Beyond Mapping III

Topic 22: Reclassifying and Overlaying Maps

Use a Map-ematical Framework for GIS Modeling — describes a conceptual structure for map analysis operations and GIS modeling

Getting the Numbers Right — describes an alternative framework based on how the map values are retrieved to classify analytical operations.

Options Seem Endless When Reclassifying Maps — discusses the basic reclassifying map operations

Overlay Operations Feature a Variety of Options — discusses the basic overlaying map operations

Computers Quickly Characterize Spatial Coincidence — discusses several human considerations in implementing GIS

Key Concepts Characterize Unique Conditions — describes a technique for handling unique combinations of map layers

Use “Shadow Maps” to Understand Overlay Errors — describes how shadow maps of certainty can be used to estimate error and its propagation

Author’s Notes: The figures in this topic use MapCalcTM software. An educational CD with online text, exercises and databases for “hands-on” experience in these and other grid-based analysis procedures is available for US$21.95 plus shipping and handling (www.farmgis.com/products/software/mapcalc/).

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(Back to the Table of Contents)

Use Map-ematical Framework for GIS Modeling

(GeoWorld, March 2004, pg. 18-19)

As GIS technology moves beyond mapping, an increasing number of analytical operations come into play. Tools for zooming, panning, colorizing and superimposing map displays are being augmented by analytical procedures like coincidence, proximity, visual exposure and optimal

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While map analysis tools might at first seem uncomfortable, they simply are extensions of traditional analysis procedures brought on by the digital nature of modern maps. Since maps are “number first, pictures later,” a map-ematical framework can be used to organize the analytical operations. Like basic math, this approach uses sequential processing of mathematical operations to perform a wide variety of complex map analyses. By controlling the order that the operations are executed, and using a common database to store the intermediate results, a mathematical-like processing structure is developed.

This “map algebra” is similar to traditional algebra where basic operations, such as addition, subtraction and exponentiation, are logically sequenced for specific variables to form equations—however, in map algebra the variables represent entire maps consisting of thousands of individual grid values. Most of traditional mathematical capabilities, plus extensive set of advanced map processing operations, comprise the map analysis toolbox.

In grid-based map analysis, the spatial coincidence and juxtapositioning of values among and within maps create new analytical operations, such as coincidence, proximity, visual exposure and optimal routes. These operators are accessed through general purpose map analysis software available in many GIS systems, such as ERDAS Imagine, GeoMedia Grid or the Spatial Analyst extension to ArcGIS.

There are two fundamental conditions required by any map analysis package—a consistent data structure and an iterative processing environment. Previous Beyond Mapping columns (July – September, 2002) described the characteristics of a grid-based data structure by introducing the concepts of an analysis frame, map stack and data types. This discussion extended the traditional discrete set of map features (points, lines and polygons) to map surfaces that characterize geographic space as a continuum of uniformly-spaced grid cells.

The second condition is the focus of this and the next couple of columns. It provides an iterative processing environment by logically sequencing map analysis operations and involves: 1) retrieval of one or more map layers from the database, 2) processing that data as specified by the user, 3) creation of a new map containing the processing results, and 4) storage of the new map for subsequent processing.

Each new map derived as processing continues aligns with the analysis frame so it is automatically geo-registered to the other maps in the database. The values comprising the derived maps are a function of the processing specified for the “input map(s).”

This cyclical processing provides an extremely flexible structure similar to “evaluating nested parentheticals” in traditional math. Within this structure, one first defines the values for each variable and then solves the equation by performing the mathematical operations on those numbers in the order prescribed by the equation. For example, the equation for calculating
percent change in your investment portfolio—

Table 1 – Equation for Calculating Percent Change

\[ \% \text{Change} = \frac{A}{C} = \left( \frac{B - C}{C} \right) \times 100 \]

...B is new valuation; C is old

...define variables

...intermediate solution

...intermediate solution

...final solution

—identifies that the variables B and C are first defined, then subtracted and the difference stored as an intermediate solution. The intermediate solution is divided by variable C to generate another intermediate solution that, in turn is multiplied by 100 to calculate the solution variable A—percent change value.

The same mathematical structure provides the framework for computer-assisted map analysis. The only difference is that the variables are represented by mapped data composed of thousands of organized numbers. Figure 1 shows a similar solution for calculating the percent change in animal activity for an area. Maps of activity in two periods serve as input; a difference map is calculated then divided by the earlier period and multiplied by 100. The procedure uses the same equation, just derives a different form of output—a map of percent change.

Figure 1. An iterative processing environment, analogous to basic math, is used to derive new map variables.

The processing steps shown in the figure are identical to the traditional solution except the calculations are performed for each grid cell in the study area and the result is a map that identifies
the percent change at each location (a decrease of 8.51% for the example location; red tones indicate decreased and green tones indicate increased animal activity).

Map analysis identifies what kind of change (termed the thematic attribute) occurred where (termed the spatial attribute). The characterization of what and where provides information needed for further GIS modeling, such as determining if areas of large increases in animal activity are correlated with particular cover types or near areas of low human activity.

