Beyond Mapping I

Epilog – Historical Overview and What Is in Store

Beyond Mapping book

Bringing the GIS Paradigm to Closure — *discusses the evolution and probable future of GIS technology*

Learning Computer-Assisted Map Analysis — *a 1986 journal article describing how "old-fashioned math and statistics can go a long way toward helping us understand GIS"*

A Mathematical Structure for Analyzing Maps — *1986 journal article establishing a comprehensive framework for map analysis/modeling using primitive spatial analysis and spatial statistics operators*

Note: The processing and figures discussed in this topic were derived using MapCalc™ software. See www.innovativegis.com to download a free MapCalc Learner version with tutorial materials for classroom and self-learning map analysis concepts and procedures.

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Bringing the GIS Paradigm to Closure

(GIS World Supplement, July 1992)

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Information has always been the cornerstone of effective decisions. Spatial information is particularly complex as it requires two descriptors—Where is What. For thousands of years the link between the two descriptors has been the traditional, manually drafted map involving pens, rub-on shading, rulers, planimeters, dot grids, and acetate sheets. Its historical use was for navigation through unfamiliar terrain and seas, emphasizing the accurate location of physical features.

More recently, analysis of mapped data has become an important part of understanding and managing geographic space. This new perspective marks a turning point in the use of maps from one emphasizing physical description of geographic space, to one of interpreting mapped data, combining map layers and finally, to spatially characterizing and communicating complex spatial relationships. This movement from "where is what" (descriptive) to "so what and why" (prescriptive) has set the stage for entirely new geospatial concepts and tools.

Since the 1960's, the decision-making process has become increasingly quantitative, and mathematical models have become commonplace. Prior to the computerized map, most spatial analyses were severely limited by their manual processing procedures. The computer has provided the means for both efficient handling of voluminous data and effective spatial analysis capabilities. From this perspective, all geographic information systems are rooted in the digital nature of the computerized map.

The coining of the term Geographic Information Systems reinforces this movement from maps as images to mapped data. In fact, information is GIS's middle name. Of course, there have been other, more descriptive definitions of the acronym, such as "Gee It's Stupid," or "Guessing Is Simpler," or my personal favorite, "Guaranteed Income Stream."

COMPUTER MAPPING

The early 1970's saw *computer mapping* automate map drafting. The points, lines and areas defining geographic features on a map are represented as an organized set of X, Y coordinates. These data drive pen plotters that can rapidly redraw the connections at a variety of colors, scales, and projections with the map image, itself, as the focus of the processing.

The pioneering work during this period established many of the underlying concepts and procedures of modern GIS technology. An obvious advantage with computer mapping is the ability to change a portion of a map and quickly redraft the entire area. Updates to resource maps which could take weeks, such as a forest fire burn, can be done in a few hours. The less obvious advantage is the radical change in the format of mapped data— from analog inked lines on paper, to digital values stored on disk.

SPATIAL DATA MANAGEMENT

During 1980's, the change in data format and computer environment was exploited. *Spatial database management systems* were developed that linked computer mapping capabilities with traditional database management capabilities. In these systems, identification numbers are assigned to each geographic feature, such as a timber harvest unit or ownership parcel. For example, a user is able to point to any location on a map and instantly retrieve information about that location. Alternatively, a user can specify a set of conditions, such as a specific forest and soil combination, then direct the results of the geographic search to be displayed as a map.

Early in the development of GIS, two alternative data structures for encoding maps were debated. The *vector data model* closely mimics the manual drafting process by representing map features (discrete spatial objects) as a set of lines which, in turn, are stores as a series of X,Y coordinates. An alternative structure, termed the *raster data model*, establishes an imaginary grid over a project area, and then stores resource information for each cell in the grid (continuous map surface). The early debate attempted to determine the universally best structure. The relative advantages and disadvantages of both were viewed in a competitive manner that failed to recognize the overall strengths of a GIS approach encompassing both formats.

By the mid-1980's, the general consensus within the GIS community was that the nature of the data and the processing desired determines the appropriate data structure. This realization of the duality of mapped data structure had significant impact on geographic information systems. From one perspective, maps form sharp boundaries that are best represented as lines. Property ownership, timber sale boundaries, and road networks are examples where lines are real and the data are certain. Other maps, such as soils, site index, and slope are interpretations of terrain conditions. The placement of lines identifying these conditions is subject to judgment and broad classification of continuous spatial distributions. From this perspective, a sharp boundary implied by a line is artificial and the data itself is based on probability.

Increasing demands for mapped data focused attention on data availability, accuracy and standards, as well as data structure issues. Hardware vendors continued to improve digitizing equipment, with manual digitizing tablets giving way to automated scanners at many GIS facilities. A new industry for map encoding and database design emerged, as well as a marketplace for the sales of digital map products. Regional, national and international organizations began addressing the necessary standards for digital maps to insure compatibility among systems. This era saw GIS database development move from project costing to equity investment justification in the development of corporate databases.

MAP ANALYSIS AND MODELING

As GIS continued its evolution, the emphasis turned from descriptive query to prescriptive analysis of maps. If early GIS users had to repeatedly overlay several maps on a light-table, an analogous procedure was developed within the GIS. Similarly, if repeated distance and bearing calculations were needed, the GIS system was programmed with a mathematical solution. The result of this effort was GIS functionality that mimicked the manual procedures in a user's daily activities. The value of these systems was the savings gained by automating tedious and repetitive operations.

