3.1 A Map-ematical Framework

As GIS technology moves beyond mapping an increasing number analytical operations come into play. Tools for zooming, panning, colorizing and superimposing map displays are being augmented by analytical procedures like coincidence, proximity, visual exposure and optimal routes.

While map analysis tools might at first seem uncomfortable they simply are extensions of traditional analysis procedures brought on by the digital nature of modern maps. Since maps are “number first, pictures later,” a map-ematical framework can be can be used to organize the analytical operations. Like basic math, this approach uses sequential processing of mathematical operations to perform a wide variety of complex map analyses. By controlling the order which the operations are executed, and using a common database to store the intermediate results, a mathematical-like processing structure is developed.

This “map algebra” is similar to traditional algebra where basic operations, such as addition, subtraction and exponentiation, are logically sequenced for specific variables to form equations—however, in map algebra the variables represent entire maps consisting of thousands of individual grid values. Most of traditional mathematical capabilities, plus extensive set of advanced map processing operations, comprise the map analysis toolbox.

As with matrix algebra (a mathematics operating on groups of numbers) new operations emerge that are based on the nature of the data. Matrix algebra’s transposition, inversion and diagonalization are examples of the extended set of techniques in matrix algebra.

In grid-based map analysis, the spatial coincidence and juxtapositioning of values among and within maps create new analytical operation, such as coincidence, proximity, visual exposure and optimal routes. These operators are accessed through general purpose map analysis software available in many GIS systems, such as GRASS, ERDAS or the Spatial Analyst extension to ArcGIS. The MapCalc Learner package included with this book is designed to provide hands-on experience with the fundamental concepts, considerations and procedures used in map analysis.

While the specific command syntax and mechanics differ among software brands, the basic analytical capabilities and spatial reasoning skills used in GIS modeling form a common foundation. Appendix C contains a cross-listing reference of commands from MapCalc to GRASS, ERDAS and Spatial Analyst. In addition, a cross-listing of commands used is identified at the end of each exercise set.

There are two fundamental conditions required by any map analysis package—a consistent data structure and an iterative processing environment. Topic 2 described the characteristics of a grid-based data structure by introducing the concepts of an analysis frame, map stack and data types. The traditional discrete set of map features (points, lines and polygons) where extended to map surfaces that characterize geographic space as a continuum of uniformly-spaced grid cells. This structure forms a framework for the map-ematics underlying map analysis and modeling.

The second condition of map analysis provides an iterative processing environment by logically sequencing map analysis operations. This involves:

— retrieval of one or more map layers from the database,
— processing that data as specified by the user,
— creation of a new map containing the processing results, and
— storage of the new map for subsequent processing.

Each new map derived as processing continues aligns with the analysis frame so it is automatically geo-
registered to the other maps in the database. The values comprising the derived maps are a function of the
processing specified for the “input maps.”

This cyclical processing provides an extremely flexible structure similar to “evaluating nested
parentheticals” in traditional math. Within this structure, one first defines the values for each variable and
then solves the equation by performing the mathematical operations on those numbers in the order
prescribed by the equation. For example, the equation for calculating percent change in your investment
portfolio—

\[
\text{\%Change} = \frac{A}{C} = \frac{(B - C)}{C} \times 100
\]

- define variables
- intermediate solution #1
- intermediate solution #2
- final solution

— identifies that the variables B and C are first defined, then subtracted and the difference stored as an
intermediate solution. The intermediate solution is divided by variable C to generate another intermediate
solution that, in turn is multiplied by 100 to calculate the solution variable A.

This same basic mathematical structure provides the framework for computer-assisted map analysis. The
only difference is that the variables are represented by mapped data composed of thousands of organized
values. Figure 3-1 shows a similar solution for calculating the percent change in animal activity.

![Image](image-url)

*Figure 3-1. An iterative processing environment, analogous to basic math, is used to derive new map variables.*

The processing steps shown in the figure are identical to the traditional solution except the calculations are
performed for each grid cell in the study area and the result is a map that identifies the percent change at
each location. Map analysis identifies what kind of change (termed the thematic attribute) occurred where
(termed the spatial attribute). The characterization of what and where provides information needed for
further GIS modeling, such as determining if areas of large increases in animal activity are correlated with particular cover types or near areas of low human activity.

Within this iterative processing structure, four fundamental classes of map analysis operations can be identified. These include:

— **Reclassifying Maps** – involving the reassignment of the values of an existing map as a function of its initial value, position, size, shape or contiguity of the spatial configuration associated with each map category.

— **Overlaying Maps** – resulting in the creation of a new map where the value assigned to every location is computed as a function of the independent values associated with that location on two or more maps.

— **Measuring Distance and Connectivity** – involving the creation of a new map expressing the distance and route between locations as straight-line length (simple proximity) or as a function of absolute or relative barriers (effective proximity).

— **Characterizing and Summarizing Neighborhoods** – resulting in the creation of a new map based on the consideration of values within the general vicinity of target locations.

Reclassification operations merely repackage existing information on a single map. Overlay operations, on the other hand, involve two or more maps and result in the delineation of new boundaries. Distance and connectivity operations are more advanced techniques that generate entirely new information by characterizing the relative positioning of map features. *Topics 4, 5, 6 and 7* are concerned with distance-based operators and their applications. Neighborhood operations summarize the conditions occurring in the general vicinity of a location. *Topics 8 and 11* focus on neighborhood operators and their applications.

The reclassifying and overlaying operations based on point processing are the backbone of current GIS applications, allowing rapid updating and examination of mapped data. However, other than the significant advantage of speed and ability to handle tremendous volumes of data, these capabilities are similar to those of manual map processing. Map-wide overlays, distance and neighborhood operations, on the other hand, identify more advanced analytic capabilities and most often do not have paper-map legacy procedures.

The mathematical structure and classification scheme of Reclassify, Overlay, Distance and Neighbors form a conceptual framework that is easily adapted to modeling spatial relationships in both physical and abstract systems. A major advantage is flexibility. For example, a model for siting a new highway could be developed as a series of processing steps. The analysis likely would consider economic and social concerns (e.g., proximity to high housing density, visual exposure to houses), as well as purely engineering ones (e.g., steep slopes, water bodies). The combined expression of both physical and non-physical concerns within a quantified spatial context is a major benefit.