Within this iterative processing structure, four fundamental classes of map analysis operations can be identified. These include:

- **Reclassify**ing Maps – involving the reassignment of the values of an existing map as a function of its initial value, position, size, shape or contiguity of the spatial configuration associated with each map category.
- **Overlay**ing Maps – resulting in the creation of a new map where the value assigned to every location is computed as a function of the independent values associated with that location on two or more maps.
- **Measuring Distance and Connectivity** – involving the creation of a new map expressing the distance and route between locations as straight-line length (simple proximity) or as a function of absolute or relative barriers (effective proximity).
- **Characterizing and Summarizing Neighborhoods** – resulting in the creation of a new map based on the consideration of values within the general vicinity of target locations.

Reclassification operations merely repackage existing information on a single map. Overlay operations, on the other hand, involve two or more maps and result in the delineation of new boundaries. Distance and connectivity operations are more advanced techniques that generate entirely new information by characterizing the relative positioning of map features. Neighborhood operations summarize the conditions occurring in the general vicinity of a location. See the Author’s Notes for links to more detailed discussions of the types of map analysis operations.

The reclassifying and overlaying operations based on point processing are the backbone of current GIS applications, allowing rapid updating and examination of mapped data. However, other than the significant advantage of speed and ability to handle tremendous volumes of data, most of these capabilities are similar to those of manual map processing. Map-wide overlays, distance and neighborhood operations, on the other hand, identify more advanced analytic capabilities and most often do not have analogous paper-map legacy procedures.

The mathematical structure and classification scheme of **Reclassify**, **Overlay**, **Distance** and **Neighbors** form a conceptual framework that is easily adapted to modeling spatial relationships in both physical and abstract systems. A major advantage is flexibility. For example, a model for siting a new highway can be developed as a series of processing steps. The analysis might consider economic and social concerns (e.g., proximity to high housing density, visual exposure to houses), as well as purely engineering ones (e.g., steep slopes, water bodies). The combined
expression of both physical and non-physical concerns within a quantified spatial context is another significant major benefit.

However, the ability to simulate various scenarios (e.g., steepness is twice as important as visual exposure and proximity to housing is four times more important than all other considerations) provides an opportunity to fully integrate spatial information into the decision-making process. By noting how often and where the proposed route changes as successive runs are made under varying assumptions, information on the unique sensitivity to siting a highway in a particular locale is described.

In the old environment, decision-makers attempted to interpret results, bounded by vague assumptions and system expressions of a specialist. Grid-based map analysis, on the other hand, engages decision-makers in the analytic process, as it both documents the thought process and encourages interaction. It’s sort of like a “spatial spreadsheet” containing map-matical equations (or recipes) that encapsulates the spatial reasoning of a problem and solves it using digital map variables.

**Getting the Numbers Right**

*(GeoWorld, May 2007, pg. 16-17)*

The concept that “maps are numbers first, pictures later” underlies all GIS processing. However in map analysis, the digital nature of maps takes on even more importance. How the map values are 1) retrieved and 2) processed establishes a basic framework for classifying all of the analytical capabilities. In obtaining map values for processing there are three basic methods— Local, Focal and Zonal (see author’s note).

While the Local/Focal/Zonal classification scheme is most frequently associated with grid-based modeling, it applies equally well to vector-based analysis— just substitute the concept of “polygon, line or point” for that of a grid “cell” as the smallest addressable unit of space providing the map values for processing.

*Local* processing retrieves a map value for a single map location independent of its surrounding values, then processes the value to derive and assign a new value to the location (figure 1). For example, an elevation value of 8250 associated with a grid cell location on an existing terrain surface is retrieved and then the contouring equation of \( \text{Interval} = \lceil \text{Integer}(\text{MapValue} - \text{ContourBase}) / \text{ContourInterval} \rceil = \lceil \text{int}((8250 + 100) / 100) \rceil = 83 \) is evaluated. The new map value of 83 is stored to indicate the 83\(^{rd}\) 100-foot contour interval (8200-8300 feet) from a sea level contour base interval of 1 (0 to 100 feet). The processing is repeated for all map locations and the resultant map is filed with the other map layers in the stack.
Figure 1. Local operations use point-by-point processing of map values that occur at each map location.

A similar operation might multiply the elevation value times 0.3048 \( [\text{ElevMeters} = \text{ElevFeet} \times 0.3048 = 8250 \times 0.3048 = 2871] \) to convert the elevation from feet to meters. In turn, a generalized atmospheric cooling relationship of 9.78 degC per 1000 meter rise can be applied \( [(2871 / 1000 \times 9.78)] \) to assign a value of 28.08 degC cooler than sea level air (termed Adiabatic Lapse Rate for those who are atmospheric physics challenged).

The lower portion of figure 1 expands the Local processing concept from a single map layer to a stack of registered map layers. For example, a point-by-point overlay process might retrieve the elevation, slope, aspect, fuel loading, weather, and other information from a series of map layers as values used in calculating wildfire risk for a location. Note that the processing is still spatially-myopic as it addresses a single map location at a time (grid cell) but obtains a string of values for that location before performing a mathematical or statistical process to summarize the values.

