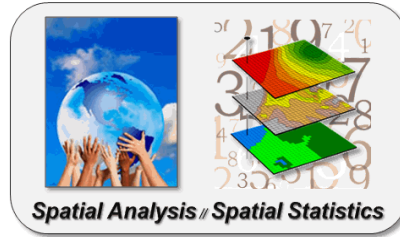


# ***Spatial/STEM:***

## ***A Mathematical/Statistical Framework for Understanding and Communicating Map Analysis and Modeling***



**Premise: There is a “map-ematics” that extends traditional math/stat concepts and procedures for the quantitative analysis of map variables (spatial data)**

This presentation provides a fresh perspective on interdisciplinary instruction at the college level by combining the philosophy and approach of *STEM* with the spatial reasoning and analytical power of grid-based Map Analysis and Modeling

*This PowerPoint with notes and online links to further reading is posted at*  
[www.innovativegis.com/basis/Courses/SpatialSTEM/](http://www.innovativegis.com/basis/Courses/SpatialSTEM/)

*Presented by*

**Joseph K. Berry**

Adjunct Faculty in Geosciences, Department of Geography, University of Denver  
Adjunct Faculty in Natural Resources, Warner College of Natural Resources, Colorado State University  
Principal, Berry & Associates // Spatial Information Systems

Email: [jberry@innovativegis.com](mailto:jberry@innovativegis.com) — Website: [www.innovativegis.com/basis](http://www.innovativegis.com/basis)

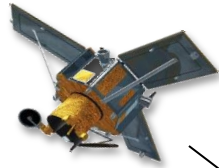
(Nanotechnology)

# Geotechnology

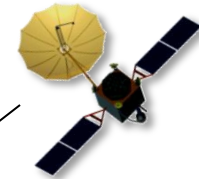
(Biotechnology)

**Geotechnology** is one of the three "mega technologies" for the 21st century and promises to forever change how we conceptualize, utilize and visualize spatial relationships in scientific research and commercial applications (U.S. Department of Labor)

## Geographic Information Systems (map and analyze)



**Remote Sensing**  
(measure and classify)



**Global Positioning System** (location and navigation)



GPS/GIS/RS

### The Spatial Triad

Computer Mapping (70s)  
Spatial Database Management (80s)

#### Technological Tool

**Mapping**  
involves precise placement (delineation) of physical features (graphical inventory)

Map Analysis (90s)  
Multimedia Mapping (00s)

#### Analytical Tool

**Modeling** involves analysis of spatial patterns and relationships (map analysis/modeling)

Where



Descriptive Mapping

is



What

Prescriptive Modeling



Why

So What

and

What If

# A Mathematical Structure for Map Analysis/Modeling

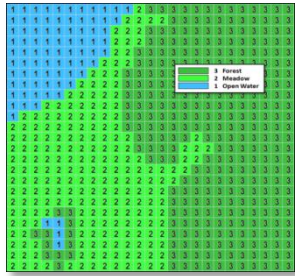
Technological Tool

Geotechnology → RS — **GIS** — GPS

Analytical Tool

Mapping/Geo-Query (*Discrete, Spatial Objects*) (*Continuous, Map Surfaces*)

**Map Analysis/Modeling**



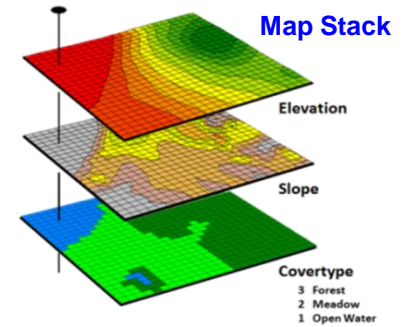
...organized set of numbers

Geo-registered  
Analysis Frame → Matrix  
of Numbers

“Map-ematics”

**Maps as Data, not Pictures**

Vector & *Raster* — Aggregated & *Disaggregated*  
Qualitative & *Quantitative*



**Spatial Analysis Operations**

Grid-based  
Map Analysis  
Toolbox

**Spatial Statistics Operations**

GISer’s Perspective:

*Reclassify and Overlay*  
*Distance and Neighbors*

GISer’s Perspective:

*Surface Modeling*  
*Spatial Data Mining*

Mathematician’s Perspective:

*Basic GridMath & Map Algebra*  
*Advanced GridMath*  
*Map Calculus*  
*Map Geometry*  
*Plane Geometry Connectivity*  
*Solid Geometry Connectivity*  
*Unique Map Analytics*



**The SpatialSTEM  
Framework**

Traditional math/stat procedures  
can be extended into  
geographic space to stimulate  
those with diverse backgrounds  
and interests for...