However, the ability to simulate various scenarios (e.g., steepness is twice as important as visual exposure and proximity to housing is four times more important than all other considerations) provides an opportunity to fully integrate spatial information into the decision-making process. By noting how often and where the proposed route changes as successive runs are made under varying assumptions, information on the unique sensitivity to siting a highway in a particular locale is described.

In addition to flexibility, there are several other advantages in developing a generalized analytical structure for map analysis. The systematic rigor of a mathematical approach forces both theorist and user to carefully consider the nature of the data being processed. Also it provides a comprehensive format for learning that is independent of specific disciplines or applications. Furthermore the flowchart of processing succinctly describes the components and weightings capsulated in an analysis.

This communication enables decision-makers to more fully understand the analytic process and actually interact with weightings, incomplete considerations and/or erroneous assumptions. These comments, in most cases, can be easily incorporated and new results generated in a timely manner. From a decision-
maker’s point of view, traditional manual techniques for analyzing maps are a distinct and separate task from the decision itself. They require considerable time to perform and many of the considerations are subjective in their evaluation.

In the old environment, decision-makers attempt to interpret results, bounded by vague assumptions and system expressions of the technician. Computer-assisted map analysis, on the other hand, engages decision-makers in the analytic process. In a sense, it both documents the thought process and encourages interaction—sort of like a “spatial spreadsheet.”

3.2 Reclassifying Map Values

The first, and in many ways the most fundamental class of analytical operations, involves the reclassification of map categories. Each operation involves the creation of a new map by assigning thematic values to the categories of an existing map. These values may be assigned as a function of the initial value, position, contiguity, size, or shape of the spatial configuration of the individual categories. Each of the reclassification operations involves the simple repackaging of information on a single map, and results in no new boundary delineation. Such operations can be thought of as the purposeful "re-coloring" of maps.

Figure 3-2 shows the result of simply reclassifying a map as a function of its initial thematic values. For display, a unique symbol is associated with each value. In the figure, the COVERTYPE map has categories of Open Water, Meadow and Forest. These features are stored as thematic values 1, 2 and 3, respectively, and displayed as separate colors. A binary map that isolates the Open Water locations can be created by simply assigning 0 to the areas of Meadow and Forest. While the operation seems trivial by itself, it has map analysis implications far beyond simply re-coloring the map categories as well soon be apparent.

A similar reclassification operation might involve the ranking or weighing of qualitative map categories to generate a new map with quantitative values. For example, a map of soil types might be assigned values that indicate the relative suitability of each soil type for residential development.

Quantitative values may also be reclassified to yield new quantitative values. This might involve a specified reordering of map categories (e.g., given a map of soil moisture content, generate a map of suitability levels for plant growth). Or, it could involve the application of a generalized reclassifying function, such as "level slicing," which splits a continuous range of map category values into discrete intervals (e.g., derivation of a contour map of just ten contour intervals from an elevation surface composed of thousands of specific elevation values).

Other quantitative reclassification functions include a variety of arithmetic operations involving map category values and a specified or computed constant. Among these operations are addition, subtraction, multiplication, division, exponentiation, maximization, minimization, normalization and other scalar mathematical and statistical operators. For example, an elevation surface expressed in feet might be
converted to meters by multiplying each map value by the appropriate conversion factor of 3.28083 feet per meter.

Reclassification operations can also relate to location, as well as purely thematic attributes associated with a map. One such characteristic is position. An overlay category represented by a single "point" location, for example, might be reclassified according to its latitude and longitude. Similarly, a line segment or area feature could be reassigned values indicating its center or general orientation.

A related operation, termed parceling, characterizes category contiguity. This procedure identifies individual "clumps" of one or more points having the same numerical value and spatially contiguous (e.g., generation of a map identifying each lake as a unique value from a generalized map of water representing all lakes as a single category).

Another location characteristic is size. In the case of map categories associated with linear features or point locations, overall length or number of points might be used as the basis for reclassifying those categories. Similarly, an overlay category associated with a planar area might be reclassified according to its total acreage or the length of its perimeter. For example, a map of water types might be reassigned values to indicate the area of individual lakes or the length of stream channels. The same sort of technique might also be used to deal with volume. Given a map of depth to bottom for a group of lakes, for example, each lake might be assigned a value indicating total water volume based on the area of each depth category.

Figure 3-3 identifies a similar processing sequence using the information derived in figure 3-2. While your eye sees two distinct blobs of water on the OPEN WATER map the computer only "sees" distinctions by different map category values. Since both water bodies are assigned the same value of 1 there isn’t a mathematical distinction and the computer cannot differentiate.

The Clump operation is used to identify the contiguous features as separate values—clump #1 (Larry’s Lake), #2 (Dry Land) and clump #3 (Peter’s Pond). The Size operation is used to calculate the size of each clump—clump #1 = 78 hectares, clump #2 = 543 ha and clump #3 = 4 ha. The final step uses the Renumber operation to isolate the large water body in the northwest portion of the project area.

Figure 3-3. A sequence of reclassification operations (renumber, clump, size and renumber) can be used to isolate large water bodies from a cover type map.
In addition to the initial value, position, contiguity, and size of features, shape characteristics also can be used as the basis for reclassifying map categories. Shape characteristics associated with linear forms identify the patterns formed by multiple line segments (e.g., dendritic stream pattern). The primary shape characteristics associated with polygonal forms include feature integrity, boundary convexity, and nature of edge.

Feature integrity relates to “intact-ness” of an area. A category that is broken into numerous "fragments" and/or contains several interior "holes" is said to have less spatial integrity than ones without such violations. Feature integrity can be summarized as the Euler Number that is computed as the number of holes within a feature less one short of the number of fragments which make up the entire feature. An Euler Number of zero indicates features that are spatially balanced, whereas larger negative or positive numbers indicate less spatial integrity.

