity measurements tend to overstate true distances for off-orthogonal/diagonal locations. However, the error becomes minimal with distance and use of smaller grids. And the utility of the added information in a proximity surface often outweighs the lack of absolute precision of simple distance measurement.

Figure 2 shows the calculation details for the remaining rings. For example, the larger inset on the left side of the figure shows ring 1 advancing into the second ring. All forward movements from the cells forming the ring into their adjacent cells are considered. Note the multiple paths that can reach individual cells. For example, movement into the top-right corner cell can be an orthogonal step from the 1.000 cell for an accumulated distance of 2.000. Or it can be reached

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by a diagonal step from the 1.414 cell for an accumulated distance of 2.828. The smaller value is stored in compliance with the idea that distance implies “shortest.” If the spatial resolution of the analysis grid is 300m then the ground distance is  $2.000 * 300\text{m}/\text{gridCell} = 600\text{m}$ . In a similar fashion, successive ring movements are calculated, added to the previous ring’s stored values and the smallest of the potential distance values being stored. The distance waves rapidly propagate throughout the project area with the shortest distance to the starting location being assigned at every location.

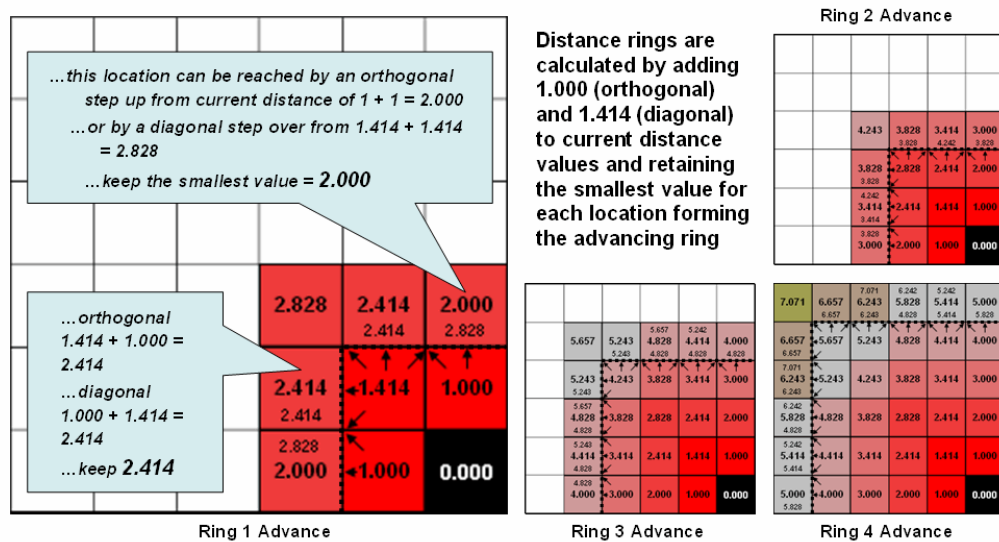


Figure 2. Simple distance rings advance by summing 1.000 or 1.414 grid space movements and retaining the minimal accumulated distance of the possible paths.

If more than one starting location is identified, the proximity surface for the next starter is calculated in a similar fashion. At this stage every location in the project area has two proximity values—the current proximity value and the most recent one (figure 3). The two surfaces are compared and the smallest value is retained for each location—distance to closest starter location. The process is repeated until all of the starter locations representing sets of points, lines or areas have been evaluated.

While the computation is overwhelming for humans, the repetitive nature of adding constants and testing for smallest values is a piece of cake for computers (millions of iterations in a few seconds). More importantly, the procedure enables a whole new way of representing relationships in spatial context involving “effective distance” that responds to realistic differences in the characteristics and conditions of movement throughout geographic space.

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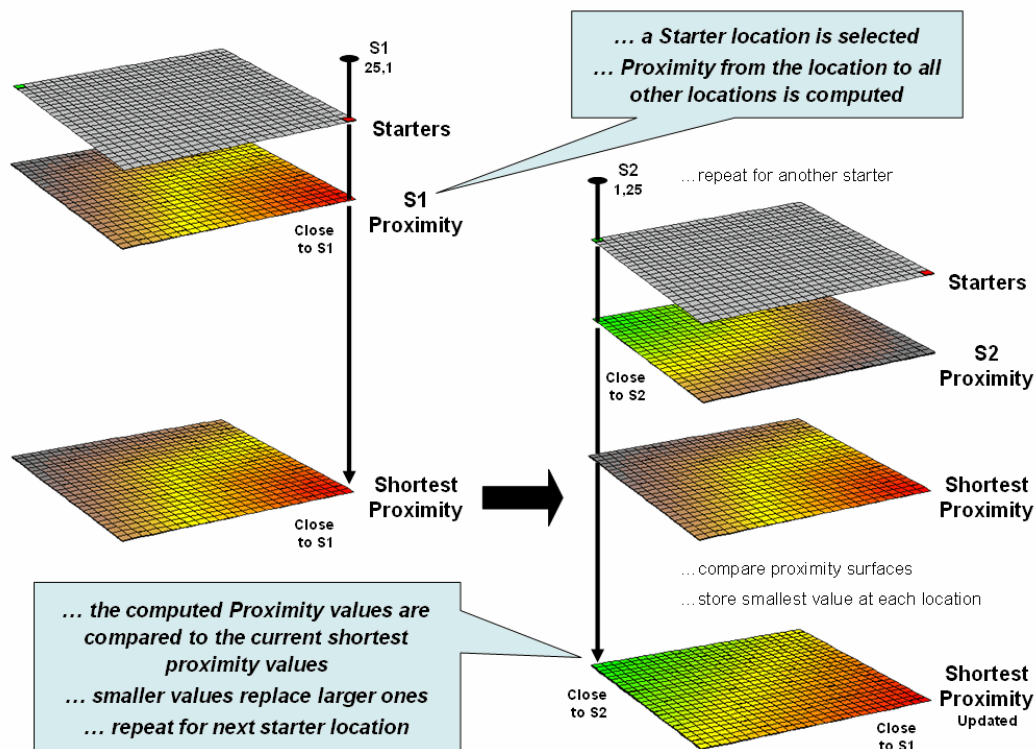


Figure 3. Proximity surfaces are compared and the smallest value is retained to identify the distance to the closest starter location.

## Extend Simple Proximity to Effective Movement

(GeoWorld, June 2005, pg. 18-19)

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Last section's discussion suggested that in many applications, the shortest route between two locations might not always be a straight-line. And even if it is straight, its geographic length may not always reflect a traditional measure of distance. Rather, distance in these applications is best defined in terms of "movement" expressed as travel-time, cost or energy that is consumed at rates that vary over time and space. Distance modifying effects involve weights and/or barriers— concepts that imply the relative ease of movement through geographic space might not always constant.

Figure 1 illustrates one of the effects of distance being affected by a *movement characteristic*. The left-side of the figure shows the simple proximity map generated when both starting locations are considered to have the same characteristics or influence. Note that the midpoint

(dark green) aligns with the perpendicular bisector of the line connecting the two points and confirms a plane geometry principle you learned in junior high school.

The right-side of the figure, on the other hand, depicts effective proximity where the two starting locations have different characteristics. For example, one store might be considered more popular and a “bigger draw” than another (Gravity Modeling). Or in old geometry terms, the person starting at S1 hikes twice as fast as the individual starting at S2—the weighted bisector identifies where they would meet. Other examples of weights include attrition where movement changes with time (e.g., hiker fatigue) and change in mode (drive a vehicle as far as possible then hike into the off-road areas).

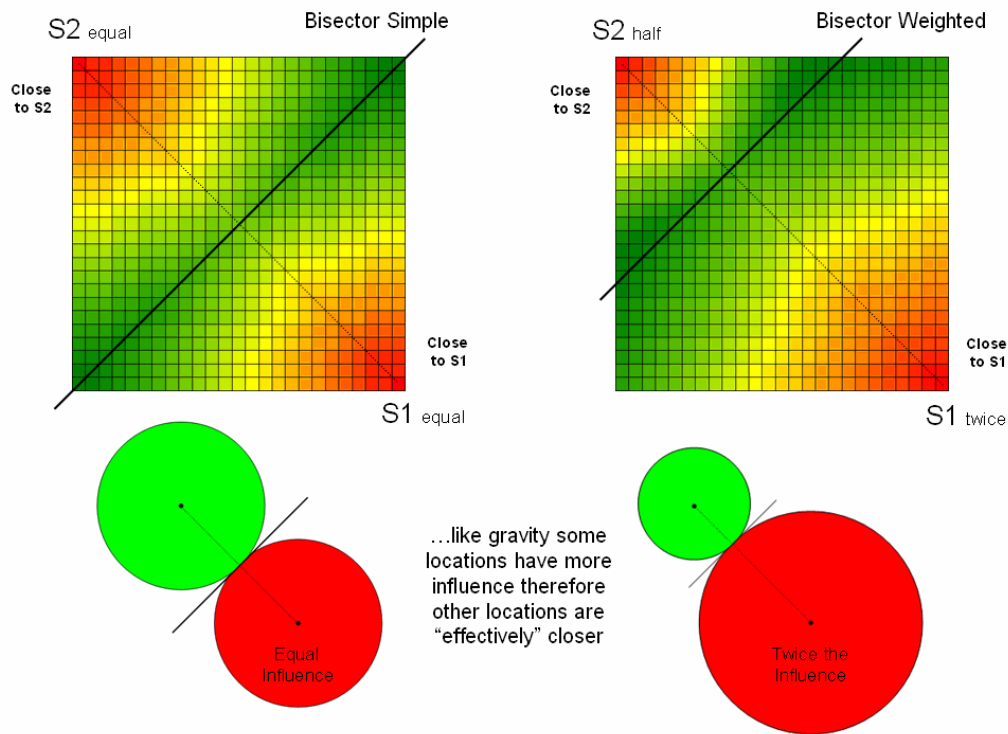


Figure 1. Weighting factors based on the characteristics of movement can affect relative distance, such as in Gravity Modeling where some starting locations exert more influence.

In addition to weights that reflect movement characteristics, effective proximity responds to intervening conditions or barriers. There are two types of barriers that are identified by their effects— absolute and relative. *Absolute barriers* are those completely restricting movement and therefore imply an infinite distance between the points they separate. A river might be regarded as an absolute barrier to a non-swimmer. To a swimmer or a boater, however, the same river might be regarded as a *relative barrier* identifying areas that are passable, but only at a cost which can be equated to an increase in geographical distance. For example, it might take five times longer to row a hundred meters than to walk that same distance.

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In the conceptual framework of tossing a rock into a pond, the waves can crash and dissipate against a jetty extending into the pond (absolute barrier; no movement through the grid spaces). Or they can proceed, but at a reduced wavelength through an oil slick (relative barrier; higher cost of movement through the grid spaces). The waves move both around the jetty and through the oil slick with the ones reaching each location first identifying *the set of shortest, but not necessarily straight-lines among groups of points*.

The shortest routes respecting these barriers are often twisted paths around and through the barriers. The GIS database enables the user to locate and calibrate the barriers; the wave-like analytic procedure enabling the computer to keep track of the complex interactions of the waves and the barriers. For example, figure 2 shows the effective proximity surfaces for the same set of starter locations discussed in the first section in this series (section 1, [figure 3](#)).