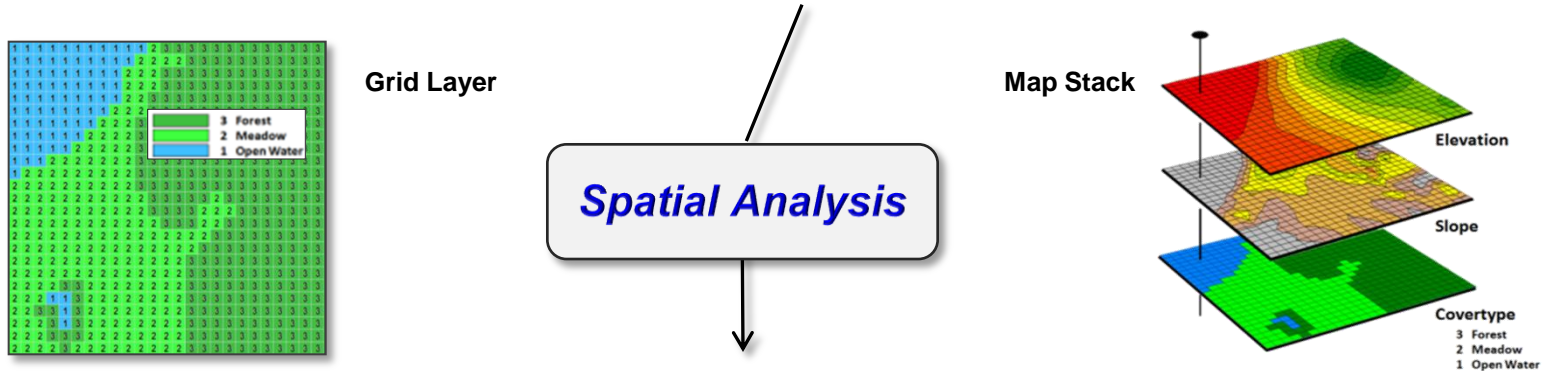
“*thinking analytically  
with maps*”

Statistician’s Perspective:

*Basic Descriptive Statistics*  
*Basic Classification*  
*Map Comparison*  
*Unique Map Statistics*  
*Surface Modeling*  
*Advanced Classification*  
*Predictive Statistics*

# Spatial Analysis Operations (Geographic Context)

GIS as “Technical Tool” (*Where is What*) vs. “**Analytical Tool**” (*Why, So What and What if*)



**Spatial Analysis** extends the basic set of discrete map features (points, lines and polygons) to map **surfaces** that represent continuous geographic space as a set of contiguous grid cells (matrix), thereby providing a **Mathematical Framework** for *map analysis* and *modeling* of the

**Contextual Spatial Relationships** within and among grid map layers

Map Analysis Toolbox



✓ Unique spatial operations

## **Mathematical Perspective:**

**Basic GridMath & Map Algebra** (+ - \* /)

**Advanced GridMath** (Math, Trig, Logical Functions)

**Map Calculus** (Spatial Derivative, Spatial Integral)

**Map Geometry** (Euclidian Proximity, Effective Proximity, Narrowness)

**Plane Geometry Connectivity** (Optimal Path, Optimal Path Density)

**Solid Geometry Connectivity** (Viewshed, Visual Exposure)

✓ **Unique Map Analytics** (Contiguity, Size/Shape/Integrity, Masking, Profile)