Convexity and edge, are other shape indices that relate to the configuration of boundaries of polygonal features. Convexity is the measure of the extent to which an area is enclosed by its background, relative to the extent to which the area encloses this background. The Convexity Index for a feature is computed by the ratio of its perimeter to its area. The most regular configuration is that of a circle which is totally convex and, therefore, not enclosed by the background at any point along its boundary.

Comparison of a feature’s computed convexity to a circle of the same area, results in a standard measure of boundary regularity. The nature of the boundary at each point can be used for a detailed description of boundary configuration. At some locations the boundary might be an entirely concave intrusion, whereas others might be at entirely convex protrusions. Depending on the "degree of edginess," each point can be assigned a value indicating the actual boundary convexity at that location.

This explicit use of cartographic shape as an analytic parameter is unfamiliar to most GIS users. However, a non-quantitative consideration of shape is implicit in any visual assessment of mapped data. Particularly promising is the potential for applying quantitative shape analysis techniques in the areas of digital image classification and wildlife habitat modeling. A map of forest stands, for example, might be reclassified such that each stand is characterized according to the relative amount of forest edge with respect to total acreage and the frequency of interior forest canopy gaps. Those stands with a large proportion of edge and a high frequency of gaps will generally indicate better wildlife habitat for many species. Topic 9 investigates some of tools for analyzing landscape patterns.

### 3.3 Overlying Maps

The general class of overlay operations can be characterized as "light-table gymnastics." These involve the creation of a new map where the value assigned to every point, or set of points, is a function of the independent values associated with that location on two or more existing map layers. In **location-specific** overlaying, the value assigned is a function of the point-by-point coincidence of the existing maps. In **category-wide** composites, values are assigned to entire thematic regions as a function of the values on other overlays that are associated with the categories. Whereas the first overlay approach conceptually involves the vertical spearing of a set of map layers, the latter approach uses one map to identify boundaries by which information is extracted from other maps.

Figure 3-4 shows an example of location-specific overlaying. Here, maps of COVERTYPE and topographic SLOPE_CLASSES are combined to create a new map identifying the particular cover/slope combination at each location. A specific function used to compute new category values from those of existing maps being overlaid can vary according to the nature of the data being processed and the specific use of that data within a modeling context. Environmental analyses typically involve the manipulation of quantitative values to generate new values that are likewise quantitative in nature. Among these are the basic arithmetic operations such as addition, subtraction, multiplication, division, roots, and exponentials.

Functions that relate to simple statistical parameters such as maximum, minimum, median, mode, majority, standard deviation or weighted average also can be applied. The type of data being manipulated dictates the appropriateness of the mathematical or statistical procedure used. For example, the addition of
qualitative maps such as soils and land use would result in mathematically meaningless sums, since their thematic values have no numerical relationship. Other map overlay techniques include several that might be used to process either quantitative or qualitative data and generate values which can likewise take either form. Among these are masking, comparison, calculation of diversity, and permutations of map categories (as depicted in figure 3-4).

More complex statistical techniques may also be applied in this manner, assuming that the inherent interdependence among spatial observations can be taken into account. This approach treats each map as a variable, each point as a case, and each value as an observation. A predictive statistical model can then be evaluated for each location, resulting in a spatially continuous surface of predicted values. The mapped predictions contain additional information over traditional non-spatial procedures, such as direct consideration of coincidence among regression variables and the ability to spatially locate areas of a given level of prediction. Topic 12 investigates the considerations in spatial data mining derived by statistically overlaying mapped data.

An entirely different approach to overlaying maps involves category-wide summarization of values. Rather than combining information on a point-by-point basis, this group summarizes the spatial coincidence of entire categories shown on one map with the values contained on another map(s). Figure 3-5 contains an example of a category-wide overlay operation. In this example, the categories of the COVERTYPE map are used to define an area over which the coincidental values of the SLOPE map are averaged. The computed values of average slope within each category area are then assigned to each of the cover type categories.

Summary statistics which can be used in this way include the total, average, maximum, minimum, median, mode, or minority value; the standard deviation, variance, or diversity of values; and the correlation, deviation, or uniqueness of particular value combinations. For example, a map indicating the proportion of undeveloped land within each of several counties could be generated by superimposing a map of county boundaries on a map of land use and computing the ratio of undeveloped land to the total land area for each county. Or a map of zip code boundaries could be superimposed over maps of demographic data to determine the average income, average age, and dominant ethnic group within each zip code.
Figure 3-5. Category-wide overlay operations summarize the spatial coincidence of map categories, such as generating the average SLOPE for each COVERTYPE category.

As with location-specific overlay techniques, data types must be consistent with the summary procedure used. Also of concern is the order of data processing. Operations such as addition and multiplication are independent of the order of processing. Other operations, such as subtraction and division, however, yield different results depending on the order in which a group of numbers is processed. This latter type of operations, termed non-commutative, cannot be used for category-wide summaries.

3.4 Characterizing Spatial Coincidence

As previously noted, GIS maps are numbers and a rigorous, quantitative approach to map analysis can be maintained. However, most of our prior experience with maps is non-quantitative, using paper maps composed of inked lines, shadings, symbols and zip-a-tone. We rarely think of map uncertainty and error propagation. And we certainly wouldn't think of demanding such capabilities in our GIS software. That is, not as of yet.

Everybody knows the 'bread and butter' of a GIS is its ability to overlay maps. To a human, map overlay involves "light-table gymnastics" where we peer through a stack of acetate sheets and interpreting the subtle hues of resulting colors. To a GIS you're asking the computer to identify the condition from each map layer for every location in a project area. From the computer's perspective, however, this is simply one of a host of ways to characterize the spatial coincidence.