### Effective Proximity Surfaces

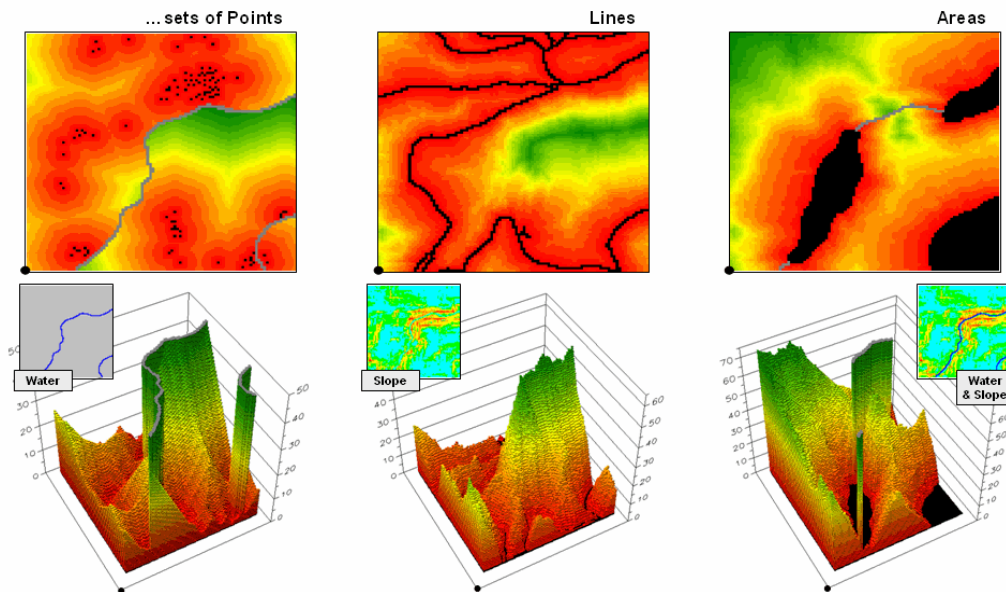


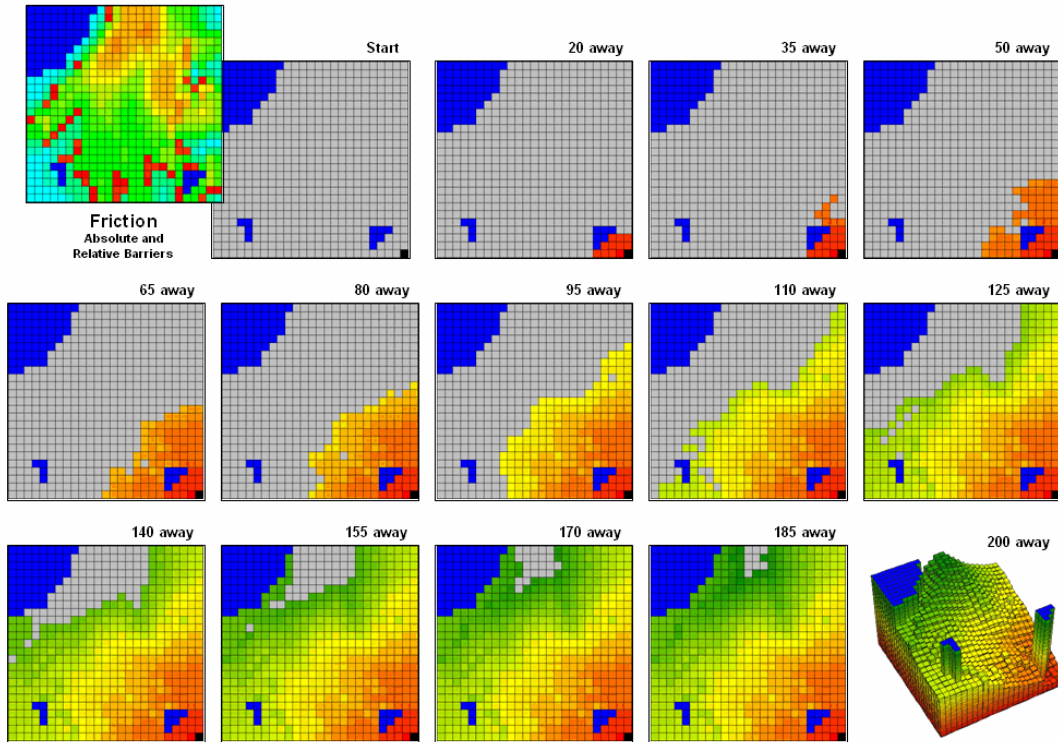
Figure 2. Effective Proximity surfaces consider the characteristics and conditions of movement throughout a project area.

The point features in the left inset respond to treating flowing water as an absolute barrier to movement. Note that the distance to the nearest house is very large in the center-right portion of the project area (green) although there is a large cluster of houses just to the north. Since the water feature can't be crossed, the closest houses are a long distance to the south.

Terrain steepness is used in the middle inset to illustrate the effects of a relative barrier. Increasing slope is coded into a friction map of increasing impedance values that make movement through steep grid cells effectively farther away than movement through gently sloped locations. Both absolute and relative barriers are applied in determining effective

proximity sensitive areas in the right inset.

The dramatic differences between the concept of distance “as the crow flies” (simple proximity) and “as the crow walks” (effective proximity) is a bit unfamiliar and counter-intuitive. However, in most practical applications, the comfortable assumption that all movement occurs in straight lines totally disregards reality. When traveling by trains, planes, automobiles, and feet there are plenty of bends, twists, accelerations and decelerations due to characteristics (weights) and conditions (barriers) of the movement.



*Figure 3. Effective Distance waves are distorted as they encounter absolute and relative barriers, advancing faster under easy conditions and slower in difficult areas.*

Figure 3 illustrates how the splash algorithm propagates distance waves to generate an effective proximity surface. The Friction Map locates the absolute (blue/water) and relative (light blue=gentle/easy through red= steep/hard) barriers. As the distance wave encounters the barriers their effects on movement are incorporated and distort the symmetric pattern of simple proximity waves. The result identifies the “shortest, but not necessarily straight” distance connecting the starting location with all other locations in a project area.

Note that the absolute barrier locations (blue) are set to infinitely far away and appear as pillars in the 3-D display of the final proximity surface. As with simple proximity, the effective distance values form a bowl-like surface with the starting location at the lowest point (zero away)

from itself) and then ever-increasing distances away (upward slope). With effective proximity, however, the bowl is not symmetrical and is warped with bumps and ridges that reflect intervening conditions—the greater the impedance the greater the upward slope of the bowl. In addition, there can never be a depression as that would indicate a location that is closer to the starting location than everywhere around it. Such a situation would violate the ever-increasing concentric rings theory and is impossible except on Star Trek where Spock and the Captain dematerialize then reappear somewhere else without physically passing through the intervening locations.

## Calculate and Compare to Find Effective Proximity

(GeoWorld, July 2005, pg. 18-19)

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The last couple of sections have focused on how effective distance is measured in a grid-based GIS. Basic to this expanded view of distance is the conceptualization of the measurement process as waves radiating from a location(s) — analogous to the ripples caused by tossing a rock into a pond. As the wave front moves through space, it first checks to see if a potential “step” is passable (absolute barrier locations are not). If not passable, the distance is set to infinitely far away. If passable, the wave front moves there and incurs the “cost” of such a movement identified on the Friction Map (relative barrier values of impedance). As the wave proceeds, all possible paths are considered and the shortest distance assigned to every location in a project area (least total impedance from the starting point).

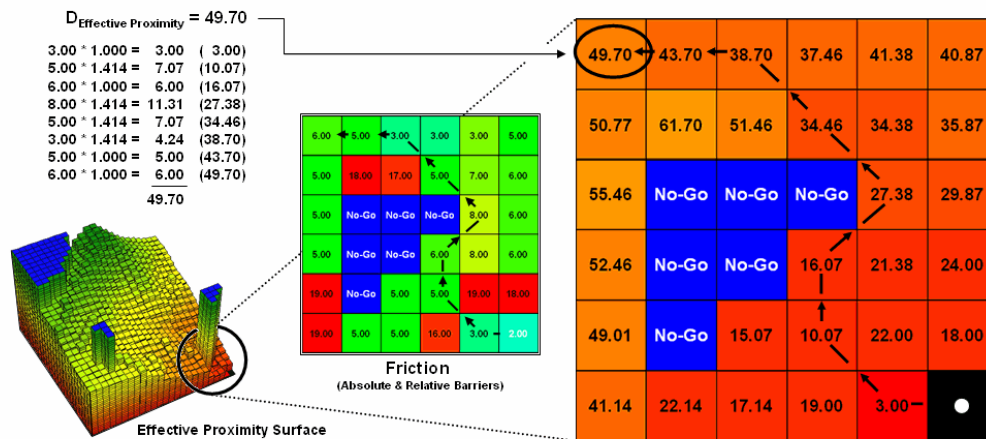


Figure 1. Effective proximity is generated by summing a series of steps that reflect the characteristics and conditions of moving through geographic space.

Figure 1 shows the effective proximity values for a small portion of the results forming the

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proximity surface discussed last month. Manual Measurement, Pythagorean Theorem and Simple Proximity all report that the geographic distance to the location in the upper-right corner is  $5.071 * 300\text{meters}/\text{gridCell} = 1521$  meters. But this simple geometric measure assumes a straight-line connection that crosses extremely high impedance values, as well as absolute barrier locations—an infeasible route that results in exhaustion and possibly death for a walking crow.

The shortest path respecting absolute and relative barriers is shown as first sweeping to the left and then passing around the absolute barrier on the right side. This counter-intuitive route is formed by summing the series of shortest steps at each juncture. The first step away from the starting location is toward the lowest friction and is computed as the impedance value times the type of step for  $3.00 * 1.000 = 3.00$ . The next step is considerably more difficult at  $5.00 * 1.414 = 7.07$  and when added to the previous step's value yields a total effective distance of 10.07. The process of determining the shortest step distance and adding it to the previous distance is repeated over and over to generate the final accumulated distance of the route.

It is important to note that the resulting value of 49.70 can't be directly compared to the 507.1 meters geometric value. Effective proximity is like applying a rubber ruler that expands and contracts as different movement conditions reflected in the Friction Map are encountered. However, the proximity values do establish a relative scale of distance and it is valid to interpret that the 49.7 location is nearly five times farther away than the location containing the 10.07 value.

If the Friction Map is calibrated in terms of a standard measure of movement, such as time, the results reflect that measure. For example, if the base friction unit was 1-minute to cross a grid cell the location would be 49.71 minutes away from the starting location. What has changed isn't the fundamental concept of distance but it has been extended to consider real-world characteristics and conditions of movement that can be translated directly into decision contexts, such as how long will it take to hike from "my cabin to any location" in a project area. In addition, the effective proximity surface contains the information for delineating the shortest route to anywhere—simply retrace to wave front movement that got there first by taking the steepest downhill path over the accumulation surface.

The calculation of effective distance is similar to that of simple proximity, just a whole lot more complicated. Figure 2 shows the set of movement possibilities for advancing from the first ring to the second ring. Simple proximity only considers forward movement whereas effective proximity considers all possible steps (both forward and backward) and the impedance associated with each potential move.

For example, movement into the top-right corner cell can be an orthogonal step times the friction value ( $1.000 * 6.00$ ) from the 18.00 cell for an accumulated distance of 24.00. Or it can be reached by a diagonal step times the friction value ( $1.414 * 6.00$ ) from the 19.00 cell for an accumulated distance of 30.48. The smaller value is stored in compliance with the idea that distance implies "shortest." The calculations in the blue panels show locations where a forward

step from ring 1 is the shortest, whereas the yellow panels show locations where backward steps from ring 2 are shorter.