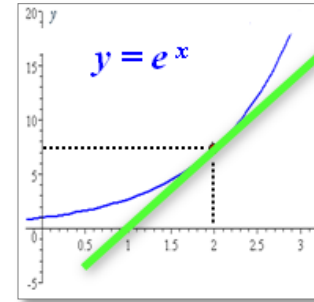
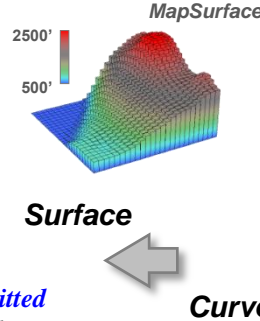
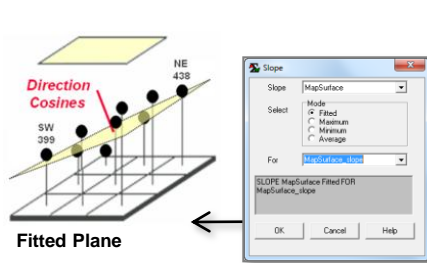
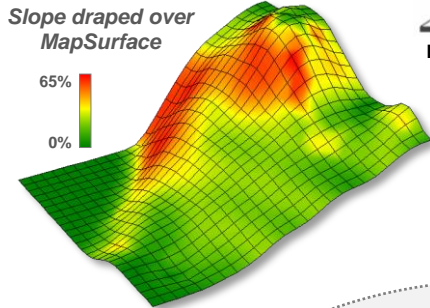
# Spatial Analysis Operations (Math Examples)



**Advanced Grid Math** — Math, Trig, Logical Functions  
**Map Calculus** — Spatial Derivative, Spatial Integral

## Spatial Derivative

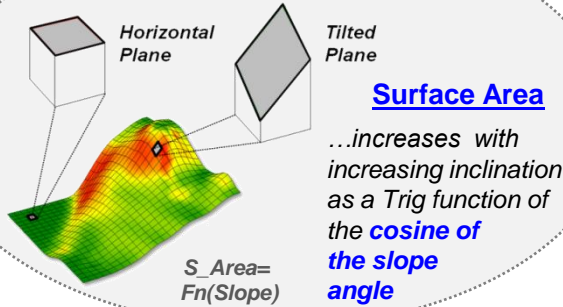
...is equivalent to the slope of the **tangent plane** at a location



The **derivative** is the instantaneous "rate of change" of a function and is equivalent to the slope of the **tangent line** at a point

$D_{z,xy}$  Elevation

## Advanced Grid Math



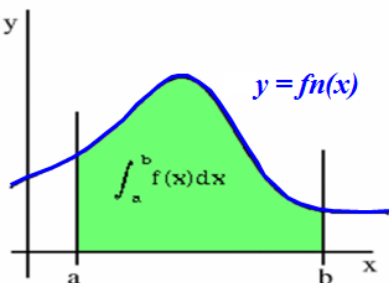
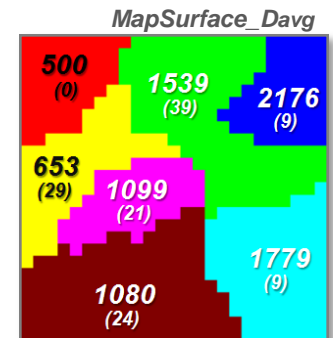
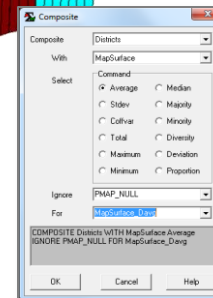
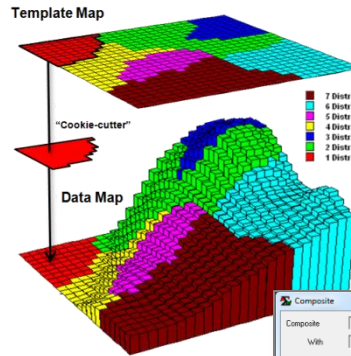
$S\_area = cellsize / \cos(D_{z,xy} \text{ Elevation})$

$\int$  Districts\_Average Elevation

## Spatial Integral

...summarizes the values on a surface for specified map areas (Total= **volume** under the surface)

COMPOSITE Districts WITH MapSurface Average FOR MapSurface\_Davg



The **integral** calculates the **area** under the curve for any section of a function.