Let's compare how you and your computer might approach the task of identifying coincidence. Your eye moves randomly about the stack, pausing for a nanosecond at each location and mentally establishing the conditions by interpreting the color. Your summary might conclude that the northeastern portion of the area is unfavorable as it has "kind of a magenta tone." This is the result of visually combining steep slopes portrayed as bright red with unstable soils portrayed as bright blue with minimal vegetation portrayed as dark green. If you want to express the result in map form, you would tape a clear acetate sheet on top and delineate globs of color differences and label each parcel with your interpretation. Whew! No wonder you want a GIS.
The GIS goes about the task in a very similar manner. In a vector system, line segments defining polygon boundaries are tested to determine if they cross. When a line on one map crosses a line on another map, a new combinatorial polygonal is indicated. Trigonometry is employed, and the X,Y coordinate of the intersection of the lines is computed. The two line segments are split into four and values identifying the combined map conditions are assigned. The result of all this crossing and splitting is the set of polygonal prodigy you so laboriously delineated by hand.

A raster system has things a bit easier. As all locations are predefined as a consistent set of cells within a matrix, the computer merely 'goes' to a location, retrieves the information stored for each map layer and assigns a value indicating the combined map conditions. The result is a new set of values for the matrix identifying the coincidence of the maps.

The big difference between ocular and computer approaches to map overlay is not so much in technique, as it is in the treatment of the data. If you have several maps to overlay you quickly run out of distinct colors and the whole stack of maps goes to an indistinguishable dark, purplish hue. One remedy is to classify each map layer into just two categories, such as suitable and unsuitable. Keep one as clear acetate (good) and shade the other as light grey (bad). The resulting stack avoids the ambiguities of color combinations, and depicts the best areas as lighter tones. However, in making the technique operable you have severely limited the content of the data—just good and bad.

The computer can mimic this technique by using binary maps. A "0" is assigned to good conditions and a "1" is assigned to bad conditions. The sum of the maps has the same information as the brightness scale you observe—the smaller the value the better. The two basic forms of logical combination can be computed. "Find those locations which have good slopes .AND. good soils .AND. good vegetative cover." Your eye sees this as the perfectly clear locations. The computer sees this as the numeric pattern 0-0-0. "Find those locations which have good slopes .OR. good soils .OR. good vegetative cover." To you this is could be any location that is not the darkest shading; to the computer it is any numeric pattern that has at least one 0. But how would you handle, "Find those locations which have good slopes .OR. good soils .AND. good vegetative cover"? You can't find them by simply viewing the stack of maps. You would have to spent a lot of time flipping through the stack. To the computer, this is simply the patterns 0-1-0, 1-0-0 and 0-0-0. It's a piece of cake from the digital perspective.

In fact any combination is easy to identify. Let's say we expand our informational scale and redefine each map from just good and bad to not suitable (0), poor (1), marginal (2), good (3) and excellent (4). We could ask the computer to INTERSECT SLOPES WITH SOILS WITH COVER COMPLETELY FOR ALL-COMBINATIONS. The result is a map indicating all combinations that actually occur among the three maps. Likely this map would be too complex for human viewing enjoyment, but it contains the detailed information basic to many application models. A more direct approach is a geographic search for the best areas invoked by asking to INTERSECT SLOPES WITH SOILS WITH COVER FOR EXCELLENT-AREAS ASSIGNING 1 TO 4 AND 4 AND 4. Any combination not assigned a value drops to 0, leaving a map with 1's indicating the excellent areas.

Another way of combining these maps is by asking to COMPUTE SLOPES MINIMIZE SOILS MINIMIZE COVER FOR WEAK-LINK. The resulting map's values indicate the minimal coincidence rating for each location. Low values indicate areas of concern and a 0 indicate areas to dismiss as not suitable from at least one map's information. There is a host of other computational operations you could invoke, such as plus, minus, times, divide, and exponentiation. Just look at the functional keys on your hand calculator. But you may wonder, "why would someone want to raise one map to the power of another"? Spatial modelers who have gone beyond mapping, that's who.

What would happen if, for each location (be it a polygon or a cell), we computed the sum of the three maps, then divided by the number of maps? That would yield the average rating for each location. Those with the higher averages are better. Right? You might want to take it a few steps further. First, in a particular
application, some maps may be more important than others in determining the best areas. Ask the computer to AVERAGE SLOPES TIMES 5 WITH SOILS TIMES 3 WITH COVER TIMES 1 FOR WEIGHTED-AVERAGE. The result is a map whose average ratings are more heavily influenced by slope and soil conditions.

Just to get a handle on the variability of ratings at each location, you can determine the standard deviation—either simple or weighted. Or for even more information, determine the coefficient of variation, which is the ratio of the standard deviation to the average, expressed as a percent. What will that tell you? It hints at the degree of confidence you should put into the average rating. A high COFFVAR indicates wildly fluctuating ratings among the maps and you might want to look at the actual combinations before making a decision.

A statistical way to summarized coincidence between maps is a cross-tab table. If you CROSSTAB FORESTS WITH SOILS a table results identifying how often each forest type jointly occurs with each soil type. In a vector system, this is the total area in each forest/soil combinations. In a raster system, this is simply a count of all the cell locations for each forest/soil combination.

![Figure 3-6. A cross-tab table statistically summarizes the coincidence among the categories on two maps.](image)

For example, reading across the first row of table in figure 3-6 notes that Forest category 1 (Deciduous) contains 303 cells distributed throughout the map. The total count for Soils category 1 (Lowland) is 427 cells. The next section of the table notes that the joint condition of Deciduous/Lowland occurs 299 times for 47.84 percent of the total map area. Contrast this result with that of Deciduous/Upland occurrence on the row below indicating only four “crosses” for less than one percent of the map. The coincidence statistics for the Conifer category is more balanced with 128 cells (20.48%) occurring with the Lowland soil and 194 cells (31.04%) occurring with the Upland soil.