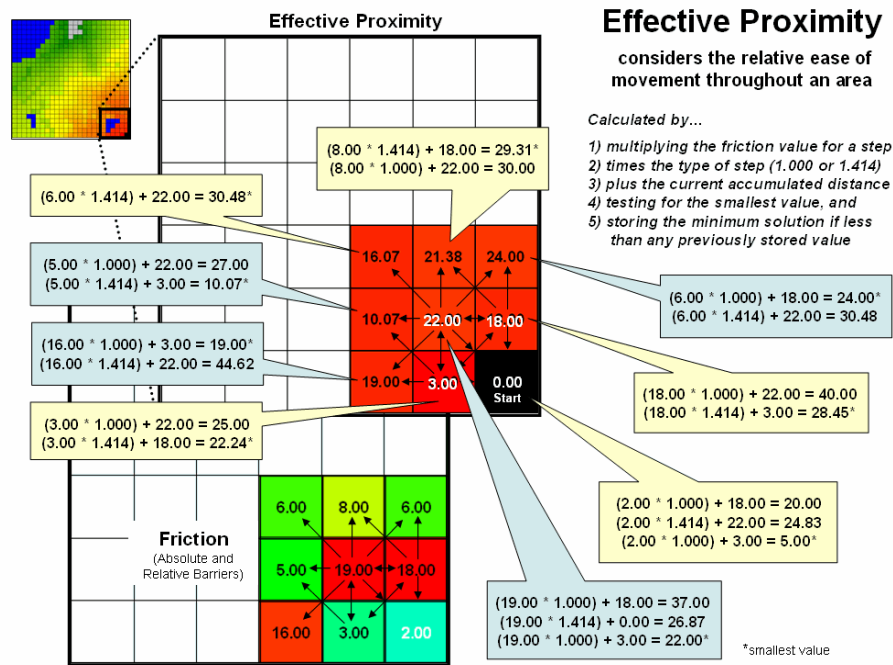


Figure 2. Effective distance rings advance by summing the friction factors times the type of grid space movements and retaining the minimal accumulated distance of the possible paths.

The explicit procedure for calculating effective distance in the example involves:

- Step 1) multiplying the friction value for a step
- Step 2) times the type of step (1.000 or 1.414)
- Step 3) plus the current accumulated distance
- Step 4) testing for the smallest value, and
- Step 5) storing the minimum solution if less than any previously stored value.

Extending the procedure to consider movement characteristics merely introduces an additional step at the beginning—multiplying the relative weight of the starter.

The complete procedure for determining effective proximity from two or more starting locations is graphically portrayed in figure 3. Proximity values are calculated from one location then another and stored in two matrices. The values are compared on a cell-by-cell basis and the shortest value is retained for each instance. The “calculate then compare” process is repeated for other starting locations with the working matrix ultimately containing the shortest distance values, regardless which starter location is closest. Piece-of-cake for a computer.

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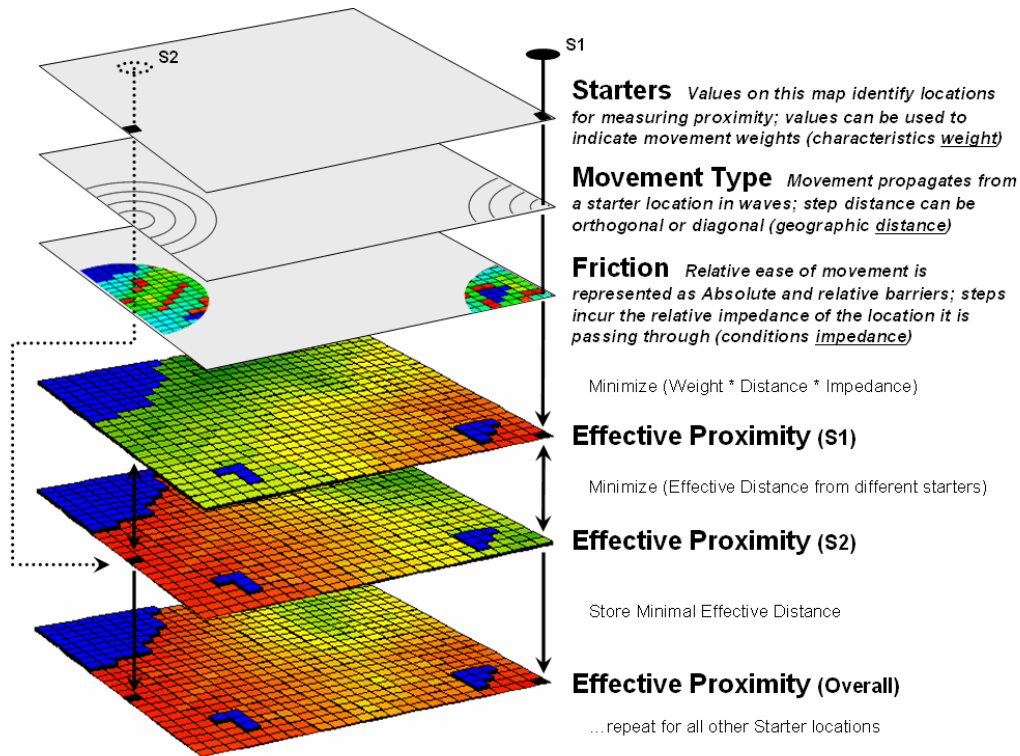


Figure 3. Effective proximity surfaces are computed respecting movement weights and impedances then compared and the smallest value is retained to identify the distance to the closest starter location.

## Taking Distance to the Edge

(GeoWorld, August 2005, pg. 18-19)

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The past series of four sections have focused on how simple distance is extended to effective proximity and movement in a modern GIS. Considerable emphasis was given to the calculations involving a propagating wave of increasing distance (algorithm) instead of our more familiar procedures of measuring with a ruler (manual) or solving the Pythagorean Theorem (mathematical).

While the computations of simple and effective proximity might be unfamiliar and appear complex, once programmed they are easily and quickly performed by modern computers. In addition, there is a rapidly growing wealth of digital data describing conditions that impact movement in the real world. It seems that all is in place for a radical rethinking and expression of distance—computers, programs and data are poised.

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However, what seems to be the major hurdle for adoption of this new way of spatial thinking lies in the experience base of potential users. Our paper map legacy suggests that the “shortest straight line between two points” is the only way to investigate spatial context relationships and anything else is disgusting (or at least uncomfortable).

This restricted perspective has led most contemporary GIS applications to employ simple distance and buffers. While simply automating traditional manual procedures might be comfortable, it fails to address the reality of complex spatial problems or fully engage the potential of GIS technology.

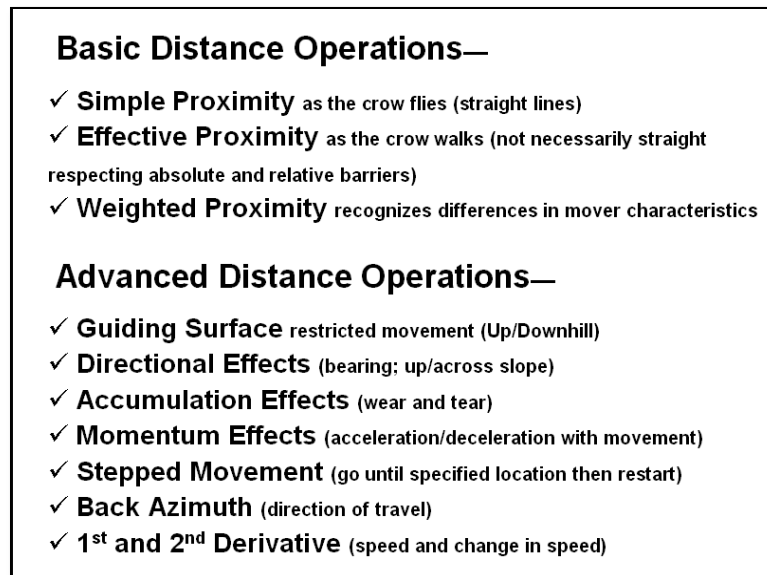


Figure 1. Extended list of advanced distance operations.

The first portion of figure 1 identifies the basic operations described in the previous sections. Our traditional thinking of distance as the “shortest, straight line between two points” is extended to **Simple Proximity** by relaxing the assumption that all movement is just between two points. **Effective Proximity** relaxes the requirement that all movement occurs in straight lines. **Weighted Proximity** extends the concept of static geographic distance by accounting for different movement characteristics, such as speed.

The result is a new definition of distance as the “shortest, not necessarily straight set of connections among all points.” While this new definition may seem awkward it is more realistic as very few things move in a straight line. For example, it has paved the way for online driving directions from your place to anywhere ...an impossible task for a ruler or Pythagoras.

In addition, the new procedures have set the stage for even more advanced distance operations (lower portion of figure 1). A **Guiding Surface** can be used to constrain movement up, down or across a surface. For example, the algorithm can check an elevation surface and only proceed to

downhill locations from a feature such as roads to identify areas potentially affected by the wash of surface chemicals applied.

The simplest *Directional Effect* involves compass directions, such as only establishing proximity in the direction of a prevailing wind. A more complex directional effect is consideration of the movement with respect to an elevation surface—a steep uphill movement might be considered a higher friction value than movement across a slope or downhill. This consideration involves a dynamic barrier that the algorithm must evaluate for each point along the wave front as it propagates.

*Accumulation Effects* account for wear and tear as movement continues. For example, a hiker might easily proceed through a fairly steep uphill slope at the start of a hike but balk and pitch a tent at the same slope encountered ten hours into a hike. In this case, the algorithm merely “carries” an equation that increases the static/dynamic friction values as the movement wave front progresses. A natural application is to have a user enter their gas tank size and average mileage into MapQuest so it would automatically suggest refilling stops along your vacation route.

A related consideration, *Momentum Effects*, tracks the total effective distance but in this instance it calculates the net effect of up/downhill conditions that are encountered. It is similar to a marble rolling over an undulating surface—it picks up speed on the downhill stretches and slows down on the uphill ones. In fact, this was one of my first spatial exercises in computer programming in the 1970s. The class had to write a program that determined the final distance and position of a marble given a starting location, momentum equation based on slope and a relief matrix ...all in unstructured FORTRAN.

The remaining three advanced operations interact with the accumulation surface derived by the wave front’s movement. Recall that this surface is analogous to football stadium with each tier of seats being assigned a distance value indicating increasing distance from the field. In practice, an accumulation surface is a twisted bowl that is always increasing but at different rates that reflect the differences in the spatial patterns of relative and absolute barriers.

*Stepped Movement* allows the proximity wave to grow until it reaches a specified location, and then restart at that location until another specified location and so on. This generates a series of effective proximity facets from the closest to the farthest location. The steepest downhill path over each facet, as you might recall, identifies the optimal path for that segment. The set of segments for all of the facets forms the optimal path network connecting the specified points.

The direction of optimal travel through any location in a project area can be derived by calculating the *Back Azimuth* of the location on the accumulation surface. Recall that the wave front potentially can step to any of its eight neighboring cells and keeps track of the one with the least “friction.” The aspect of the steepest downhill step (N, NE, E, SE, S, SW, W or NW) at any location on the accumulation surface therefore indicates the direction of the best path



through that location. In practice there are two directions—one in and one out for each location.

An even more bazaar extension is the interpretation of the *1<sup>st</sup> and 2<sup>nd</sup> Derivative* of an accumulation surface. The 1<sup>st</sup> derivative (rise over run) identifies the change in accumulated value (friction value) per unit of geographic change (cell size). On a travel-time surface, the result is the speed of optimal travel across the cell. The second derivative generates values whether the movement at each location is accelerating or decelerating.

Chances are these extensions to distance operations seem a bit confusing, uncomfortable, esoteric and bordering on heresy. While the old “straight line” procedure from our paper map legacy may be straight forward, it fails to recognize the reality that most things rarely move in straight lines.

Effective distance recognizes the complexity of realistic movement by utilizing a procedure of propagating proximity waves that interact with a map indicating relative ease of movement. Assigning values to relative and absolute barriers to travel enable the algorithm to consider locations to favor or avoid as movement proceeds. The basic distance operations assume static conditions, whereas the advanced ones account for dynamic conditions that vary with the nature of the movement.

So what’s the take home from this series describing effective distance? Two points seem to define the bottom line. First, that the digital map is revolutionizing how we perceive distance, as well as how calculate it. It is the first radical change since Pythagoras came up with his theorem about 2,500 years ago. Secondly, the ability to quantify effective distance isn’t limited by computational power or available data; rather our difficulties in understanding accepting the concept. Hopefully the discussions have shed some light on this rethinking of distance measurement.