While the examples might not directly address your application interests, the assertion that you can add, subtract, multiply, divide and otherwise “crunch the numbers” ought to alert you to the map-ematical nature of GIS. It suggests a map calculator with all of the buttons, rights and privileges of your old friendly handheld calculator—except a map calculator operates on entire map layers composed of thousands upon thousands of geo-registered map values.

The underlying “cyclical” structure of \( \text{Retrieve} \rightarrow \text{Process} \rightarrow \text{Store} \rightarrow \text{File} \) also plays upon our traditional math experience. You enter a number or series of numbers into a calculator, press a function button and then store the intermediate result (calculator memory or scrap of paper) to be used as input for subsequent processing. You repeat the cycle over and over to solve a complex expression or model in a “piece-by-piece” fashion—whether traditional scaler mathematics or...
spatial map-ematics.

Figure 2 outlines a different class of analytical operators based on how the values for processing are obtained. *Focal* processing retrieves a set of map values within a neighborhood/vicinity around a location. For example, a 3x3 window could be used to identify the nine adjacent elevations at a location, and then apply a slope function to the data to calculate terrain steepness. The derived slope value is stored for the location and the process repeated over and over for all other locations in a project area.

![FOCAL Processing Operations Diagram](image-url)

**Figure 2.** Focal operations use a vicinity-context to retrieve map values for summary.

The concept of a fixed window of neighboring map values can be extended to other spatial contexts, such as effective distance, optimal paths, viewsheds, visual exposure and narrowness for defining the influence or “reach” around a map location. For example, a travel-time map considering the surrounding street network could be used to identify the total number of customers within a 10-minute drive. Or the total number of houses that are visually connected to a location within a half-mile could be calculated.

While Focal processing defines an “effective reach” to retrieve surrounding map values for processing, *Zonal* processing uses a predefine “template” to identify map values for summary (figure 3). For example, a wildlife habitat unit might serve as a template map to retrieve slope values from a data map of terrain steepness. The average of all of the coincident slope values is computed and then stored for all of the locations defining the template.

Similarly, a map of total sales (data map) can be calculated for a set of sales management districts (template map). The standard set of statistical summaries is extended to spatial operations such as contiguity and shape of individual map features.
Figure 3. Zonal operations use a separate template map to retrieve map values for summary.

The Local/Focal/Zonal organization scheme addresses how analytic operations work and is particularly appropriate for GIS developers and programmers. The Reclassify/Overlay/Distance/Neighbors scheme I have used throughout the Beyond Mapping series uses a different perspective—one based on the information derived and its utility (see, Use a Map-ematical Framework for GIS Modeling, GeoWorld, March 2004, pg 18-19).

However, both the “how it works” and “what it is” perspectives agree that all analytical operations require retrieving and processing numbers within a cyclical map-ematical environment. The bottom line being that maps are numbers and map analysis crunches the numbers in challenging ways well outside our paper-map legacy.

Author’s Note: Local, Focal and Zonal processing classes were first suggested by Dana Tomin in his doctoral dissertation “Geographic Information Systems and Cartographic Modeling” (Yale University, 1980) and partially used in organizing the Spatial Analyst/Grid modules in ESRI’s ArcGIS software.

Options Seem Endless When Reclassifying Maps
(geoWorld, April 2004, pg. 18-19)

The previous section described a map-ematical framework for GIS modeling. In the discussion four fundamental classes of analytical operations were identified—Reclassify, Overlay, Distance...
and Neighbors. Reclassifying maps, in many ways, represent the simplest group as they address the categories on a single map and do not change the spatial distribution of the data.

The reassignment of existing values can be made as a function of the initial value, position, contiguity, size, or shape of the spatial configuration of the individual map categories. Each reclassification operation involves the simple repackaging of information on a single map, and results in no new boundary delineation. Such operations can be thought of as the purposeful "re-coloring" of maps.

![Figure 1. Areas of meadow and forest on a COVERTYPE map can be reclassified to isolate large areas of open water.](image)

Figure 1 shows the result of simply reclassifying a map as a function of its initial thematic values. For display, a unique symbol is associated with each value. In the figure, the cover type map has categories of Open Water, Meadow and Forest. These features are stored as thematic values 1, 2 and 3, respectively, and displayed as separate colors blue, light green and dark green.

The binary map on the right side of the figure isolates the Open Water locations by simply assigning zero to the areas of Meadow and Forest and displaying as the categories as grey. Although the operation seems trivial by itself, it has map analysis implications far beyond simply re-coloring the map categories. And it graphically demonstrate the basic characteristic of reclassify operations—values change but the spatial pattern of the data doesn’t.

A similar reclassification operation might involve the ranking or weighing of qualitative map categories to generate a new map with quantitative values. For example, a map of soil types could be assigned values that indicate the relative suitability of each soil type for residential development.

Quantitative values also might be reclassified to yield new quantitative values. This might involve a specified reordering of map categories (e.g., given a map of soil moisture content, generate a map of suitability levels for plant growth). Or, it could involve the application of a generalized reclassifying function, such as "level slicing," which splits a continuous range of map category
values into discrete intervals (e.g., derivation of a contour map of just 10 contour intervals from an elevation surface composed of thousands of specific elevation values).