Surface

Curve

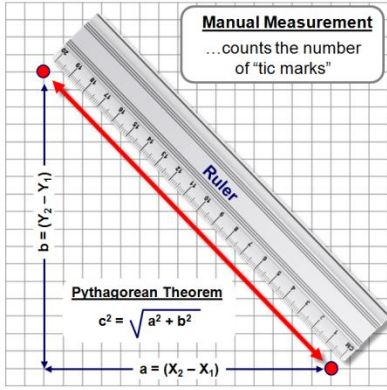
# Spatial Analysis Operations *(Distance Examples)*

**Map Geometry** — (Euclidian Proximity, Effective Proximity, Narrowness)

**Plane Geometry Connectivity** — (Optimal Path, Optimal Path Density)

**Solid Geometry Connectivity** — (Viewshed, Visual Exposure)

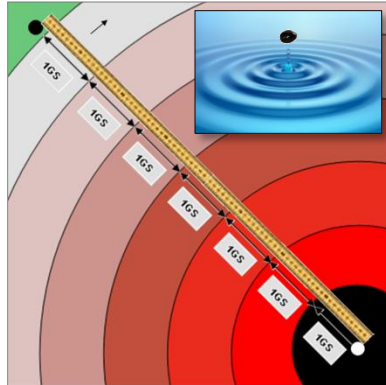
## Distance



**Mathematical Measurement**  
...solves the Pythagorean Theorem

Shortest straight line between two points...

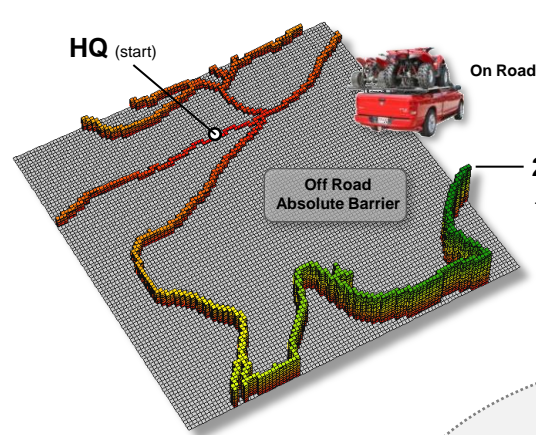
## Euclidean Proximity



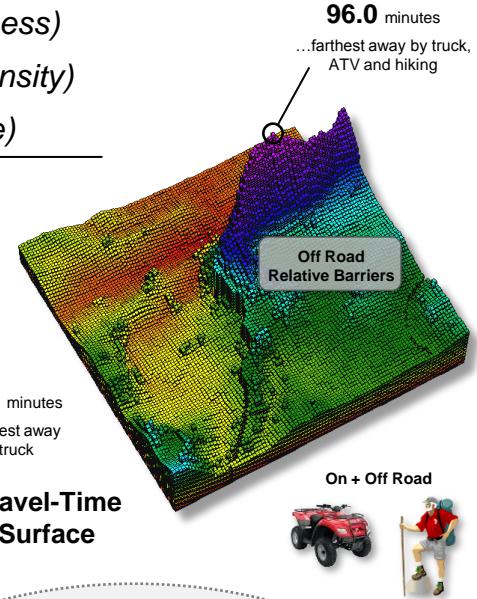
...from a point to everywhere...

...not necessarily straight lines (movement)

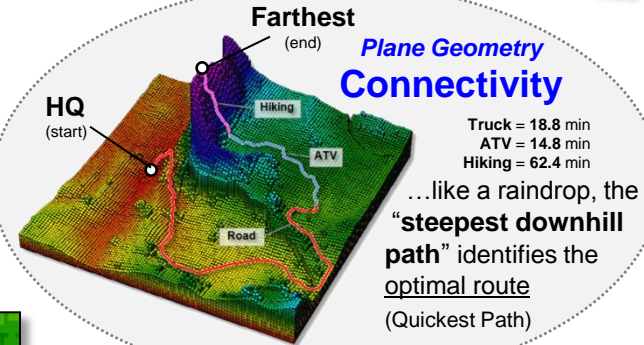
## Effective Proximity



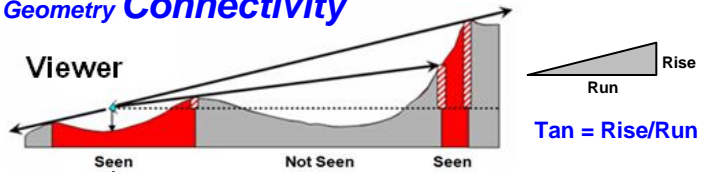
...like a raindrop, the "steepest downhill path" identifies the optimal route (Quickest Path)