## ***Advancing the Concept of Effective Distance***

*(GeoWorld, February 2011)*

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The previous section described several advanced distance procedures. This and the next section expand on those discussions by describing the algorithms used in implementing the advanced grid-based techniques.

The top portion of figure 1 shows the base maps and procedure used in deriving Static Effective Distance. The “Starter” map identifies the locations from which distance will be measured, and their row, column coordinates are entered into a data stack for processing. The “Friction,” or

discrete cost map, notes conditions that impede movement within a project area—“absolute” barriers prohibit, while “relative” barriers restrict movement.

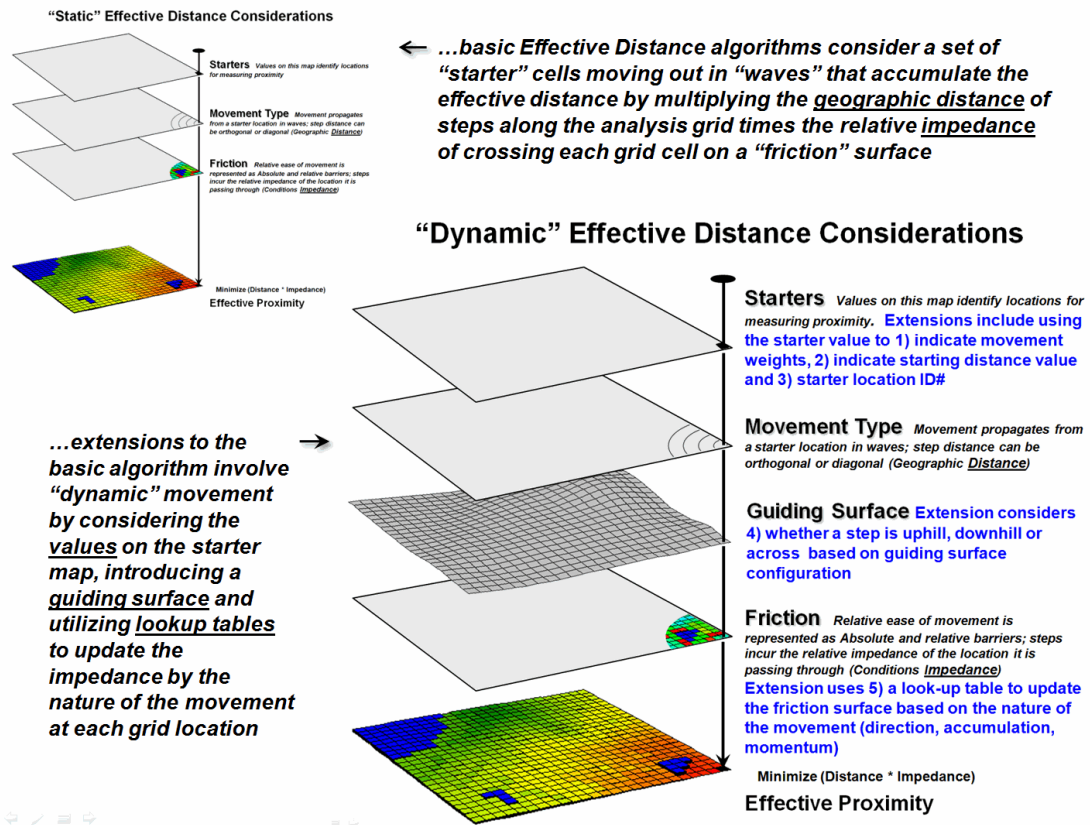


Figure 1. The five most common Dynamic Effective Distance extensions to traditional “cost distance” calculations.

Briefly stated, the basic algorithm pops a location off the Starter stack, then notes the nature of the geographic movement to adjacent cells— orthogonal= 1.000 and diagonal= 1.414. It then checks the impedance/cost for moving into each of the surrounding cells. If an absolute barrier exists, the effective distance for that location is set to infinity. Otherwise, the geographic movement type is multiplied by the impedance/cost on the friction map to calculate the accumulated cost. The procedure is repeated as the movement “wave” continues to propagate like tossing a rock into a still pond. If a location can be accessed by a shorter wave-front path from the Starter cell, or from a different Starter cell, the minimum effective distance is retained.

The “*minimize (distance \* impedance)*” wave propagation repeats until the Starter stack is exhausted. The result is a map surface of the accumulated cost to access anywhere within a project area from its closest Starter location. The solution is expressed in friction/cost units (e.g., minutes are used to derive a travel-time map).

The bottom portion of figure 1 identifies the additional considerations involved in extending the algorithm for Dynamic Effective Distance. Three of the advanced techniques involve special handling of the values associated with the Starter locations—1) weighted distance, 2) stepped accumulation and 3) back-link to closest Starter location. Other extensions utilize 4) a guiding surface to direct movement and 5) look-up tables to update relative impedance based on the nature of the movement.

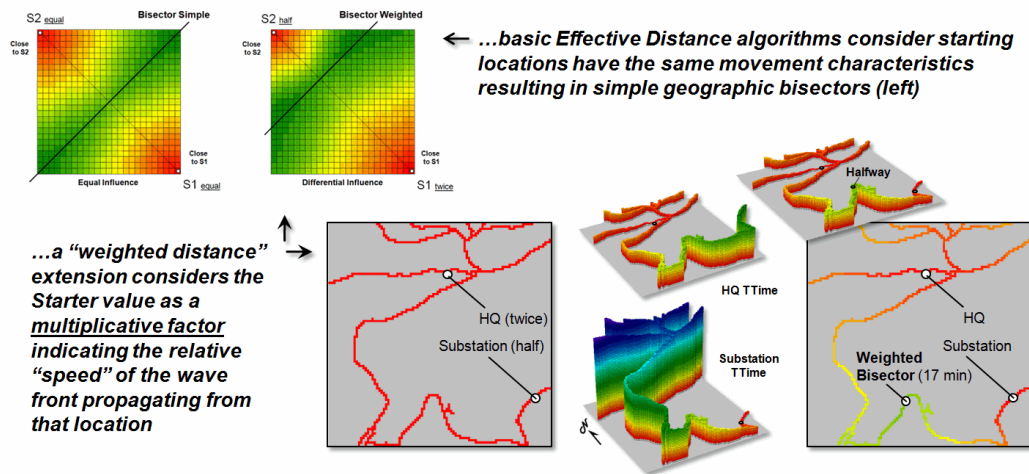


Figure 2. Weighted distance takes into account differences in the relative movement (e.g., speeds) away from different Starter locations.

Figure 2 shows the results of “weighted distance” that considers differences in *movement characteristics*. Most distance algorithms assume that the character of movement is the same for all Starter locations and that the solution space between two Starter locations will be a true halfway point (perpendicular bisector). For example, if there were two helicopters flying toward each other, where one is twice as fast as the other, the “effective halfway” meeting is shifted to an off-center, weighted bisector (upper left). Similarly, two emergency vehicles traveling at different speeds will not meet at the geographic midpoint along a road network (lower right).

Weighted distance is fairly easy to implement. When a Starter location is popped off the stack, its value is used to set an additional weighting factor in the effective distance algorithm—*minimize ((distance \* impedance) \* Starter weight)*. The weight stays in effect throughout a Starter location’s evaluation and then updated for the next Starter location.

Figure 3 shows the results of “stepped accumulation” distance that considers a series of sequenced *movement steps* (see Author’s Note). In the example, on-road travel-time is first calculated along the road network from the headquarters Starter location with off-road travel treated as an absolute barrier. The next step assumes starting anywhere along the roads and proceeding off-road by ATV with relative barriers determined by terrain steepness and absolute barriers set to locations exceeding ATV operating limits (<40% slope). The final step propagates the distance wave into the very steep areas assuming hiking travel.

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Stepped distance is a bit more complicated to implement. It involves a series of calls to the effective distance algorithm with the sequenced Starter maps values used to set the accumulation distance counter— *minimize [Starter value + (distance \* impedance)]*. The Starter value for the first call to calculate effective distance by truck from the headquarters is set to one (or a slightly larger value to indicate “scramble time” to get to the truck). As the wave front propagates each road location is assigned a travel-time value.

...a “stepped surface” extension considers the Starter value as an initial distance and continues accumulating distance using a different “friction surface” calibration set

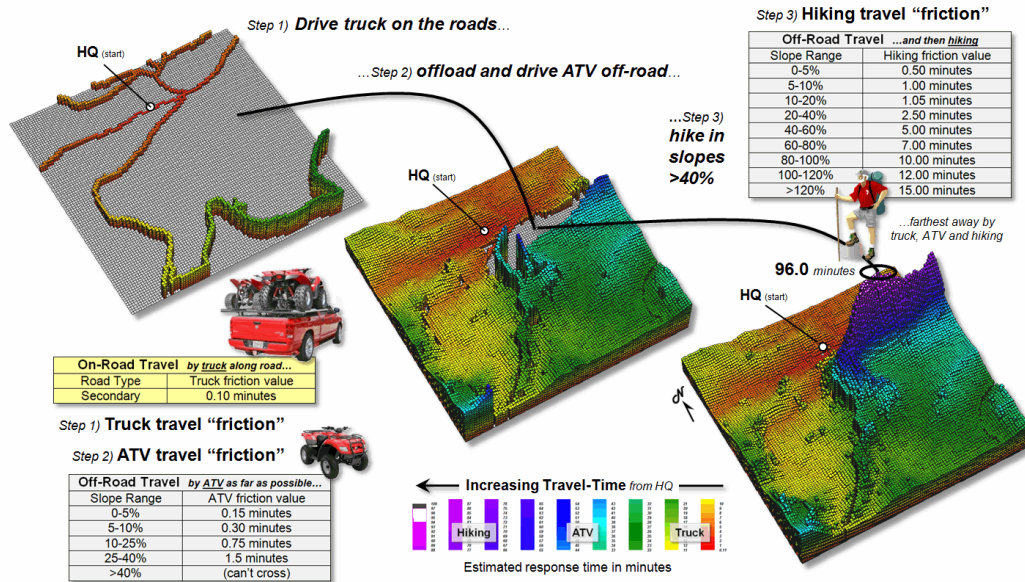


Figure 3. A stepped accumulation surface changes the relative/absolute barriers calibrations for different modes of travel.

The second call uses the accumulated travel-time at each road location to begin off-road ATV movement. In essence the algorithm picks up the wave propagation where it left off and a different friction map is utilized to reflect the relative and absolute barriers associated with ATV travel. Similarly, the third step picks up where ATV travel left off and distance wave continues into the very steep slopes using the hiking friction map calibrations. The final result is a complete travel-time surface identifying the minimum time to reach any grid location assuming the best mix of truck, ATV and hiking movement.

A third way that Starter value can be used is as an ID number to identify the Starter location with the minimum travel-time. In this extension, as the wave front propagates the unique Starter ID is assigned to the corresponding grid cell for every location that “beats” (minimizes) all of the preceding paths that have been evaluated. The result is a new map that identifies the effectively closest Starter location to any accessible grid location within a project area. This new map is commonly referred to a “back-link” map.

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In summary, the value on the Starter map can be used to model weighted effective distance, stepped movement and back-linked to the closest starting location. The next section considers the introduction of a *guiding surface* to direct movement and use of look-up tables to change the friction “on-the-fly” based on the *nature of the movement* (direction, accumulation and momentum).

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**Author’s Note:** For more information on backcountry emergency response, see [www.innovativegis.com/basis/MapAnalysis/Topic29/Topic29.htm](http://www.innovativegis.com/basis/MapAnalysis/Topic29/Topic29.htm), Topic 29, “Spatial Modeling in Natural Resources,” subsection on “E911 for the Backcountry.”