These data may cause you to jump to some conclusions, but you had better consider the last section of the Table before you do. This section normalizes the coincidence count to the total number of cells in each category. For example, the 299 Deciduous/Lowland coincidence accounts for 98.68 percent of all occurrences of Deciduous trees ((299/303)*100). That’s a very strong relationship. However, from Lowland soil occurrence the 299 Deciduous/Lowland coincidence is a bit weaker as it accounts for only 70.02 percent of all occurrences of Lowland soils ((299/427)*100). In a similar vein, the Conifer/Upland coincidence is very strong as it accounts for 97.98 percent of the occurrence of all Upland soil occurrences. Both columns of coincidence percentages must be considered as a single high percent might be merely the result of the other category occurring just about everywhere.
There are still a couple of loose ends before we can wrap-up point-by-point overlay summaries. One is direct map comparison, or change detection. For example, if you encode a series of land use maps for an area, then subtract each successive pair of maps; the locations that underwent change will appear as non-zero values for each time step. In GIS-speak, you would enter COMPUTE LANDUSE-T2 MINUS LANDUSE-T1 FOR CHANGE-T2&1 for a map of the land use change.

If you are real tricky and think map-ematically you will assign a binary progression to the land use categories (1,2,4,8,16, etc.), as the differences will automatically identify the nature of the change. The only way you can get a 1 is 2-1; a 2 is 4-2; a 3 is 4-1; a 6 is 8-2; etc. A negative sign indicates the opposite change, and now all bases are covered.

The last point-by-point operation is a weird one—covering. This operation is truly spatial and has no traditional math counterpart. Imagine you prepared two acetate sheets by coloring all of the forested areas opaque green on one sheet and all of the roads an opaque red on the other sheet. Now overlay them on a light-table. If you place the forest sheet down first the red roads will “cover” the green forests and you will see the roads passing through the forests. If the roads map goes down first, the red lines will stop abruptly at the green forest globs.

In a GIS, however, the colors become numbers and the clear acetate is assigned zero. The command COVER FORESTS WITH ROADS causes the computer to go to a location and assess the shish kabob of values it finds. If the kabob value for roads is 0 (clear), keep the forest value underneath it. If the road value is non-zero, place that value at the location, regardless of the value underneath.

### 3.5 A Simple Habitat Model

A simple habitat model can be developed using only reclassify and overlay operations. For example, a Hugag is a curious mythical beast (see figure 3-7) with strong preferences for terrain configuration:

- **Prefers low elevations** (severe nose bleeds at higher altitudes)
- **Prefers gentle slopes** (fear of falling over and unable to get up)
- **Prefers southerly aspects** (a place in the sun)

![Hugag](Fearsome_Creatures_of_the_Lumberwoods_by_William_T._Cox_1910)

The Hugag is a huge animal of the Lake States. Its range includes western Wisconsin, northern Minnesota, and a territory extending indefinitely northward in Canadian wilds toward Hudson Bay. In size the Hugag may be compared to the moose, and in form it somewhat resembles that animal. Very noticeable, however, are its jointless legs, which compel the animal to remain on its feet, and its long upper lip, which prevents it from grazing. If it tried that method of feeding it would simply tramp it upper lip into the dirt. Its head and neck are leathery and hairless; its strangely corrugated ears flop downward; its four-toed feet, long bushy tail, shaggy coat and general make-up give the beast an unmistakably prehistoric appearance. The hugag has a perfect mania for travelling, and few hunters who have taken up its trail ever came up with the beast or back to camp. It is reported to keep going all day long, browsing on twigs, flopping it lip around trees, and skipping back as occasion offers, and at night, since it cannot lie down, it leans against a tree, bracing it hind legs and marking time with its front ones. The most successful hugag hunters have adopted the practice of notchting trees so that they are almost ready to fail, and when the hugag leans up against one both the tree and the animal come down. In its helpless condition it is easily dispatched. The last one killed, so far as known, was on the Turtle River, in northern Minnesota, where a young one, weighing 1,800 pounds, was found stuck in the mud. It was knocked in the head by Mike Flynn, of Cass lake.

**Figure 3-7. The Hugag prefers low elevations, gentle slopes and southerly aspects.**

A binary habitat model of Hugag preferences is the simplest to conceptualize and implement. It is analogous to the manual procedures for map analysis popularized in the book *Design with Nature*, by Ian L. McHarg, first published in 1969. This seminal work was the forbearer of modern map analysis by describing an overlay procedure involving paper maps, transparent sheets and pens.
For example, if avoiding steep slopes was an important decision criterion, a draftsperson would tape a transparent sheet over a topographic map, delineate areas of steep slopes (contour lines close together) and fill-in the precipitous areas with an opaque color. The process is repeated for other criteria, such as the hugag’s preference to avoid areas that are northerly-oriented and at high altitudes. The annotated transparencies then are aligned on a light-table and the “clear” areas showing through identify acceptable habitat for the animal.

An analogous procedure can be implemented in a computer by using the value 0 to represent the unacceptable areas (opaque) and 1 to represent acceptable habit (clear). As shown in figure 3-8, an Elevation map is used to derive a map of terrain steepness (Slope_map) and orientation (Aspect_map). A value of 0 is assigned to locations hugags want to avoid—

- Greater than 1800 feet elevation = 0 …too high
- Greater than 30% slope = 0 …too steep
- North, northeast and northwest = 0 …to northerly

—with all other locations assigned a value of 1 to indicate acceptable areas. The individual binary habit maps are shown in 3D and 2D displays on the right side of figure 3-8. The dark red portions identify unacceptable areas that are analogous to McHarg’s opaque colored areas delineated on otherwise clear transparencies.

A Binary Suitability map of Hugag habitat is generated by multiplying the three individual binary preference maps (left side of figure 3-9). If a zero is encountered on any of the map layers, the solution is sent to zero (bad habitat). For the example location on the right side of the figure, the preference string of values is 1 * 1 * 0 = 0 (Bad). Only locations with 1 * 1 * 1 = 1 (Good) identify areas with out any limiting factors——good elevations, good slopes and good orientation. These areas are analogous to the clear areas showing through the stack of transparencies on a light-table.