## Plane Geometry Connectivity



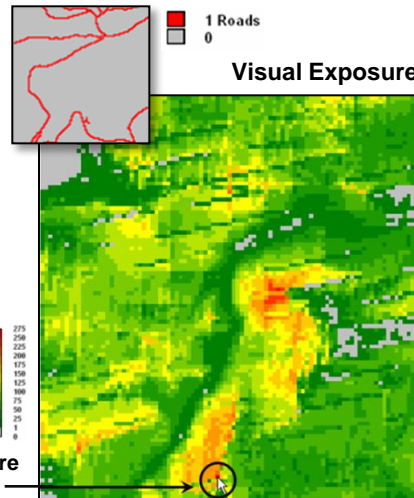
## Solid Geometry Connectivity



Seen if **new tangent exceeds all previous tangents** along the line of sight



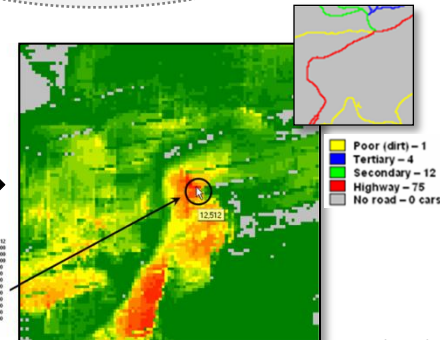
270/621 = 43% of the entire road network is connected



Counts  
# Viewers

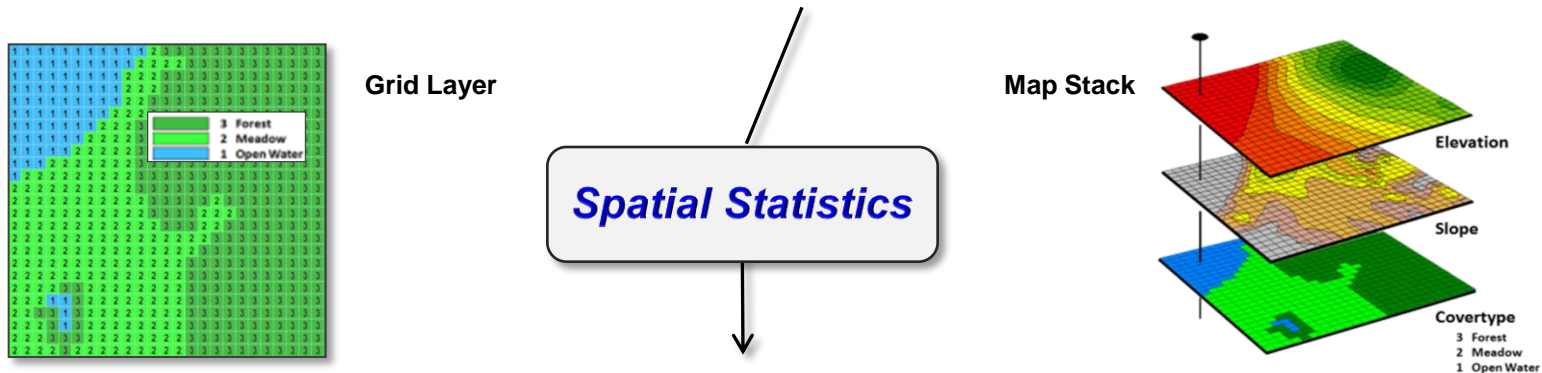
Sums  
Viewer Weights

Highest Weighted Exposure



# Spatial Statistics Operations (Numeric Context)

GIS as “Technical Tool” (*Where is What*) vs. “**Analytical Tool**” (*Why, So What and What if*)