Other quantitative reclassification functions include a variety of arithmetic operations involving map category values and a specified or computed constant. Among these operations are addition, subtraction, multiplication, division, exponentiation, maximization, minimization, normalization and other scalar mathematical and statistical operators. For example, an elevation surface expressed in feet could be converted to meters by multiplying each map value by the appropriate conversion factor of 3.28083 feet per meter.

Reclassification operations can also relate to location, as well as purely thematic. One such characteristic is position. An overlay category represented by a single "point" location, for example, might be reclassified according to its latitude and longitude. Similarly, a line segment or area feature could be reassigned values indicating its center or general orientation.

A related operation, termed parceling, characterizes category contiguity. This procedure identifies individual "clumps" of one or more points that have the same numerical value and are spatially contiguous (e.g., generation of a map identifying each lake as a unique value from a generalized map of water representing all lakes as a single category).

Another location characteristic is size. In the case of map categories associated with linear features or point locations, overall length or number of points might be used as the basis for reclassifying the categories. Similarly, an overlay category associated with a planar area could be reclassified according to its total acreage or the length of its perimeter.

A map of water types, for example, could be reassigned values to indicate the area of individual lakes or the length of stream channels. The same sort of technique also could be used to deal with volume. Given a map of depth to bottom for a group of lakes, each lake might be assigned a value indicating total water volume based on the area of each depth category.

Figure 2 identifies a similar processing sequence using the information derived in figure 1. Although your eye sees two distinct blobs of water on the OPEN WATER map, the computer only "sees" distinctions by different map category values. Because both water bodies are assigned the same value of 1, there isn’t a map-ematical distinction that the computer cannot see the distinction.
Figure 2. A sequence of reclassification operations (renumber, clump, size and renumber) can be used to isolate large water bodies from a cover type map.

The Clump operation is used to identify the contiguous features as separate values—clump #1 (Larry’s Lake), #2 (Dry Land) and clump #3 (Peter’s Pond). The Size operation is then calculates the size of each clump—clump #1 = 78 hectares, clump #2 = 543 and clump #3 = 4. The final step uses the Renumber operation to isolate the large water body in the northwest portion of the project area.

In addition to the initial value, position, contiguity, and size of features, shape characteristics also can be used as the basis for reclassifying map categories. Shape characteristics associated with linear forms identify the patterns formed by multiple line segments (e.g., dendritic stream pattern). The primary shape characteristics associated with polygonal forms include feature integrity, boundary convexity, and nature of edge.

Feature integrity relates to an area’s “intact-ness”. A category that is broken into numerous "fragments" and/or contains several interior "holes" is said to have less spatial integrity than categories without such violations. Feature integrity can be summarized as the Euler Number that’s computed as the number of holes within a feature less one short of the number of fragments. Euler Numbers of zero indicates features that are spatially balanced, whereas larger negative or positive numbers indicate less spatial integrity—either broken into more pieces or poked with more holes.

Convexity and edge, are other shape indices that relate to the configuration of polygonal features’ boundaries. The Convexity Index for a feature is computed by the ratio of its perimeter to its...
area. The most regular configuration is that of a circle which is totally convex and, therefore, not enclosed by the background at any point along its boundary.

Comparison of a feature's computed convexity to a circle of the same area, results in a standard measure of boundary regularity. The nature of the boundary at each point can be used for a detailed description of boundary configuration. At some locations the boundary might be an entirely concave intrusion, whereas others might be at entirely convex protrusions. Depending on the "degree of edginess," each point can be assigned a value indicating the actual boundary convexity at that location.

This explicit use of cartographic shape as an analytic parameter is unfamiliar to most GIS users. However, a non-quantitative consideration of shape is implicit in any visual assessment of mapped data. Particularly promising is the potential for applying quantitative shape analysis techniques in the areas of digital image classification and wildlife habitat modeling.

A map of forest stands, for example, could be reclassified such that each stand is characterized according to the relative amount of forest edge with respect to total acreage and the frequency of interior forest canopy gaps. Stands with a large proportion of edge and a high frequency of gaps will generally indicate better wildlife habitat for many species. In any event, reclassify operations simply assign new values to old category values—some times seeming trivial and some times a bit conceptually complex.

**Overlay Operations Feature a Variety of Options**

*GeoWorld, May 2004, pg. 18-19*

The general class of overlay operations can be characterized as "light-table gymnastics." These involve the creation of a new map where the value assigned to every point, or set of points, is a function of the independent values associated with that location on two or more existing map layers. In **location-specific** overlaying, the value assigned is a function of the point-by-point coincidence of the existing maps. In **category-wide** composites, values are assigned to entire thematic regions as a function of the values on other overlays that are associated with the categories. Whereas the first overlay approach conceptually involves the vertical spearing of a set of map layers, the latter approach uses one map to identify boundaries by which information is extracted from other maps.