While this procedure mimics manual map processing, it is limited in the information it generates. The solution is binary and only differentiates acceptable and unacceptable locations. But isn’t an area that is totally bad (0 * 0 * 0 = 0) different from one that is just limited by one factor (1 * 1 * 0 = 0)? Two factors are acceptable thus making it “nearly good.”
The right side of figure 3-9 shows a **Ranking Suitability** map of Hugag habitat. In this instance the individual binary maps are simply added together for a count of the number of acceptable locations. Note that the areas of perfectly acceptable habitat (light grey) on both the binary and ranking suitability maps have the same geographic pattern. However, the unacceptable area on the ranking suitability map contains values 0 through 2 indicating how many acceptable factors occur at each location. The zero value for the area in the northeastern portion of the map identifies very bad conditions ($0 + 0 + 0 = 0$). The example location, on the other hand, is nearly good ($1 + 1 + 0 = 2$).

The ability to report the degree of suitability stimulated a lot of research into using additive colors for the manual process. Physics suggests that if blue, green and red are used as semi-opaque colors on the transparencies, the resulting color combinations should be those indicated in the left side of the table in figure 3-10. However, in practice the technique collapsed to an indistinguishable brownish-purple glob of colors when three or more map layers were overlaid.

Computer overlay, on the other hand, can accurately differentiate all possible combinations as shown on the right side of figure 3-10. The trick is characterizing the unacceptable areas on each map layer as a binary progression of values—1, 2, 4, 8, 16, 32, etc. In the example in the figure, 1 is assigned to areas that are too high, 2 assigned to areas that are too steep and 4 assigned to areas that are too northerly oriented. When adding a binary progression of values each combination of values results in a unique sum. The result is termed a **Ranking Combination** suitability map.

In the three-map example, all possible combinations are contained in the number range 0 (best) to 7 (bad) as indicated. A value of 4 can only result from a map layer sequence of $0 + 0 + 4$ that corresponds to OK.
Elevation, OK Steepness and Bad Aspect. If a fourth map was added to the stack, its individual binary map value for unacceptable areas would be 8 and the resulting range of values for the four map-stack would be from 0 (best) to 15 (bad) with a unique value for each combination of habitat conditions.

The *binary, ranking and ranking combinations* approaches to suitability mapping share a common assumption—that each habitat criterion can be discretely classified as either acceptable or unacceptable. However, some wildlife experts believe Hugags are complex animals and actually perceive a continuum of preference. For example, their distain for high elevations isn’t a sharp cutoff at 1800 feet. Rather they strongly prefer low land areas, venture sometimes into transition elevations but absolutely detest the higher altitudes.

Figure 3-11 shows a **Rating Suitability** map of Hugag habitat. In this case the three criteria maps are “graded” on a scale of 1 (bad) to 9 (best). For example, the elevation-based conditions are calibrated as:

- > 1800 feet = 1 (bad)
- 1400-1800 feet = 3 (marginal)
- 1250-1400 feet = 5 (OK)
- 900-1250 feet = 7 (better)
- 0-900 feet = 9 (best)

The other two criteria of slope and orientation are similarly graded on the same scale as shown in the figure. This process is analogous to a professor grading student exams for an overall course grade. Each map layer is like a test, each grid cell is like a student and each map value at a location is like the grade for test. To determine the overall grade for a semester a professor averages the individual test scores. An overall habitat score is calculated in a similar manner—take the average of the three calibrated map layers.

```
\[(9 + 7 + 1) / 3\] = 5.67 \text{ OK to Better}
```

Figure 3-11. A Rating Suitability map is derived by averaging a series of “graded” maps representing individual habitat criteria.

The example location in this instance is assigned an average habitat value of 5.67 that is somewhere between OK and Better habitat conditions. The rating was derived by averaging the three calibrated values of—

- 9 \[\ldots\text{64 feet assigned Best elevation condition of between 0 and 900 feet}\]
- 7 \[\ldots\text{7\% assigned Better steepness condition of between 5 and 15\%}\]
- 1 \[\ldots\text{northwest assigned Bad aspect condition}\]

\[17 / 3 = 5.67\text{ average habitat rating}\]

Note the increased information provided by a rating suitability map. All three of the extended techniques (ranking, ranking combination and rating) consistently identify the completely bad area in the northeast and southeast. What changes with the different techniques is the information about subtle differences in the
marginal through better areas. The rating suitability map contains the most information as it uses a consistent scale and reports habitat “goodness” values to the decimal point.

That brings the discussion full circle and reinforces the concept of a map-ematical framework for analyzing spatial relationships. Generally speaking, map analysis procedures that take full advantage of the digital nature of maps tend to provide more information for better decision-making. The remaining topics in this book describe a myriad of map-ematical operations that take the historical perspective of maps and their applications well beyond traditional mapping. While GIS technology’s plane of reference might be paper maps, pens and rulers, its future lays in the innovative application of new analysis tools supporting better understanding and communication of complex spatial interactions.
3.6 Exercises

Access *MapCalc* using the *Tutor25.rgs* by selecting **Start** → **Programs** → **MapCalc Learner** → **MapCalc Learner** → **Open existing map set** → **NR_MapCalc Data** → **Tutor25.rgs**. The following set of exercises utilizes this database.

3.6.1 Spatial Analysis Operations

Press the **Map Analysis** button to access the set of analysis side of MapCalc. The Map Analysis window has three main areas—**Main Menu**, **Analysis Operations** and the **Command Scripting Area**.

The Analysis Operations toolbar is the gateway to the map analysis and modeling capabilities.

The toolbar is divided into a series of drop-down menus for selecting individual operations. The first four menus access **Reclassify**, **Overlay**, **Distance** and **Neighbors** operations addressing the contextual relationships within and among maps. The **Statistical** operations investigate numerical relationships, **Import/Export** operations involve data exchange and **Macro** operations are used in constructing scripts of command sequences.

The basic spatial analysis classes of **Reclassify** and **Overlay** are used in the exercises for this topic. Specific techniques include Reclassify operations of **Clump**, **Configure**, **Renumber**, **Slice** and **Size**, and Overlay operations of **Calculate**, **Composite**, **Compute**, **Cover**, **Crosstab** and **Intersect**.

3.6.2 Reclassifying Map Features

Use the **View** button to display the **Covertype** map.