**Spatial Statistics** seeks to map the variation in a data set instead of focusing on a single typical response (central tendency), thereby providing a **Statistical Framework** for *map analysis* and *modeling* of the

Numerical Spatial Relationships within and among grid map layers

## Statistical Perspective:

Map Analysis Toolbox



**Basic Descriptive Statistics** (*Min, Max, Median, Mean, StDev, etc.*)

**Basic Classification** (*Reclassify, Contouring, Normalization*)

**Map Comparison** (*Joint Coincidence, Statistical Tests*)

✓ **Unique Map Statistics** (*Roving Window and Regional Summaries*)

✓ **Surface Modeling** (*Density Analysis, Spatial Interpolation*)

**Advanced Classification** (*Map Similarity, Maximum Likelihood, Clustering*)

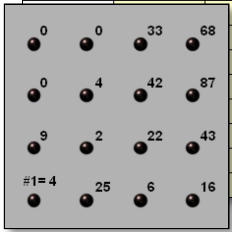
**Predictive Statistics** (*Map Correlation/Regression, Data Mining Engines*)

✓ **Unique spatial operations**

# Spatial Statistics *(Linking Data Space with Geographic Space)*

## Geo-registered Sample Data

Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
	1000	1000	10	6
	1500	17	22	
	2000	34	42	
	2500	22	33	
	1000	20	16	
	1500	28	43	
	2000	42	87	
	2500	34	68	



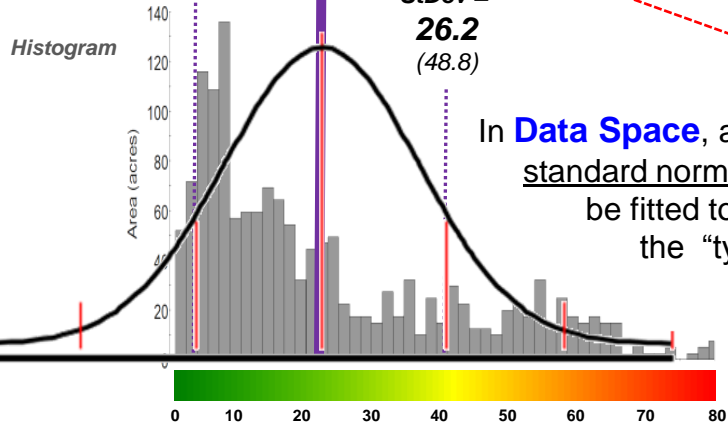
Non-Spatial Statistics

## Standard Normal Curve

Average = 22.6

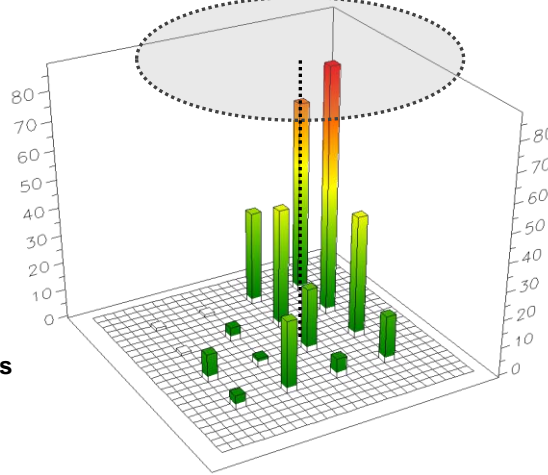
StDev = 26.2 (48.8)

In **Data Space**, a standard normal curve can be fitted to the data to identify the "typical value" (average)



## Numeric Distribution

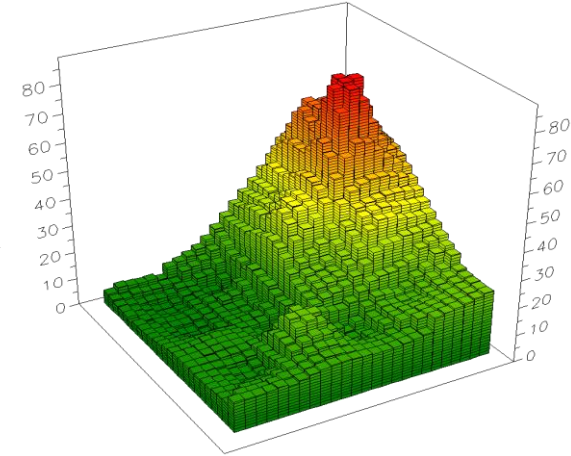
## Roving Window (weighted average)



## Discrete Sample Map

Spatial Statistics

## Spatial Distribution



## Continuous Map Surface

**Surface Modeling** techniques are used to derive a continuous map surface from discrete point data— fits a Surface to the data (maps the variation).