## A Dynamic Tune-up for Distance Calculations

(GeoWorld, March 2011)

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Last section described three ways that a “Starter value” can be used to extend traditional effective distance calculations—by indicating movement weights (*gravity model*), indicating a starting/continuing distance value (*stepped-accumulation*) and starter ID# for identifying which starter location is the closest (*back-link*). All three of these extensions dealt with differences in the nature of the movement itself as it emanates from a location.

The other two extensions for dynamic effective distance involve differences in the nature of the intervening conditions—*guiding surface* redirection and *dynamic impedance* based on accumulation, momentum and direction. Figure 1 identifies a “guiding surface” responding to whether a movement step is uphill, downhill or across based on the surface’s configuration.

Inset a) on the left-side of the figure shows a constrained proximity surface that identifies locations that are up to 200 meters “downhill” from roads. The result forms a “variable-width buffer” around the roads that excludes uphill locations. The downhill locations within the buffer are assigned proximity values indicating how close each location is to the nearest road cell above it. Also note that the buffer is “clipped” by the ocean so only on-island buffer distances are shown.

Inset b) uses a different kind of guiding surface— a tilted plane characterizing current flow from the southwest. In this case, downhill movement corresponds to “down-current” flows from the two adjacent watersheds. While a simple tilted plane ignores the subtle twists and turns caused by winds and bathymetry differences, it serves as a first order current movement characterization.

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...a “guiding surface” extension considers the value on the surface to determine whether a potential step is up, down or across a surface

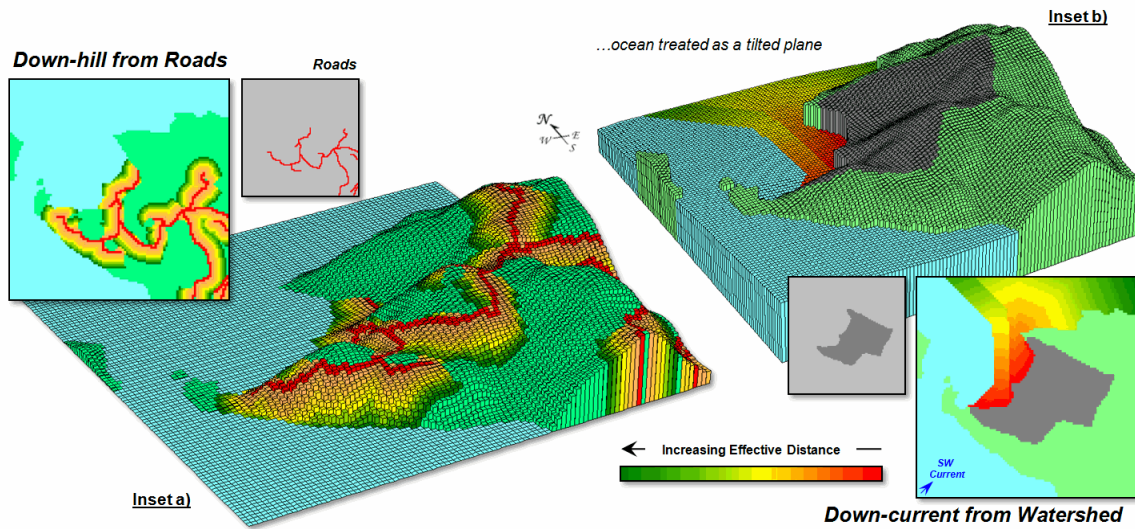


Figure 1. A Guiding Surface can be used to direct or constrain movement within a project area.

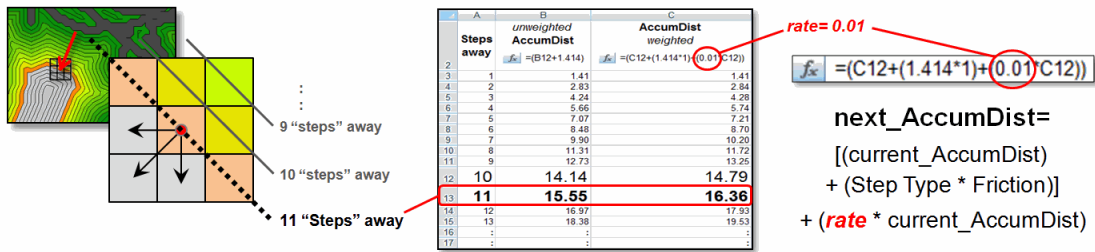
A similar, yet more detailed guiding surface, is a barometric map derived from weather station data. A “down-wind” map tracks the down surface (barometric gradient) movement from each project location to areas of lower atmospheric pressure. Similarly, “up-surface” movement from any location on a pollution concentration surface can help identify the probable pathway flow from a pollution source (highest concentration).

“Dynamic impedance” involves changes with respect to increasing distance (accumulation), net movement force (momentum) and interactions between a movement path and its intervening conditions (direction). The top portion of figure 2 outlines the use of an “additive factor equation” to dynamically slow down movement in a manner analogous to compound interest of a savings account. As a distance wave propagates from a Starting location, the effective distance of each successive step is slightly more impeded, like a tired hiker’s pace decreasing with increasing distance—the last mile of a 20 mile trek seems a lot farther.

The example shows the calculations for the 11<sup>th</sup> step of a SW moving wave front (orthogonal step type= 1.414) with a constant impedance (friction= 1) and a 1% compounding impedance (rate= .01). The result is an accumulated hindrance effectively farther by about 25 meters (16.36 – 15.55=.81 \* 30m cell size).

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## 1) Accumulation (increasing distance; Additive Factor equation)



## 2) Momentum (net accumulation; Additive Factor table)

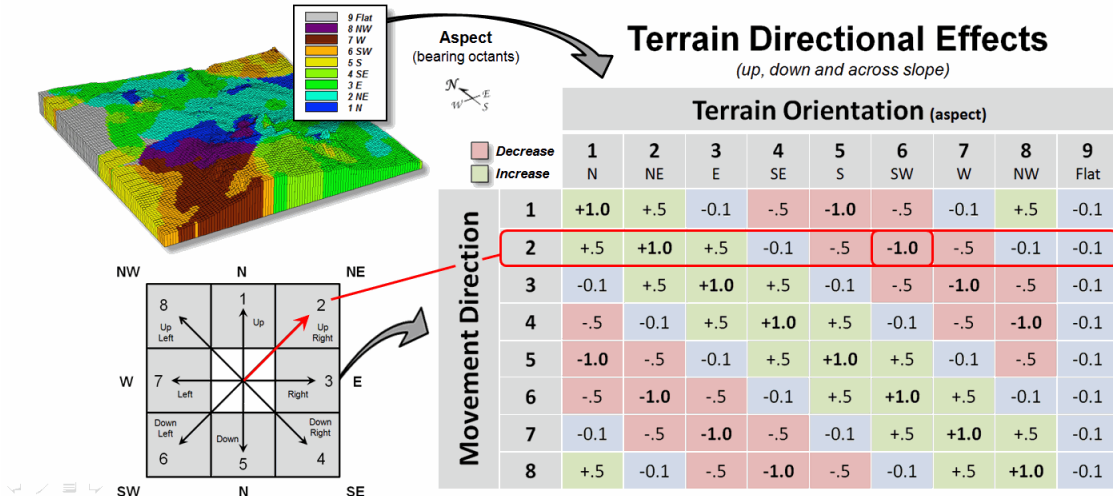


Figure 2. Accumulation and Momentum can be used to account for dynamic changes in the nature of intervening conditions and assumptions about movement in geographic space.

The bottom portion of figure 2 shows the approach for assessing the net accumulation of movement (momentum). This brings back a very old repressed memory of a lab exercise in a math/programming course I attempted over 30 years ago. We were given a terrain-like surface and coefficients of movement (acceleration and deceleration) of a ball under various uphill and downhill situations. Our challenge was to determine the location to drop the ball so it would roll the farthest ... the only thing I really got was “dropping the ball.” In looking back, I now realize that an “additive factor table” could have been a key to the solution.

The table in the figure shows the “costs/payments” of downhill, across and uphill movements. For this simplified example, imagine a money exchange booth at each grid location—the toll or payout is dependent on the direction of the wave front with respect to the orientation of the surface. If you started somewhere with a \$10 bag of money, depending on your movement path and surface configuration, you would collect a dollar for going straight downhill (+1.0) but lose a dollar for going straight uphill (-1.0).

The table summarizes the cost/payout for all of the movement directions under various terrain

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conditions. For example, a NE step is highlighted (direction= 2) that corresponds to a SW terrain orientation (aspect= 6) so your movement would be straight uphill and cost you a dollar. The effective net accumulation from a given Starter cell to every other location is the arithmetic sum of costs/payments encountered—the current amount in the bag at location is your net accumulation; stop when your bag is empty (\$0). In the real-world, the costs/payments would be coefficients of exacting equations to determine the depletions/additions at each step.

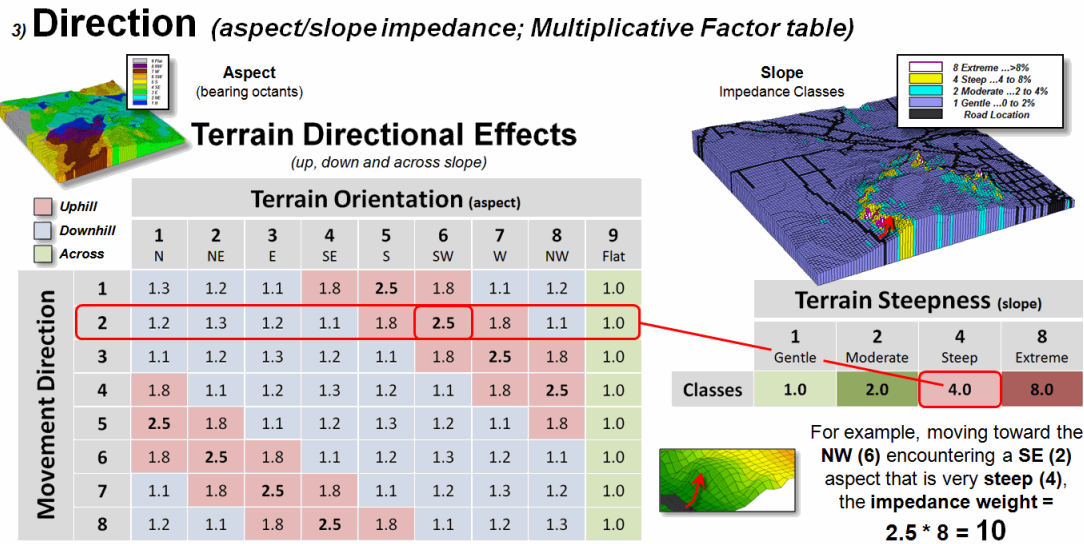


Figure 3. Directional effects of movement with respect to slope/aspect variations can be accounted for “on-the-fly.”

Figure 3 extends the consideration of dynamic movement through the use of a “multiplicative factor table” based on two criteria—terrain aspect and steepness. All trekkers know that hiking up, down or across slope are radically different endeavors, especially on steep slopes. Most hiking time solutions, however, simply assign a “typical cost” (friction) that assumes “the steeper the terrain, the slower one goes” regardless of the direction of travel. But that is not always true, as it is about as easy to negotiate across a steep slope as it is to traverse a gentle uphill slope.

The table in figure 3 identifies the multiplicative weights for each uphill, downhill or across movement based on terrain aspect. For example, as a wave front considers stepping into a new location it checks its movement direction (NE= 2) and the aspect of the cell (SW= 6), identifies the appropriate multiplicative weight in the table (2,6 position= 2.5), then checks the “typical” steepness impedance (steep= 4.0) and multiplies them together for an overall friction value ( $2.5*4.0= 10.0$ ); if movement was NE on a gentle slope the overall friction value would be just 1.1.