Figure 1 shows an example of location-specific overlaying. Here, maps of COVERTYPE and topographic SLOPE_CLASSES are combined to create a new map identifying the particular cover/slope combination at each location. A specific function used to compute new category
values from those of existing maps being overlaid can vary according to the nature of the data being processed and the specific use of that data within a modeling context. Environmental analyses typically involve the manipulation of quantitative values to generate new values that are likewise quantitative in nature. Among these are the basic arithmetic operations such as addition, subtraction, multiplication, division, roots, and exponentials.

Functions that relate to simple statistical parameters such as maximum, minimum, median, mode, majority, standard deviation or weighted average also can be applied. The type of data being manipulated dictates the appropriateness of the mathematical or statistical procedure used. For example, the addition of qualitative maps such as soils and land use would result in mathematically meaningless sums, since their thematic values have no numerical relationship. Other map overlay techniques include several that might be used to process either quantitative or qualitative data and generate values which can likewise take either form. Among these are masking, comparison, calculation of diversity, and permutations of map categories (as depicted in figure 1).

More complex statistical techniques may also be applied in this manner, assuming that the inherent interdependence among spatial observations can be taken into account. This approach treats each map as a variable, each point as a case, and each value as an observation. A predictive statistical model can then be evaluated for each location, resulting in a spatially continuous surface of predicted values. The mapped predictions contain additional information over traditional non-spatial procedures, such as direct consideration of coincidence among regression variables.
and the ability to spatially locate areas of a given level of prediction. Topic 12 investigates the considerations in spatial data mining derived by statistically overlaying mapped data.

An entirely different approach to overlaying maps involves category-wide summarization of values. Rather than combining information on a point-by-point basis, this group summarizes the spatial coincidence of entire categories shown on one map with the values contained on another map(s). Figure 2 contains an example of a category-wide overlay operation. In this example, the categories of the COVERTYPE map are used to define an area over which the coincidental values of the SLOPE map are averaged. The computed values of average slope within each category area are then assigned to each of the cover type categories.

Summary statistics which can be used in this way include the total, average, maximum, minimum, median, mode, or minority value; the standard deviation, variance, or diversity of values; and the correlation, deviation, or uniqueness of particular value combinations. For example, a map indicating the proportion of undeveloped land within each of several counties could be generated by superimposing a map of county boundaries on a map of land use and computing the ratio of undeveloped land to the total land area for each county. Or a map of zip code boundaries could be superimposed over maps of demographic data to determine the average income, average age, and dominant ethnic group within each zip code.

Figure 2. Category-wide overlay operations summarize the spatial coincidence of map categories, such as generating the average SLOPE for each COVERTYPE category.
As with location-specific overlay techniques, data types must be consistent with the summary procedure used. Also of concern is the order of data processing. Operations such as addition and multiplication are independent of the order of processing. Other operations, such as subtraction and division, however, yield different results depending on the order in which a group of numbers is processed. This latter type of operations, termed non-commutative, cannot be used for category-wide summaries.

**Computers Quickly Characterize Spatial Coincidence**

*(GeoWorld, June 2004, pg. 18-19)*

As previously noted, GIS maps are numbers and a rigorous, quantitative approach to map analysis can be maintained. However, most of our prior experience with maps is non-quantitative, using paper maps composed of inked lines, shadings, symbols and zip-a-tone. We rarely think of map uncertainty and error propagation. And we certainly wouldn't think of demanding such capabilities in our GIS software. That is, not as of yet.

Everybody knows the 'bread and butter' of a GIS is its ability to overlay maps. To a human, map overlay involves "light-table gymnastics" where we peer through a stack of acetate sheets and interpreting the subtle hues of resulting colors. To a GIS you're asking the computer to identify the condition from each map layer for every location in a project area. From the computer's perspective, however, this is simply one of a host of ways to characterize the spatial coincidence.

Let's compare how you and your computer might approach the task of identifying coincidence. Your eye moves randomly about the stack, pausing for a nanosecond at each location and mentally establishing the conditions by interpreting the color. Your summary might conclude that the northeastern portion of the area is unfavorable as it has "kind of a magenta tone." This is the result of visually combining steep slopes portrayed as bright red with unstable soils portrayed as bright blue with minimal vegetation portrayed as dark green. If you want to express the result in map form, you would tape a clear acetate sheet on top and delineate globs of color differences and label each parcel with your interpretation. Whew! No wonder you want a GIS.

The GIS goes about the task in a very similar manner. In a vector system, line segments defining polygon boundaries are tested to determine if they cross. When a line on one map crosses a line on another map, a new combinatorial polygonal is indicated. Trigonometry is employed, and the X,Y coordinate of the intersection of the lines is computed. The two line segments are split into four and values identifying the combined map conditions are assigned. The result of all this crossing and splitting is the set of polygonal prodigy you so laboriously delineated by hand.
A raster system has things a bit easier. As all locations are predefined as a consistent set of cells within a matrix, the computer merely 'goes' to a location, retrieves the information stored for each map layer and assigns a value indicating the combined map conditions. The result is a new set of values for the matrix identifying the coincidence of the maps.

The big difference between ocular and computer approaches to map overlay is not so much in technique, as it is in the treatment of the data. If you have several maps to overlay you quickly run out of distinct colors and the whole stack of maps goes to an indistinguishable dark, purplish hue. One remedy is to classify each map layer into just two categories, such as suitable and unsuitable. Keep one as clear acetate (good) and shade the other as light grey (bad). The resulting stack avoids the ambiguities of color combinations, and depicts the best areas as lighter tones. However, in making the technique operable you have severely limited the content of the data—just good and bad.