Press the **Map Analysis** button and select **Reclassify** → **Renumber** then complete the dialog box as shown below to isolate the **Open Water** features.

Select “**Covertype**” as the input map.

Specify the renumbering as assigning 0 (new value) to 2 (old value) through 3 (old upper value).

Press the “Add” button to register the reassignment.

Enter “**Open_water**” as the output map name.
Press OK and the command “RENUMBER Covertype ASSIGNING 0 TO 2 THRU 3 FOR Open_water” is entered into the script and executed and the new map “Open_water” is generated.

To temporarily hide the Map Analysis window, click on the Minimize button (“minus” icon) in the upper right corner. Click and drag the minimized bar to the tool bar area of MapCalc as shown below.

To restore the Map Analysis window you can click on the Restore button on the minimized tool bar or press the Map Analysis button.

Create a better display of the results by pressing the Shading Manager button (or double-clicking on the map legend) and entering Open Water as the label for map value 1. Change the display color by clicking on the color box and selecting blue from the pallet.

Note that the value 1 is associated with all locations classified as Open Water. Your eye detects that there are two distinct groupings of open water—a portion of a large lake at the top and a small pond at the bottom. While you “see” the different clumps, the computer only “sees” the value 1 and can’t distinguish the two separate groups from each other.

Press the Map Analysis button and select Reclassify Clump to assign separate values to the two different groups of open water—

CLUMP Open_water AT 1 Diagonally FOR Water_bodies

...and create a better display using the Shading Manager to label the map and assign dark blue to Larry’s Lake, light grey to Dry Land and light blue to Peter’s Pond.

Press the Map Analysis button and select Reclassify Size to assign the size (in # cells; each cell = 1 hectare) to each map category—

SIZE Water_bodies FOR Water_body_size

Press the Map Analysis button and select Reclassify Renumber to isolate the large water body (Larry’s Lake)—
3.6.3 Calculating Change

The basic math capabilities of a simple pocket calculator can be applied to a map stack. For example, a difference map in animal activity between two periods can be calculated by evaluating a simple equation...

\[
\text{Diff} = (\text{Period2} - \text{Period1})
\]

\[
\text{Diff} = (\text{Animal_activity2} - \text{Animal_activity1}) = \text{Difference}
\]

Compare the processing sequence you just completed with the sequence outlined in figure 3.3 and discussed in section 3.2 Reclassifying Maps. While the end product isolating Larry’s lake isn’t very exciting, the sequencing of map analysis operations illustrates the most fundamental element of map analysis—logical sequencing of operations to derive a series of maps that reflects spatial reasoning.

Note that the difference in animal activity between the two periods ranges from a decrease of -10.7 to an increase of 45.0 animals per 24-hour sampling period. Also note that the increased activity is concentrated in the northeastern portion of the project area.
Create a better display of the results by pressing the Shading Manager button (or double-clicking on the map legend) and specifying User-Defined Ranges as the Calculation Mode and 13 as the Number of Ranges. Enter the interval cutoff values in the Min [>=] column and the maximum value of 45 at the top of the Max [<] column as shown below—

Click on the color box for the -10.7 to -5 interval and set it to red. Set the 40 to 45 interval to green and the 0 to 1 interval to yellow. All of the other intervals should have the color lock off so the display pallet will form a color gradient from red to yellow to green.

In this display red tones indicate areas that animal activity decreased; green tones areas that increased. The intensity of red or green indicates the level of decreased of increased activity.

On your own, extend the analysis to generate a Percent Change map that evaluates the following equation—

\[
\% \Delta = \left( \frac{\text{Period 2} - \text{Period 1}}{\text{Period 1}} \right) * 100
\]

3.6.4 Summarizing Map Regions

Region-wide overlay is used to summarize the data on one map (data map) within areas identified on another map (template map). Use the COMPOSITE command to create a map that
shows the average slope for the different cover types by completing—

On your own, use the Clump command to assign a unique value to each of the cover type groups then calculate the average slope for each of the individual cover type groupings.

On your own, create a map that identifies the average animal activity in period 1 (Animal_activity1 map) for each of the administrative districts (Districts map).

### 3.6.5 Characterizing Map Coincidence

Use the View button to generate displays of the Covertype and Slope maps.

Press the Map Analysis button to pop-up the Map Analysis dialog box, select the Overlay class of operators and choose the Crosstab command. Complete the dialog box as shown below to generate a cross-tabular table summarizing the
The first four columns of the Crosstab Table identify the individual map categories and their frequency of occurrence. For example the first row identifies that Covertype category 1 (Open Water) is assigned to 82 grid cells and Slope_classes category 1 (Gentle) is assigned to 182 grid cells.

The next column indicates how often the two categories coexist—81 cells of Open Water and Gentle slopes coincidence representing 12.96% of the total map area.

The second row of the table identifies that there is 1 cell of Open Water coincident with Moderate slopes (15-30%) which is very counter intuitive as it is physically difficult to keep a body of water clinging to an incline. The condition occurs because a portion of the lakeshore line abuts a fairly steep slope.

Press the Map Analysis button to pop-up the Map Analysis dialog box, select the Overlay class of operators and choose the Intersect command.

Complete the form as shown below to create a map of the coincidence between the Covertype and Slope_classes maps.

Note that the Completely option is chosen to identify all of the pairwise combinations of the two maps categories. The same result is obtained if you explicitly entered the two-map reclassification assignments—

ASSIGNING 1 TO 1 AND 1 (Water/Gentle)
ASSIGNING 2 TO 1 AND 2 (Water/Moderate)
ASSIGNING 3 TO 2 AND 1 (Meadow/Gentle)
ASSIGNING 4 TO 2 AND 2 (Meadow/Moderate)
ASSIGNING 5 TO 2 AND 3 (Meadow/Steep)
ASSIGNING 6 TO 3 AND 1 (Forest/Gentle)
ASSIGNING 7 TO 3 AND 2 (Forest/Moderate)
ASSIGNING 8 TO 3 AND 3 (Forest/Steep)

Display the Coincidence completely map and use the Shading Manager button to enter labels for the map categories as indicated above.