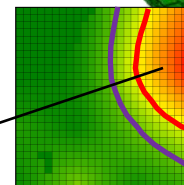
In **Geographic Space**, the typical value forms a horizontal plane implying the average is everywhere to form a horizontal plane

$\bar{X} = 22.6$

...lots of NE locations exceed Mean + 1StDev

$$\begin{aligned} \bar{X} + 1\text{StDev} &= 22.6 + 26.2 \\ &= 48.8 \end{aligned}$$

Unusually high values

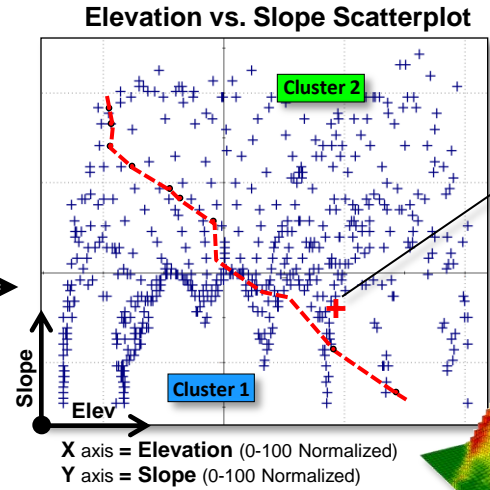
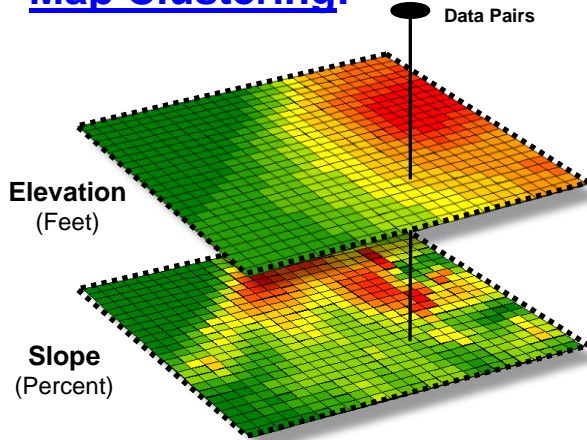


+StDev  
Average

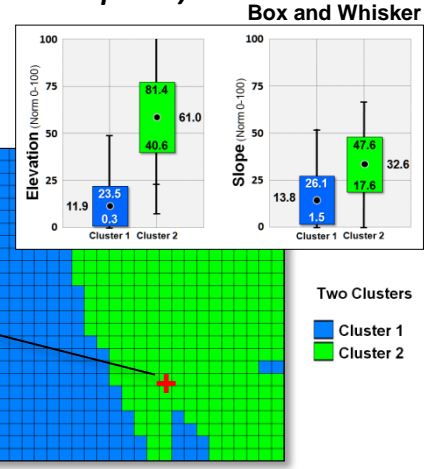


# Spatial Statistics Operations *(Data Mining Examples)*

## Map Clustering:



Plots here in...  
**Data Space**  
 →  
**Geographic Space**

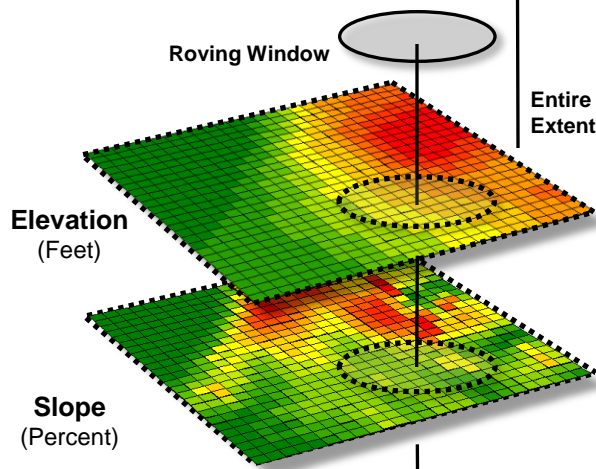


## Advanced Classification *(Clustering)*

## Data Space

## Geographic Space

## Map Correlation:



## Spatially Aggregated Correlation

**Scalar Value** – one value represents the overall non-spatial relationship between the two map surfaces

...1 large data table  
 with 25 rows x 25 columns =  
 625 map values for map wide summary