The computer can mimic this technique by using binary maps. A "0" is assigned to good conditions and a "1" is assigned to bad conditions. The sum of the maps has the same information as the brightness scale you observe—the smaller the value the better. The two basic forms of logical combination can be computed. "Find those locations which have good slopes .AND. good soils .AND. good vegetative cover." Your eye sees this as the perfectly clear locations. The computer sees this as the numeric pattern 0-0-0. "Find those locations which have good slopes .OR. good soils .OR. good vegetative cover." To you this is could be any location that is not the darkest shading; to the computer it is any numeric pattern that has at least one 0. But how would you handle, "Find those locations which have good slopes .OR. good soils .AND. good vegetative cover"? You can't find them by simply viewing the stack of maps. You would have to spent a lot of time flipping through the stack. To the computer, this is simply the patterns 0-1-0, 1-0-0 and 0-0-0. It's a piece of cake from the digital perspective.

In fact any combination is easy to identify. Let's say we expand our informational scale and redefine each map from just good and bad to not suitable (0), poor (1), marginal (2), good (3) and excellent (4). We could ask the computer to INTERSECT SLOPES WITH SOILS WITH COVER COMPLETELY FOR ALL-COMBINATIONS. The result is a map indicating all combinations that actually occur among the three maps. Likely this map would be too complex for human viewing enjoyment, but it contains the detailed information basic to many application models. A more direct approach is a geographic search for the best areas invoked by asking to INTERSECT SLOPES WITH SOILS WITH COVER FOR EXCELLENT-AREAS ASSIGNING 1 TO 4 AND 4 AND 4. Any combination not assigned a value drops to 0, leaving a map with 1's indicating the excellent areas.

Another way of combining these maps is by asking to COMPUTE SLOPES MINIMIZE SOILS MINIMIZE COVER FOR WEAK-LINK. The resulting map's values indicate the minimal coincidence rating for each location. Low values indicate areas of concern and a 0 indicate areas to dismiss as not suitable from at least one map's information. There is a host of other computational operations you could invoke, such as plus, minus, times, divide, and
exponentiation. Just look at the functional keys on your hand calculator. But you may wonder, “why would someone want to raise one map to the power of another”? Spatial modelers who have gone beyond mapping, that's who.

What would happen if, for each location (be it a polygon or a cell), we computed the sum of the three maps, then divided by the number of maps? That would yield the average rating for each location. Those with the higher averages are better. Right? You might want to take it a few steps further. First, in a particular application, some maps may be more important than others in determining the best areas. Ask the computer to AVERAGE SLOPES TIMES 5 WITH SOILS TIMES 3 WITH COVER TIMES 1 FOR WEIGHTED-AVERAGE. The result is a map whose average ratings are more heavily influenced by slope and soil conditions.

Just to get a handle on the variability of ratings at each location, you can determine the standard deviation—either simple or weighted. Or for even more information, determine the coefficient of variation, which is the ratio of the standard deviation to the average, expressed as a percent. What will that tell you? It hints at the degree of confidence you should put into the average rating. A high COFFVAR indicates wildly fluctuating ratings among the maps and you might want to look at the actual combinations before making a decision.

A statistical way to summarized coincidence between maps is a cross-tab table. If you CROSSTAB FORESTS WITH SOILS a table results identifying how often each forest type jointly occurs with each soil type. In a vector system, this is the total area in each forest/soil combinations. In a raster system, this is simply a count of all the cell locations for each forest/soil combination.

For example, reading across the first row of table in figure 1 notes that Forest category 1 (Deciduous) contains 303 cells distributed throughout the map. The total count for Soils category 1 (Lowland) is 427 cells. The next section of the table notes that the joint condition of Deciduous/Lowland occurs 299 times for 47.84 percent of the total map area. Contrast this result with that of Deciduous/Upland occurrence on the row below indicating only four “crosses” for less than one percent of the map. The coincidence statistics for the Conifer category is more balanced with 128 cells (20.48%) occurring with the Lowland soil and 194 cells (31.04%) occurring with the Upland soil.
Figure 1. A cross-tab table statistically summarizes the coincidence among the categories on two maps.

These data may cause you to jump to some conclusions, but you had better consider the last section of the Table before you do. This section normalizes the coincidence count to the total number of cells in each category. For example, the 299 Deciduous/Lowland coincidence accounts for 98.68 percent of all occurrences of Deciduous trees ((299/303)*100). That's a very strong relationship. However, from Lowland soil occurrence the 299 Deciduous/Lowland coincidence is a bit weaker as it accounts for only 70.02 percent of all occurrences of Lowland soils ((299/427)*100). In a similar vein, the Conifer/Upland coincidence is very strong as it accounts for 97.98 percent of the occurrence of all Upland soil occurrences. Both columns of coincidence percentages must be considered as a single high percent might be merely the result of the other category occurring just about everywhere.