In effect, moving uphill on steep slopes is considered nearly 10 times more difficult than traversing across a gentle slope ...that makes a lot of sense. But very few map analysis packages



handle any of the “dynamic movement” considerations (gravity model, stepped-accumulation, back-link, guiding surface and dynamic impedance) ...that doesn't make sense.

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**Author's Note:** For more information on effective distance procedures,(both static and dynamic) see [www.innovativegis.com/basis/MapAnalysis/Topic25/Topic25.htm](http://www.innovativegis.com/basis/MapAnalysis/Topic25/Topic25.htm), online book *Beyond Mapping III*, Topic 25, “Calculating Effective Distance and Connectivity.” Instructors see readings, lecture and exercise for Week 4, “Calculating Effective Distance” online course materials at [www.innovativegis.com/basis/Courses/GMcourse10/](http://www.innovativegis.com/basis/Courses/GMcourse10/).

## A Narrow-minded Approach

(GeoWorld, June 2009)

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In the previous sections, advanced and sometimes unfamiliar concepts of distance have been discussed. The traditional definition of “shortest straight line between two points” for *Distance* was extended to the concept of *Proximity* by relaxing the “two points” requirement; and then to the concept of *Movement* that respects absolute and relative barriers by relaxing the “straight line” requirement (Author's Note 1).

The concept of *Connectivity* is the final step in this expansion referring to how locations are connected in geographic space. In the case of effective distance, it identifies the serpentine route around absolute and through relative barriers moving from one location to another by the “shortest” effective path—shortest, the only remaining requirement in the modern definition of distance. A related concept involves straight rays in 3-dimensional space (line-of-sight) to determine visual connectivity among locations considering the intervening terrain and vegetative cover (Author's Note 2).

However, there is yet another concept of connectivity—*Narrowness* defined as the “shortest cord through a location connecting opposing edges.” As with all distance-related operations, the computer first generates a series of concentric rings of increasing distance from an interior point (grid cell). This information is used to assign distance to all edge locations. Then the computer moves around the edge totaling the distances for opposing edges until it determines the minimum—the shortest cord. The process is repeated for all map locations to derive a continuous map of narrowness.

For a boxer, a map of the boxing ring would have values at every location indicating how far it is to the ropes with the corners being the narrowest (minimum cord distance). Small values indicate poor boxing habitat where one might get trapped and ruthlessly bludgeoned without escape. For a military strategist, narrow locations like the Khyber Pass can prove to be inhospitable habitat as well.

Bambi and Mama Bam can have a similar dread of the narrow portions of an irregularly shaped meadow (see figure 1, insets *a* and *b*). Traditional analysis suggests that the meadow's acreage times the biomass per acre determines the herd size that can be supported. However, the spatial

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arrangement of these acres might be just as important to survival as the caloric loading calculations. The entire meadow could be sort of a Cordon Bleu of deer fodder with preference for the more open portions, an ample distance away from the narrow forest edge where danger may lurk. But much of the meadow has narrow places where patient puma prowl and pounce, imperiling baby Bambi. How can new age wildlife managers explain that to their kids—survival is just a simple calculation of acres times biomass that is independent of spatial arrangement, right?

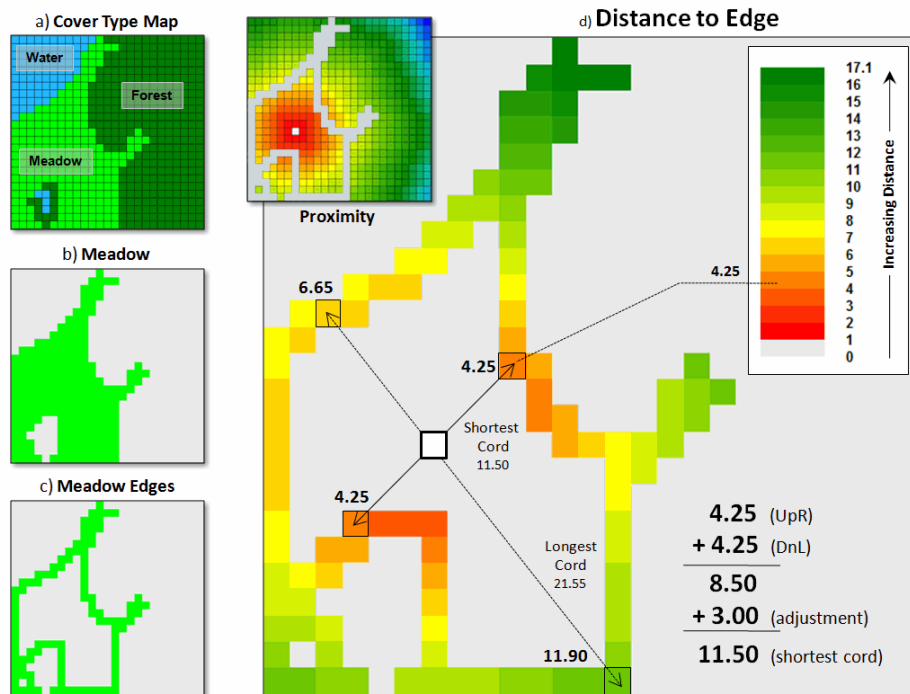


Figure 1. Narrowness determines constrictions within a map feature as the shortest cord connecting opposing edges.

Many GIS applications involve more than simple inventory mastication—extent (spatial table) times characteristic/condition (attribute table). So what is involved in deriving a narrowness map? ...how can it be summarized? ...how might one use a narrowness map and its summary metrics?

The first step is to establish a simple proximity map from a location and then transfer this information to the edge cells of the parcel containing the location (figure 1, insets c and d). The algorithm then begins at an edge cell, determines its opposing edge cell along a line passing through the location, sums the distances and applies an adjustment factor to account for the center cell and edge cell lengths. In the example, the shortest cord is the sum of the upper-right distance and its lower-left opposing distance plus the adjustment factor ( $4.25 + 4.25 + 3.00 = 8.50$ ). All other cords passing through the location are longer (e.g.,  $6.65 + 11.90 + 3.00 = 21.55$  for the longest cord). Actually, the calculations are a bit dicier as they need to adjust for off-

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orthogonal configurations ... a nuance for the programmers among you to consider.

Once the minimum cord is determined the algorithm stores the value and moves to the next location to evaluate; this is repeated until the narrowness of all of the desired locations have been derived (figure 2 inset *e* for just the meadow and *f* for the entire area). Notice that there are two dominant kidney-shaped open areas (green tones)—one in the meadow and one in the forest. Keep in mind that the effect of the “artificial edges” of the map extent in this constrained example would be minimal in a landscape level application.

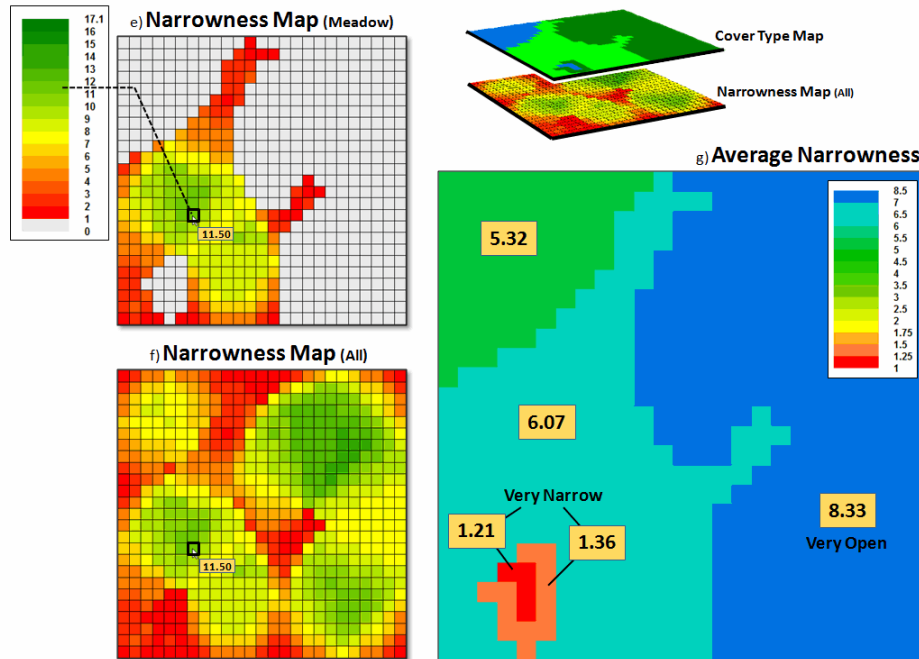


Figure 2. Summarizing average narrowness for individual parcels.

The right side of figure 2 (inset *g*) illustrates the calculation of the average narrowness for each of the cover type parcel. Narrowness determines constrictions within a map feature (polygon) as the shortest cord connecting opposing edges, such as a forest opening. It uses a region-wide overlay technique that computes the average of the narrowness values coinciding with each parcel. A better metric of relative narrowness would be the ratio of the number of narrow cells (red-tones) to the total number of cells defining a parcel. For a large perfectly circular parcel the ratio would be zero with increasing ratios to 1.0 for very narrow shapes, such as very small or ameba-shaped polygons.

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**Author's Notes:** for background discussion, see 1) Topic 25, *Calculating Effective Distance and Connectivity* and 2) Topic 15, *Deriving and Using Visual Exposure Maps* in the online book *Beyond Mapping III* at [www.innovativegis.com/basis/MapAnalysis/](http://www.innovativegis.com/basis/MapAnalysis/).

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# Narrowing-in on Absurd Gerrymanders

(GeoWorld, July 2012)

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In light of the current political circus, I thought a bit of reflection is in order on how GIS has impacted the geographic stage for the spectacle—literally drawing the lines in the sand. Since the 1990 census, GIS has been used extensively to “redistrict” electoral territories in light of population changes, thereby fueling the decennary turf wars between the Democrats and Republicans.

*Redistricting* involves redrawing of U.S. congressional district boundaries every ten years in response to population changes. In developing the subdivisions, four major considerations come into play—

- 1) equalizing the population of districts,
- 2) keeping existing political units and communities within a single district,
- 3) creating geographically compact, contiguous districts, and
- 4) avoiding the drafting of boundaries that create partisan advantage or incumbent protection.

*Gerrymandering*, on the other hand, is the deliberate manipulation of political boundaries for electoral advantage with minimal regard for the last three guidelines. The goal of both sides is to draw district boundaries that achieve the most political gain.

Three strategies for gerrymandering are applied—

- 1) attempt to concentrate the voting power of the opposition into just a few districts, to dilute the power of the opposition party outside of those districts (termed “excess vote”),
- 2) diffuse the voting power of the opposition across many districts, preventing it from having a majority vote in as many districts as possible (“wasted vote”), and
- 3) link distant areas into specific, party-in-power districts forming spindly tentacles and ameba-line pseudopods (“stacked”).

For example, the 4th Congressional District of Illinois is one of the most strangely drawn and gerrymandered congressional districts in the country (figure 1). Its bent barbell shape is the poster-child of “stacked” gerrymandering, but Georgia’s flying pig, Louisiana’s stacked scorpions and North Carolina’s praying mantis districts have equally bizarre boundaries.