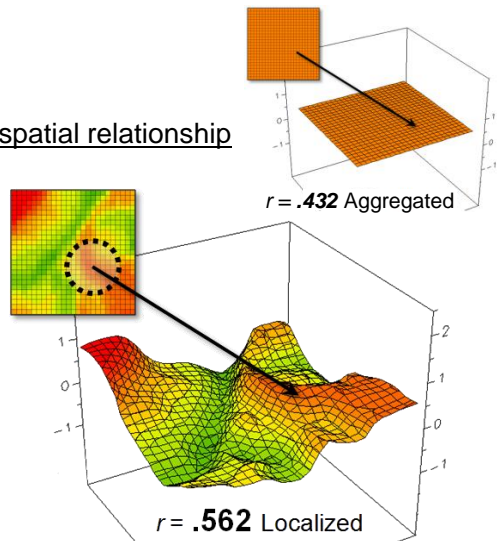
$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{([n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2])}}$$

...where **x** = Elevation value and **y** = Slope value  
 and **n** = number of value pairs

...625 small data tables  
 within 5 cell reach =  
 81 map values for localized summary

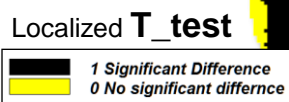
## Localized Correlation

**Map Variable** – continuous quantitative surface represents the localized spatial relationship between the two map surfaces



## Predictive Statistics *(Correlation)*

## Map Comparison *(Statistical Tests)*



# PowerPoint and Online Book Chapter on SpatialSTEM

**Part 1) Overview**

**Part 2) Spatial Analysis (contextual relationships)**

**Part 3) Spatial Statistics (numerical relationships)**

This PowerPoint with notes and online links to further reading is posted at...  
[www.innovativegis.com/basis/Courses/SpatialSTEM/](http://www.innovativegis.com/basis/Courses/SpatialSTEM/)

**Online book chapter...**

A series of Beyond Mapping columns in GeoWorld compiled into **Topic 30, "SpatialSTEM: A Math/Stat Framework for Grid-based Map Analysis and Modeling"** in the online book Beyond Mapping III posted at...  
[www.innovativegis.com/basis/MapAnalysis/Topic30/Topic30.htm/](http://www.innovativegis.com/basis/MapAnalysis/Topic30/Topic30.htm/)

## Further Reading

**Making a Case for SpatialSTEM** — a 15-page white paper describing a framework for grid-based map analysis and modeling concepts and procedures

[http://www.innovativegis.com/basis/Papers/Other/SpatialSTEM/SpatialSTEM\\_case.pdf](http://www.innovativegis.com/basis/Papers/Other/SpatialSTEM/SpatialSTEM_case.pdf)

**SpatialSTEM: Extending Traditional Mathematics and Statistics to Grid-based Map Analysis and Modeling** — white paper describing an innovative approach for teaching map analysis and modeling fundamentals within a mathematical/statistical context

[http://www.innovativegis.com/basis/Papers/Other/SpatialSTEM/SpatialSTEM\\_extendedcase.pdf](http://www.innovativegis.com/basis/Papers/Other/SpatialSTEM/SpatialSTEM_extendedcase.pdf)

**Further SpatialSTEM Readings** — a comprehensive appendix to the SpatialSTEM "extended readings" with URL links to over 125 additional readings on the grid-based map analysis/modeling concepts, terminology, considerations and procedures described in the papers on SpatialSTEM

<http://www.innovativegis.com/Basis/Courses/SpatialSTEM/sSTEMreading.pdf>

## Part 4) Future Directions