There are still a couple of loose ends before we can wrap-up point-by-point overlay summaries. One is direct map comparison, or change detection. For example, if you encode a series of land use maps for an area, then subtract each successive pair of maps; the locations that underwent change will appear as non-zero values for each time step. In GIS-speak, you would enter COMPUTE LANDUSE-T2 MINUS LANDUSE-T1 FOR CHANGE-T2&1 for a map of the land use change.

If you are real tricky and think map-ematically you will assign a binary progression to the land use categories (1,2,4,8,16, etc.), as the differences will automatically identify the nature of the change. The only way you can get a 1 is 2-1; a 2 is 4-2; a 3 is 4-1; a 6 is 8-2; etc. A negative sign indicates the opposite change, and now all bases are covered.

The last point-by-point operation is a weird one—covering. This operation is truly spatial and has no traditional math counterpart. Imagine you prepared two acetate sheets by coloring all of the
forested areas opaque green on one sheet and all of the roads an opaque red on the other sheet. Now overlay them on a light-table. If you place the forest sheet down first the red roads will “cover” the green forests and you will see the roads passing through the forests. If the roads map goes down first, the red lines will stop abruptly at the green forest globs.

In a GIS, however, the colors become numbers and the clear acetate is assigned zero. The command COVER FORESTS WITH ROADS causes the computer to go to a location and assess the shish kabob of values it finds. If the kabob value for roads is 0 (clear), keep the forest value underneath it. If the road value is non-zero, place that value at the location, regardless of the value underneath.

**Key Concepts Characterize Unique Conditions**

*GeoWorld April, 2006, pg. 14-15*

Back in the days of old GIS (a couple of decades ago) computer power was but a fraction of what sits on most desks today. As a result, CPU cycles and online storage for map analysis were severely rationed. Maps of a megabyte were relegated to super computers behind glass windows in air conditioned rooms.

In these austere conditions programmers searched for algorithms and data structures that saved nanoseconds and kilobytes. The concept of a Universal Polygon Coverage was a mainstay in vector-based processing. By smashing a stack of relatively static map layers together a single map was generated that contained all of the son and daughter polygons. This “compute once/use many” approach had significant efficiency gains as coincidence overlay was solved once then table query and math simply summarized the combinations as needed.

The top portion of figure 1 shows a simplified schematic of the Universal Polygon approach. The lines defining the individual polygons on the three vector maps are intersected (analogous to throwing spaghetti on a wall) to identify various combinations of the input conditions as depicted in the large map in the middle.
Figure 1. Comparison of the related concepts of Universal Polygon Coverage (vector) and Unique Conditions Matrix (grid).

For example, consider the unique combination of A2, B1 and C1. The intersections of the lines identify nodes that split the parent polygon boundaries into the segments defining the six son and daughter polygons of the combined conditions. The result is a single spatial table containing all of the original delineations plus the derived coincidence overlay information. Its corresponding attribute table can be easily searched for any combination of conditions and/or mathematically manipulated to generate a new field in the table—fast and efficient.

This pre-processing technique also works for grid-based data (bottom portion of figure 1). The Unique Conditions Matrix smashes a stack of grid layers together assigning a unique value for each possible combination of conditions. For example, the A2, B1 and C1 combination (termed a cohort) is defined by the grid cell block shown in the right portion of the figure. Grid location column= 12 and row= 12 is one of the forty member cohort.

Figure 2 depicts a natural extension of the procedure involving two tables—a Value Attribute Table and a Cohort Membership Table. For example, given grid layers for travel-time for six stores classified into proximity classes (1= close, 2, 3 and 4= far) results in 4096 possible combinations. Each combination, in turn, forms a single record (row) in the Value Attribute Table (VAT) file. The record contains a unique cohort ID number and a field (column) for each
of the base map conditions (figure 2, step 1). The user can operate on this table to derive new information, such as the minimum travel-time to the closest store, which is appended as a new field (step 2).

![Figure 2. Steps linking unique conditions and cohort membership tables for efficient and fast coincidence analysis.](image)

Note that the VAT records identify unique conditions occurring anywhere a project area whether the grid cells occur in few large clumps or widely dispersed. The Cohort Membership Table (CMT) contains a string of values identifying the column/row coordinates for each grid cell in the list. Thoughtful organization of the list can implicitly carry information about the spatial patterns within and among the cohorts of cells.

Step 3 links the attribute and spatial tables through their common ID numbers. Any of the fields in the VAT can be mapped by assigning the derived values to the cells defining each cohort in the CMT (step 4).

Sounds easy and straightforward but there are a few caveats. First, the technique only addresses point-by-point coincidence overlay and myopically ignores surrounding conditions. Also, the conditions need to be fairly stable, such as proximity and slope. Finally, it requires continuous data to be reassigned into few discrete classes on a relatively small number of map layers or the number of combinations explodes. For example, 20 map layers with only four classification

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categories on each, results in over sixteen million possible cohorts; a number that challenges most database tables.