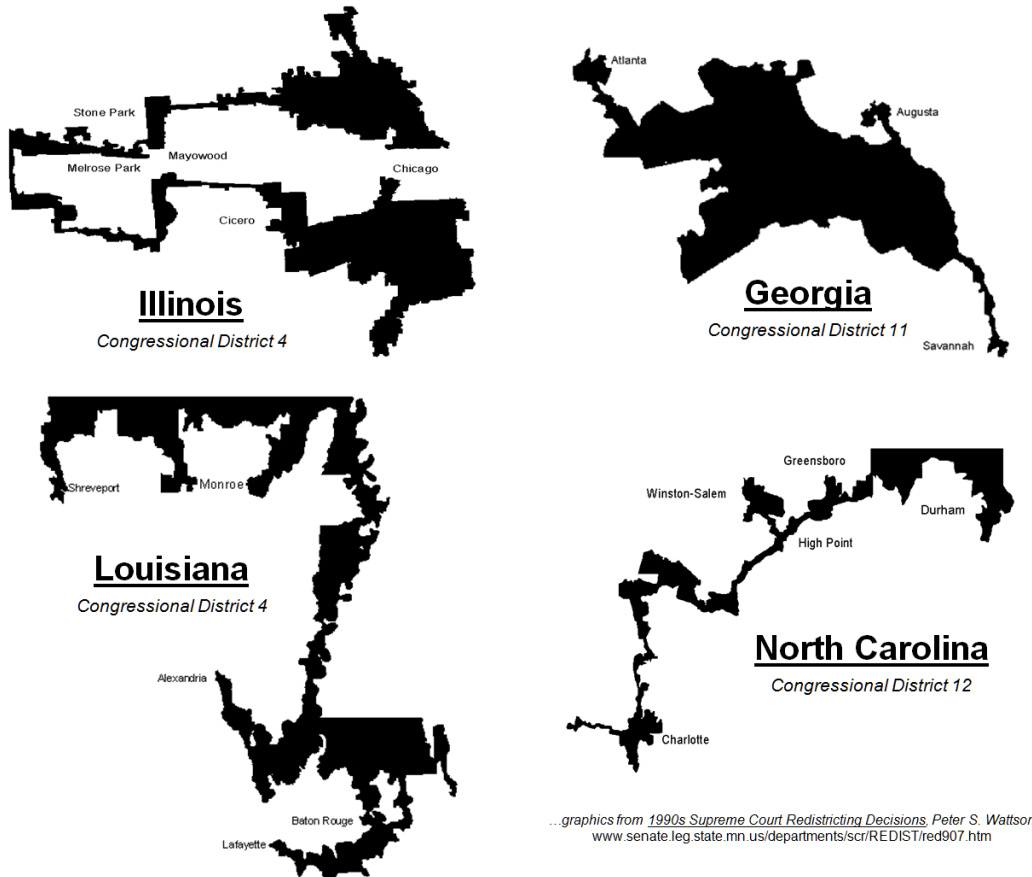


Figure 1. Examples of gerrymandered congressional districts with minimal compactness.

Coupled with census and party affiliation data, GIS is used routinely to gerrymander congressional districts. But from another perspective, it can be used to assess a district's shape and through legislative regulation could impose indices that encourage compactness. A "convexity index" (CI) and a "narrowness index" (NI) are a couple of possibilities that could rein-in bazaar gerrymanders.

The boundary configuration of any feature can be identified as the ratio of its perimeter to its area (see author's notes 1 and 2). In planimetric space, the circle has the least amount of perimeter per unit area. Any other shape has more perimeter (see figure 2), and as a result, a different Convexity Index.

In the few GIS software packages having this capability, the index uses a "fudge factor" ( $k$ ) to account for mixed units (e.g.,  $m$  for P and  $m^2$  for A) to produce a normalized range of values from 1 (very irregularly shaped) to 100 (very regularly shaped). A theoretical index of zero indicates an infinitely large perimeter around an infinitesimally small area (e.g., a line without perimeter or area, just length). At the other end, an index of 100 is interpreted as being 100

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percent similar to a perfect circle. Values in between define a continuum of boundary regularity that could be used to identify a cutoff of minimal irregularity that would be allowed in redistricting.

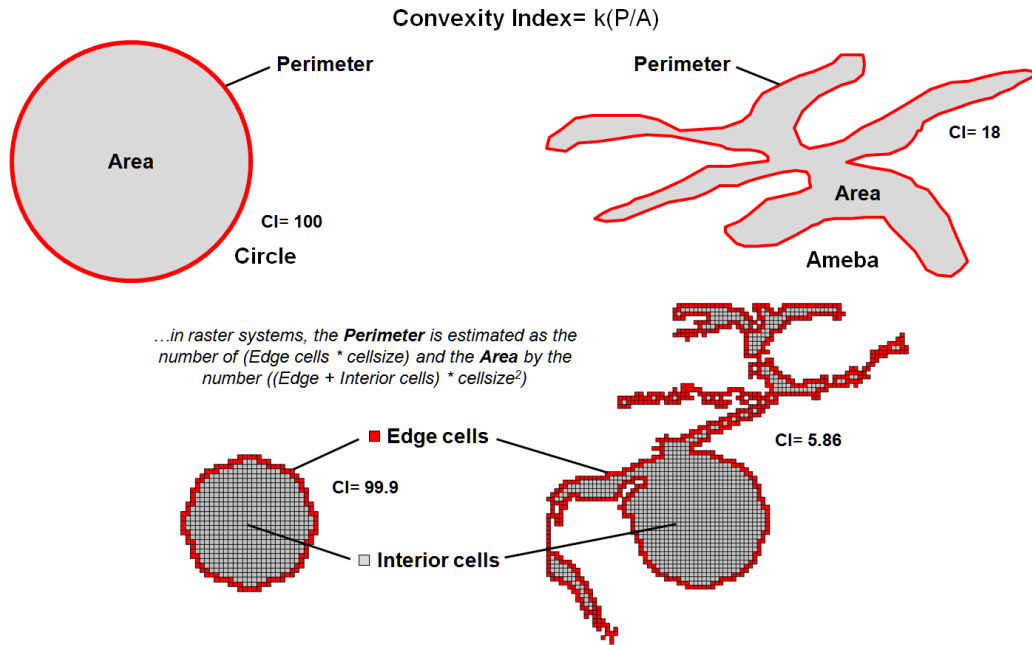


Figure 2. Convexity is characterized as the normalized ratio of a feature’s perimeter to its area.

Another metric for assessing shape involves calculating “narrowness” within a map feature. Narrowness can be defined as the “shortest cord passing through a location that connects opposing edges” (see author’s note 3). In practice, narrowness is calculated to a specified maximum distance. Locations with cords exceeding this distance are simply identified as “open areas.”

In figure 3, the narrow locations are shown as a color gradient from the most narrow locations (red=1 cell length= 30m) to minimally narrow (green= 9.9999 \*30m= 299.9m) to open areas (grey=  $\geq 300m$ ). Note the increasing number of narrow locations as the map features become increasingly less compact.

A Narrowness Index can be calculated as the ratio of the number of narrow cells to the number of open cells. For the circle in the figure, NI= 152/557= .273 with nearly four times as many open cells than narrow cells. The bug shape ratio is .848 and the spindly Medusa shape with a ratio of 2.232 has more than twice as many narrow cells as open cells.

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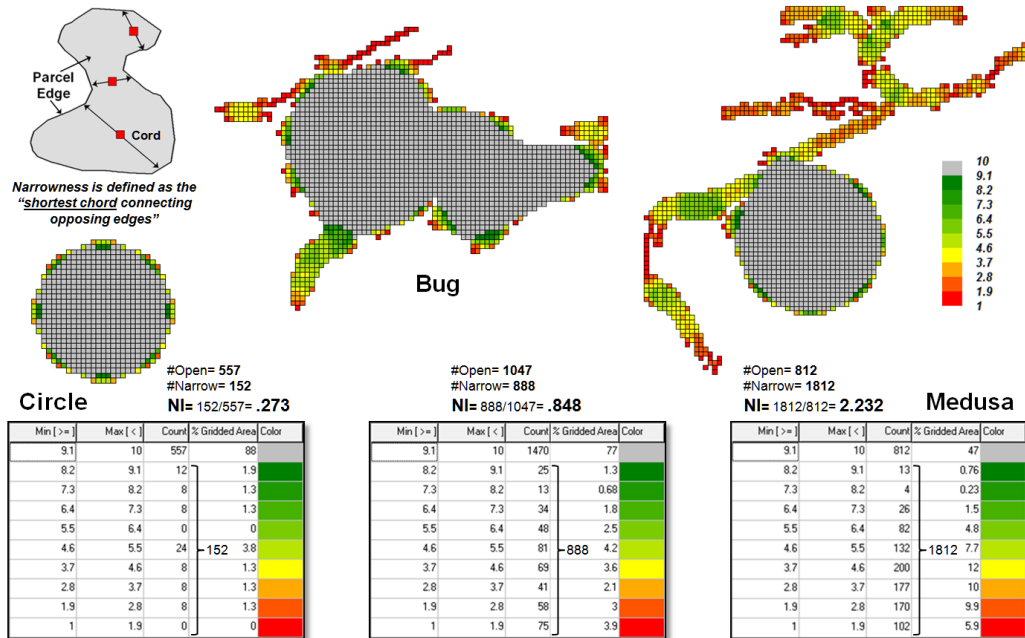


Figure 3. Narrowness is characterized as the shortest cord connecting opposing edges.

Both the convexity index and the narrowness index quantify the degree of irregularity in the configuration of a map feature. However, they provide dramatically different assessments. CI is a non-spatial index as it summarizes the overall boundary configuration as an aggregate ratio focusing on a feature's edge and can be solved through either vector or raster processing. NI on the other hand, is a spatial index as it characterizes the degree and proportion of narrowness throughout a feature's interior and only can be solved through raster processing. Also, the resulting narrowness map indicates where narrow locations occur, that is useful in refining alternative shapes.

To date, the analytical power of GIS has been instrumental in gerrymandering congressional districts that forge political advantage for whichever political party is in control after a census. In engineering an optimal partisan solution the compactness criterion often is disregarded.

On the other side of the coin, the convexity and narrowness indices provide a foothold for objective, unbiased and quantitative measures that assess proposed district compactness. Including acceptable CI and NI measures into redistricting criteria would insure that compactness is addressed— gentlemen (and ladies), start your GIS analytic engines.

**Author's Notes:** 1) *Beyond Mapping* column on Feature Shape Indices, September 1991, posted at [www.innovativegis.com/basis/BeyondMapping\\_I/Topic5/BM\\_I\\_T5.htm#Forest\\_trees](http://www.innovativegis.com/basis/BeyondMapping_I/Topic5/BM_I_T5.htm#Forest_trees); 2) PowerPoint on Gerrymandering and Legislative Efficiency by John Mackenzie, Director of Spatial Analysis Lab, University of Delaware posted at [www.udel.edu/johnmack/research/gerrymandering.ppt](http://www.udel.edu/johnmack/research/gerrymandering.ppt); 3) Narrowness is discussed in the online book *Beyond Mapping III*, Topic 25, Calculating Effective Distance and Connectivity, posted at [www.innovativegis.com/basis/MapAnalysis/Topic25/Topic25.htm#Narrowness](http://www.innovativegis.com/basis/MapAnalysis/Topic25/Topic25.htm#Narrowness).

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# Just How Crooked Are Things?

(GeoWorld, November 2012)

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In a heated presidential election month this seems to be an apt title as things appear to be twisted and contorted from all directions. Politics aside and from a down to earth perspective, how might one measure just how spatially crooked things are? My benchmark for one of the most crooked roads is Lombard Street in San Francisco—it's not only crooked but devilishly steep. How might you objectively measure its crookedness? What are the spatial characteristics? Is Lombard Street more crooked than the eastern side of Colorado's Independence Pass connecting Aspen and Leadville?