Compare the Shading Manager and the Crosstab Table summaries noting that the “count” column...
information is the same as the “# of crosses” information. Particularly interesting is that the Intersect map locates the one location of Open Water on Moderate slope occurs at the top of the map (light-green).

Press the Map Analysis button to pop-up the Map Analysis dialog box, select the Overlay class of operators and choose the Calculate command.

Complete the dialog box as shown below to generate a map that identifies the coincidence of the Covertype and Slope_classes maps as a two-digit code. Hint: click on the Maps menu item to select from a pop-up of existing maps as you type in the equation.

The procedure moves the information on the Covertype map from the “ones” to the “tens” digit (10= Open Water, 20= Meadow and 30= Forest), then adds the values to the Slope_classes map values (1= Gentle, 2= Moderate and 3= Steep).

Note that in interpreting the values on the Cover_Slope_code map the value 11 is not equivalent to an “eleven” but a 2-digit code with the first digit indicating the Covertype as 10= Open Water and the second digit indicating the Slope_class as 1= gentle. What is the correct interpretation of areas assigned a “thirty three?”

Note that the 2-digit code contains the same map coincidence information as the Crosstab Table and Intersect map. For example, all three report that there are 81 cells assigned the joint condition Open water and Gentle slope.

Complete the following processing to first generate a map moving the Covertype information to the tens digit then replacing the Covertype information with Water map information.

Note that in interpreting the values on the Cover_Slope_code map the value 11 is not equivalent to an “eleven” but a 2-digit code with the first digit indicating the Covertype as 10=
Topic 3 – Reclassifying and Overlying Maps

Analyzing Geo-Spatial Resource Data

3.6.6 Evaluating a Simple Habitat Model

Given Hugag preferences of lower elevations, gentle slopes and southern exposures can you visually recognize general areas that the animal might prefer? Could you accurately delineate good habitat within the project area? Over an entire watershed? Or an entire region?

Derive maps of terrain steepness (Slope_map) and terrain orientation (Aspect_map) by completing the following processing.

Press the Map Analysis button to pop-up the Map Analysis dialog box. Select Neighbors† Slope to pop-up the Slope dialog box. Complete the form as shown below.

Re-display the Elevation map (Window‡ Elevation) then select Map‡ Overlay† Slope_map to “drape” the Slope_map over the terrain surface and note that it is classified as steep where appears steep on 3D surface.
Press the **Map Analysis** button to pop-up the **Map Analysis** dialog box. Select **Neighbors** → **Orient** to pop-up the Orient dialog box. Complete the form as shown below.

You have already completed the first two command lines...

**SLOPE Elevation Fitted FOR Slope_map**

**ORIENT Elevation Octants FOR Aspect_map**

The next series of commands creates a **Binary Suitability** map of habitat as described in the text section 3.5.

Access the **Habitat.scr** script file by...

Press the **Map Analysis** button to pop-up the **Map Analysis** dialog box.

On the **Map Analysis** menu click on **Scripts** → **Open** → navigate to the **...\Scripts** folder and select the **Habitat.scr** file.

Double-click on the next command line...

**RENUMBER Elevation**

**ASSIGNING 0 TO 1800 THRU 2500**

**ASSIGNING 1 TO 500 THRU 1800**

**FOR E_Pref**

...to pop-up the **Renumber** dialog box for this step of the habitat command script.

Note the command assignments and interpret what processing is involved. Press **OK** to submit the command and generate the specified map. View the map that was created and interpret what the map values represent...

0 = **Unsuitable**

1 = **Suitable** for Hugags (not too high as <1800 feet)

Repeat the procedure to call-up the next command line; review the command specification and interpret the new map’s values.

0 = **Unsuitable**

1 = **Suitable** for Hugags (not too steep as <30% slope)

Repeat for the aspect preference...

0 = **Unsuitable**
1= Suitable for Hugags (E, SE, S, SW, W and flat)

Create a Binary preference map that locates suitable areas by entering…

\[
\text{COMPUTE } S_{\text{Pref}} \times A_{\text{Pref}} \times E_{\text{Pref}} \text{ FOR B_{Habitat}}
\]

Where the solution is…

0= Unsuitable

1= Suitable for Hugags (1 \times 1 \times 1 = 1)

Now create a Ranking Suitability map of habitat quality by completing the next section of the script…

…be sure you look at the map values that are generated and can interpret what they mean.

Create a Ranking Combination Suitability map of habitat by completing the next section of the script…

…be sure you look at the map values that are generated and can interpret what they mean.

Create a Rating Suitability map of habitat by double-clicking then entering the following sequence of commands…

…be sure you look at the map values that are generated and can interpret what they mean.

Since Hugags can’t swim we need to eliminate areas of open water by double-clicking then entering the following sequence of commands…

Finally, use region-wide overlay to determine the average habitat suitability rating for each of the management districts in the project area. The Districts map serves as the “template map” that summarizes (average) the suitability values on the “data map” within each of the regions.

As time permits, investigate the other Reclassify (Clump, Configure, Size, Slice) and Overlay (Cover, Intersect) commands.

On your own, generate a new and improved Binary Suitability map that recognizes that there really is minimal aspect difference between flat areas and those with slight slopes.

Use your new skills with reclassify and overlay commands to update the aspect map with this thinking by entering—

1) Use Renumber to create a binary map that identifies very gentle slopes (0 to 10%) as map value 5 and all other slopes as 0.

2) Use the COVER command to update the Aspect_map such that the aspect values for very gently sloped areas are replaced with the value 5 indicating essentially flat areas.

3) Use the Aspect_map_updated to create a new Binary Suitability map of Hugag habitat.

4) Use one of the overlay commands to identify areas that changed classification between the original B_Habitat map and the new B_Habitat_updated map you just created.

Prepare a brief report describing the processing you developed to implement the update reflecting the assumption that there is minimal difference between flat areas and those with slight slopes. If available, use Snagit screen capture software to embed screen capture of the maps used in the processing.
Topic 3 - Reclassifying and Overlying Maps