The advantage of the Unique Conditions technique in appropriate application settings is that you calculate once (VAT) and use many (CMT)—efficient and fast grid processing. In the early years this was imperative for even the basic GIS models. Today it is becoming imperative again due to the extent and complexity of sophisticated models. For example, deriving a wildfire risk map for California at a 25 meter scale involves over 500 million grid cells. Saving a few billion redundant computations and a few hundred gigabytes or so of storage can actually add up—even in today’s “super computer on a desk” environment.

**Use “Shadow Maps” to Understand Overlay Errors**

*(GeoWorld, September 2004, pg. 18-19)*

The past discussions have focused on map overlay by describing some of the procedures, considerations and applications. However, now is the moment of atonement—the pitfalls of introduced error and its propagation. Keep in mind that there are two broad types of errors in GIS: those that are present in the encoded base maps and those that arise during map analysis. Better field collection, encoding and handling of base data address the former and improved processing algorithms and procedures address the latter.

Be realistic—soil or forest maps are just estimates of the actual conditions and geographic patterns of these features. No one used a transit to survey the precise and sharp boundary lines that delineate the implied distinct spatial objects. Soil and forest parcels aren’t discrete things in either space or time and significant judgment is used in forcing boundaries around them. Under some conditions the guesses are pretty good; under other conditions, they can be pretty bad. So how can the computer “see” where things are good and bad?

That’s where the concept of a “shadow map of certainty” comes in and directs attention to both the amount and pattern of map certainty. The left side of figure 1 depicts such an information sandwich of a typical soil map with a shadow map of certainty glued to its bottom. In this way the relative certainty of the classification is known for each map location—look at the top map to see the soil classification, then peer through to the bottom map to see how likely that classification is at that location.

In this instance, any location with 100 meters of a soil boundary is assigned only 50% certainty (orange) of correct classification while interior areas of large features are assigned 100% certainty (grey). The assumption reflects the thought that “there is a soil boundary around here somewhere, but I am just not sure exactly where.”
A similar shadow map of certainty can be developed for the forest information (right side of figure 1). This simple estimate of certainty accounts for one photo interpreter reaching out to add a tree along the edge and another interpreter deciding not to. Both interpreters “see” the interior of the forest parcel (certain) but discretion is used to form its border (less certain). The 100m certainty buffer reflects a bit of wiggle-room for interpretation.

Figure 1. Thematic maps with their corresponding shadow maps of certainty. The orange areas indicate “less certain” areas (.50 probability) that are adjacent to a boundary.

Remote sensing classification, however, provides a great deal more information about map certainty. For example, the “maximum likelihood classifier” determines the probability that a location is one of a number of different forest types using a library of “spectral signatures” and multivariate statistics. In a sense the computer thinks “given the color pattern for an area and knowledge of the various color patterns for forest types in the area, which color most closely matches the appearance of the area”—analogous to the manual interpretation process.

After a nanosecond or so, the computer decides which known pattern is the closest, classifies it as that forest type, and then moves on to consider the next grid cell. For example, riparian and aspens tend to reflect lighter greens while pines and firs tend to reflect darker greens. The important point is that the computer isn’t “seeing” subtle color differences; it is analyzing numerical values to calculate the probability that a location is one of any of the possible choices.

Traditionally, the approach simply chooses the most likely spectral signature, classifies it, and then throws away the information on relative certainty of the classification. Heck, the historical objective was to produce a map, not data. In addition, a spline function often is used to inscribe a seemingly precise line around groups of similar classified cells so the results look more like a manually drafted map. The result is a set of spatially discrete objects implying perfect data for a warm and fuzzy feeling, but disregarding the spatial resolution and classification probabilities employed—sort of like throwing the bathwater out with the baby.

As GIS technology matures, the ability to discern map certainty will become increasingly
important. There are several mechanisms for viewing a shadow map of certainty information. The simplest is to include the certainty value as a mouse-over that displays the certainty value as the cursor moves about a map. Another sets the hue/color for different forest types (i.e., red for pine) then drives the brightness gun based on the certainty value at each location. You see a gradient of washed-out pink for areas of very uncertain pine classification through a bright red for areas whose classification are very certain.

However, the real power of a shadow map of certainty is in addressing processing errors that occur during map overlay. Common sense suggests that if you overlay a fairly uncertain map with another fairly uncertain map, chances are the resulting coincidence map is riddled with even more uncertainty. But the problem is more complex, as it is dependent on the intersection of the unique spatial patterns of certainty of the two (or more) maps.

Consider overlaying the soils and forest maps as depicted in figure 2. A simple map overlay considers just the thematic map information and summarizes the coincidence between the two maps. Note that both of the “speared” locations identify Lowland soils occurring with Aspen forest cover.

Now consider the effect of certainty propagation. The location on the left is certain on both

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Page 24
maps, so the coincidence is certain (1.00 * 1.00 = 1.00). However, the location on the right is less certain on both maps, so the chance that it contains both lowland soils and aspen is very uncertain certain (.50 * .50 = .25).

The ability to quantify and account for map certainty is a critical step in moving GIS beyond mapping. As map analysis becomes more entrenched in advanced applications, the ability to tell where GIS products are good or bad becomes at least as important as their graphical display and Internet availability. Heck, there might a bunch of good looking maps out there but can you stake your professional life on them?