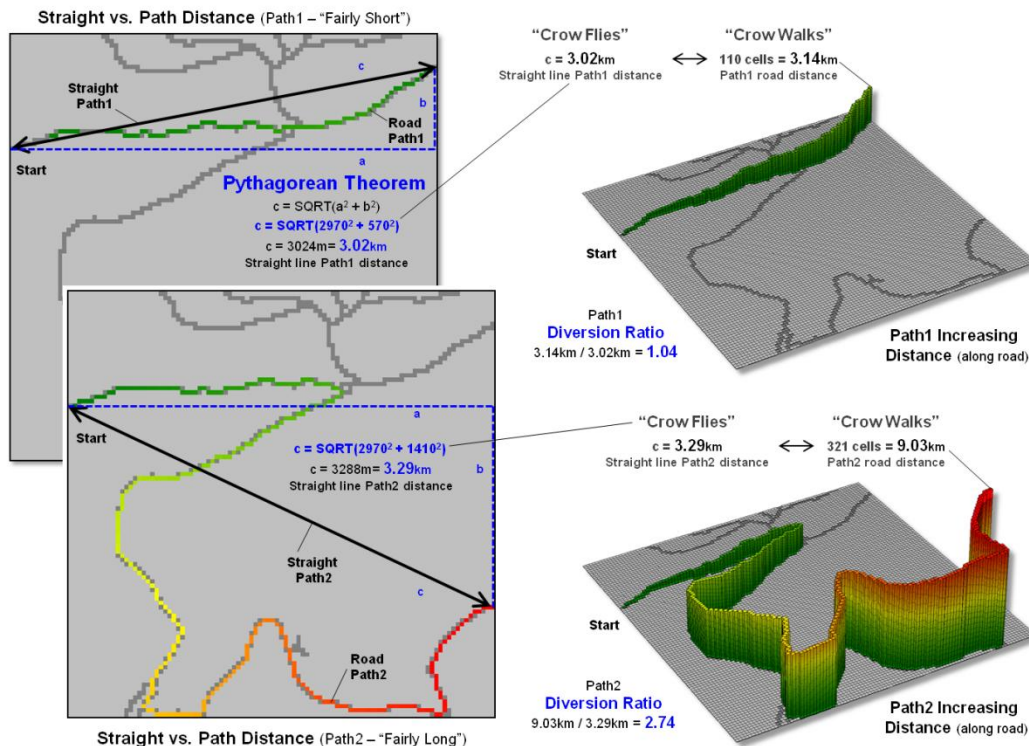


Figure 1. A Diversion Ratio compares a route's actual path distance to its straight line distance.

Webster's Dictionary defines crooked as "not straight" but there is a lot more to it from a technical perspective. For example, consider the two paths along a road network shown in figure 1. A simple crooked comparison characteristic could compare the "crow flies" distance (straight line) to the "crow walks" distance (along the road). The straight line distance is easily measured using a ruler or calculated using the Pythagorean Theorem. The on-road distance can be manually assessed by measuring the overall length as a series of "tick marks" along the edge of a

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sheet of paper successively shifted along the route. Or in the modern age, simply ask Google Maps for the route's distance.

The vector-based solution in Google Maps, like the manual technique, sums all of the line segments lengths comprising the route. Similarly, a grid-based solution counts all of the cells forming the route and multiplies by an adjusted cell length that accounts for orthogonal and diagonal movements along the sawtooth representation. In both instances, a *Diversion Ratio* can be calculated by dividing the crow walking distance (crooked) by the crow flying distance (straight) for an overall measurement of the path's diversion from a straight line.

As shown in the figure the diversion ratio for Path1 is 3.14km / 3.02km = 1.04 indicating that the road distance is just a little longer than the straight line distance. For Path2, the ratio is 9.03km / 3.29km = 2.74 indicating that the Path2 is more than two and a half times longer than its straight line. Based on crookedness being simply "not straight," Path2 is much more crooked.

Figure 2 depicts an extension of the diversion ratio to the entire road network. The on-road distance from a starting location is calculated to identify a crow's walking distance to each road location (employing Spatial Analyst's Cost Distance tool for the Esri-proficient among us). A straight line proximity surface of a crow's flying distance from the start is generated for all locations in a study area (Euclidean Distance tool) and then isolated for just the road locations. Dividing the two maps calculates the diversion ratio for every road cell.

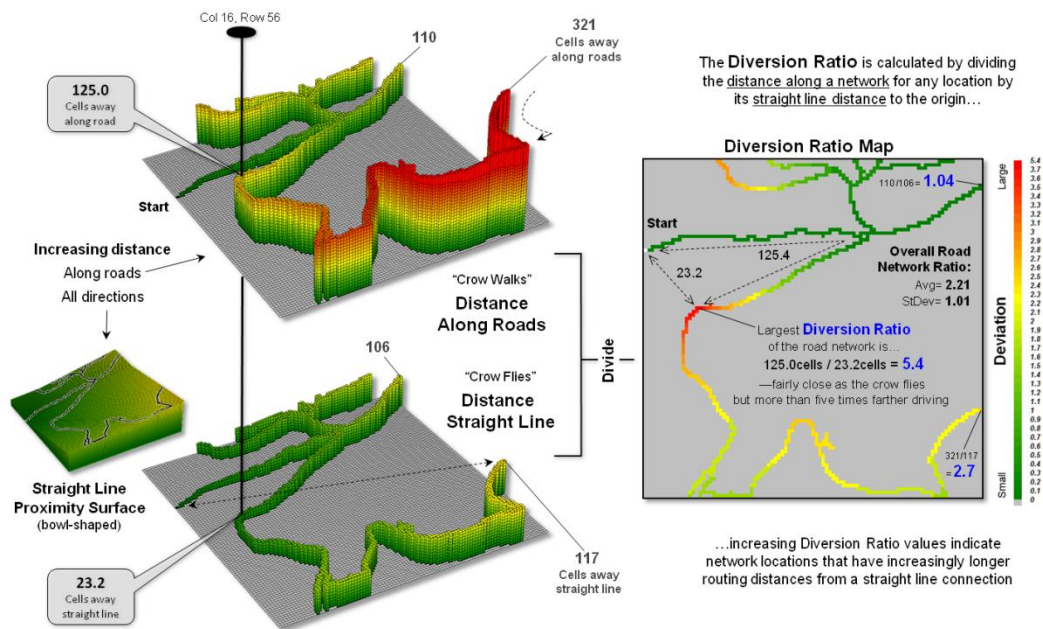


Figure 2. A *Diversion Ratio Map* identifies the comparison of path versus straight line distances for every location along a route.

The ratio for the farthest away road location is 321 cells /117 cells = 2.7, essentially the same value as computed using the Pythagorean Theorem for the straight line distance. Use of the straight line proximity surface is far more efficient than repeatedly evaluating the Pythagorean Theorem, particularly when considering typical project areas with thousands upon thousands of road cells.

In addition, the spatially disaggregated approach carries far more information about the crookedness of the roads in the area. For example, the largest diversion ratio for the road network is 5.4—crow walking distance nearly five and a half times that of crow flying distance. The average ratio for the entire network is 2.21 indicating a lot of overall diversion from straight line connection throughout the set of roads. Summaries for specific path segments are easily isolated from the overall Diversion Ratio Map— compute once, summarize many. For example, the US Forest Service could calculate a Diversion Ratio Map for each national forest’s road system and then simply “pluck-off” crookedness information for portions as needed in harvest or emergency-response planning.

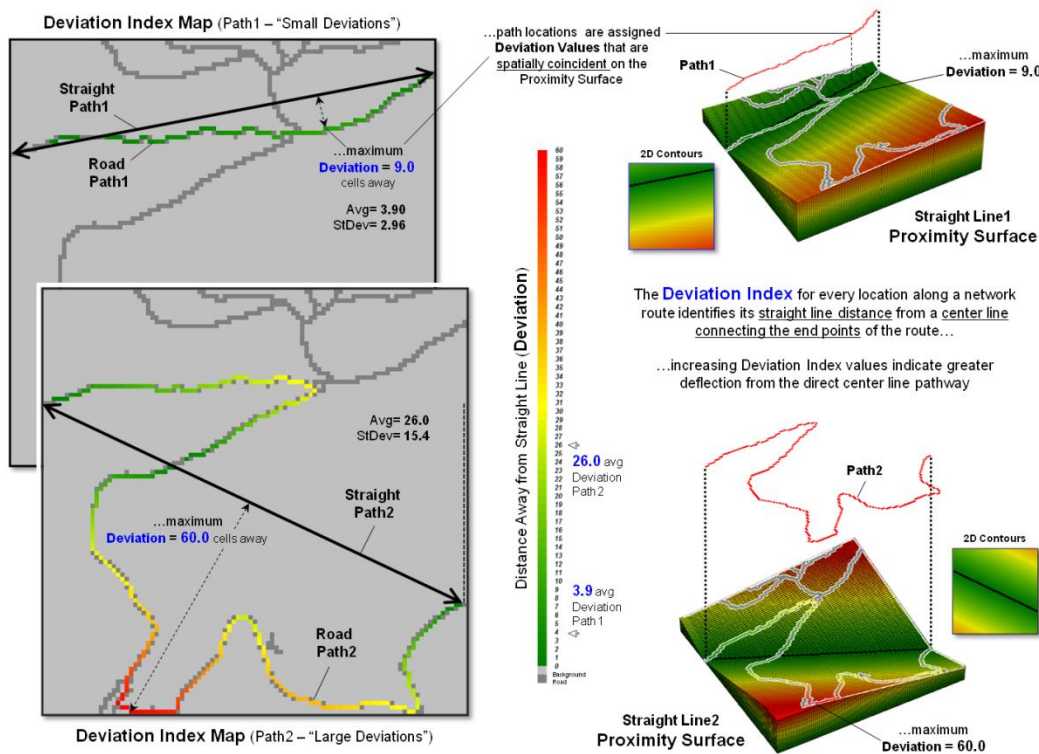


Figure 3. A Deviation Index identifies for every location along a route the deflection from a path’s centerline.

The Deviation Index shown in figure 3 takes an entirely different view of crookedness. It compares the deviation from a straight line connecting a path’s end points for each location along the actual route. The result is a measure of the “deflection” of the route as the

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perpendicular distance from the centerline. If a route is perfectly straight it will align with the centerline and contain no deflections (all deviation values= 0). Larger and larger deviation values along a route indicate an increasingly non-straight path.

The left side of figure 3 shows the centerline proximity for Paths 1 and 2. Note the small deviation values (green tones) for Path 1 confirming that it is generally close to the centerline. This confirms that it is much straighter than Path 2 with a lot of deviation values greater than 30 cells away (red tones). The average deflection (overall Deviation Index) is just 3.9 cells for Path1 and 26.0 cells for Path2.

But crookedness seems more than just longer diverted routing or deviation from a centerline. It could be that a path simply makes a big swing away from the crow's beeline flight—a smooth curve not a crooked, sinuous path. Nor is the essence of crookedness simply counting the number of times that a path crosses its direct route. Both paths in the examples cross the centerline just once but they are obviously very different patterns. Another technique might be to keep track of the above/below or left/right deflections from the centerline. The sign of the arithmetic sum would note which side contains the majority of the deflections. The magnitude of the sum would report how off-center (unbalanced) a route is. Or maybe a roving window technique could be used to summarize the deflection angles as the window is moved along a route.

The bottom line (pun intended) is that spatial analysis is still in its infancy. While non-spatial math/stat procedures are well-developed and understood, quantitative analysis of mapped data is very fertile turf for aspiring minds ...any bright and inquiring grad students out there up to the challenge?

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**Author's Note:** For a related discussion of characterizing the configuration of landscape features, see the online book *Beyond Mapping I, Topic 5: Assessing Variability, Shape, and Pattern of Map Features* posted at [www.innovativegis.com/basis/BeyondMapping\\_I/Topic5/](http://www.innovativegis.com/basis/BeyondMapping_I/Topic5/).

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Additional discussion of distance, proximity, movement and related measurements in GIS technology is online in the book *Map Analysis* by Berry posted at <http://www.innovativegis.com/basis/>.

- *Topic 25, Calculating Effective Proximity*
- *Topic 20, Surface Flow Modeling*
- *Topic 19, Routing and Optimal Paths*
- *Topic 17, Applying Surface Analysis*
- *Topic 15, Deriving and Using Visual Exposure Maps*
- *Topic 14, Deriving and Using Travel-Time Maps*
- *Topic 13, Creating Variable-Width Buffers*
- *Topic 6, Analyzing In-Store Shopping Patterns*
- *Topic 5, Analyzing Accumulation Surfaces*

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