By the mid-1980's, the bulk of descriptive query operations were available in most GIS systems and attention turned to a comprehensive theory of map analysis. The dominant feature of this theory is that spatial information is represented numerically, rather than in analog fashion as inked lines on a map. These digital maps are frequently conceptualized as a set of "floating maps" with a common registration, allowing the computer to "look" down and across the stack of digital maps. The spatial relationships of the data can be summarized (database queries) or mathematically manipulated (analytic processing). Because of the analog nature of traditional map sheets, manual analytic techniques are limited in their quantitative processing. Digital representation, on the other hand, makes a wealth of quantitative (as well as qualitative) processing possible. The application of this new theory to mapping was revolutionary and its application takes two forms—spatial statistics and spatial analysis.

Meteorologists and geophysicists have used *spatial statistics* for decades to characterize the geographic distribution, or pattern, of mapped data. The statistics describe the spatial variation in the data, rather than assuming a typical response is everywhere. For example, field measurements of snow depth can be made at several plots within a watershed. Traditionally, these data are analyzed for a single value (the average depth) to characterize an entire watershed. Spatial statistics, on the other hand, uses both the location and the measurements at sample locations to generate a map of relative snow depth throughout the watershed. This numeric-based processing is a direct extension of traditional non-spatial statistics.

Spatial analysis applications, on the other hand, involve context-based processing. For example, forester's can characterize timber supply by considering the relative skidding and log-hauling accessibility of harvesting parcels. Wildlife managers can consider such factors as proximity to roads and relative housing density to map human activity and incorporate this information into habitat delineation. Land planners can assess the visual exposure of alternative sites for a facility to sensitive viewing locations, such as roads and scenic overlooks.

Spatial mathematics has evolved similar to spatial statistics by extending conventional concepts. This "map algebra" uses sequential processing of spatial operators to perform complex map analyses. It is similar to traditional algebra in which primitive operations (e.g., add, subtract, exponentiate) are logically sequenced on variables to form equations. However in map algebra, entire maps composed of thousands or millions of numbers represent the variables of the spatial equation.

Most of the traditional mathematical capabilities, plus an extensive set of advanced map processing operations, are available in modern GIS packages. You can add, subtract, multiply, divide, exponentiate, root, log, cosine, differentiate and even integrate maps. After all, maps in a GIS are just organized sets of numbers. However, with map-ematics, the spatial coincidence and juxtaposition of values among and within maps create new operations, such as effective distance, optimal path routing, visual exposure density and landscape diversity, shape and pattern. These new tools and modeling approach to spatial information combine to extend record-keeping systems and decision-making models into effective decision support systems.

SPATIAL REASONING AND DIALOG

The future also will build on the cognitive basis, as well as the databases, of GIS technology. Information systems are at a threshold that is pushing well beyond mapping, management, modeling, and multimedia to spatial reasoning and dialogue. In the past, analytical models have focused on management options that are technically optimal— the scientific solution. Yet in reality, there is another set of perspectives that must be considered— the social solution. It is this final sieve of management alternatives that most often confounds geographic-based decisions. It uses elusive measures, such as human values, attitudes, beliefs, judgment, trust and understanding. These are not the usual quantitative measures amenable to computer algorithms and traditional decision-making models.

The step from technically feasible to socially acceptable is not so much increased scientific and econometric modeling, as it is communication. Basic to effective communication is involvement of interested parties throughout the decision process. This new participatory environment has two main elements— consensus building and conflict resolution.

Consensus Building involves technically-driven communication and occurs during the alternative formulation phase. It involves a specialist's translation of various considerations raised by a decision team into a spatial model. Once completed, the model is executed under a wide variety of conditions and the differences in outcome are noted.

From this perspective, an individual map is not the objective. It is how maps change as the different scenarios are tried that becomes information. "What if avoidance of visual exposure is more important than avoidance of steep slopes in siting a new electric transmission line? Where does the proposed route change, if at all?" What if slope is more important? Answers to these analytical queries (scenarios) focus attention on the effects of differing perspectives. Often, seemingly divergent philosophical views result in only slightly different map views. This realization, coupled with active involvement in the decision process, can lead to group consensus.

However, if consensus is not obtained, mechanisms for resolving conflict come into play. *Conflict Resolution* extends the Grateful Dead's lyrics, "nobody is right, if everybody is wrong," by seeking an acceptable management action through the melding of different perspectives. The socially-driven communication occurs during the decision formulation phase.

It involves the creation of a "conflicts map" which compares the outcomes from two or more competing uses. Each map location is assigned a numeric code describing the actual conflict of various perspectives. For example, a parcel might be identified as ideal for a wildlife preserve, a campground and a timber harvest. As these alternatives are mutually exclusive, a single use must be assigned. The assignment, however, involves a holistic perspective which simultaneously considers the assignments of all other locations in a project area.

Traditional scientific approaches rarely are effective in addressing the holistic problem of conflict resolution. Even if a scientific solution is reached, it often is viewed with suspicion by less technically-versed decision-makers. Modern resource information systems provide an alternative approach involving human rationalization and tradeoffs.

This process involves statements like, "If you let me harvest this parcel, I will let you set aside that one as a wildlife preserve." The statement is followed by a persuasive argument and group discussion. The dialogue is far from a mathematical optimization, but often comes closer to an acceptable decision. It uses the information system to focus discussion away from broad philosophical positions, to a specific project area and its unique distribution of conditions and potential uses.

CRITICAL ISSUES

The technical hurdles surrounding GIS have been aggressively tackled over the past four decades. Comprehensive spatial databases are taking form, GIS applications are accelerating and even office automation packages are including a "mapping button." So what is the most pressing issue confronting GIS in the next millennium?

Calvin, of the Calvin and Hobbes comic strip, puts it in perspective: "Why waste time learning, when ignorance is instantaneous?" Why should time be wasted in GIS training and education? It's just a tool, isn't it? The users can figure it out for themselves. They quickly grasped the operational concepts of the toaster and indoor plumbing. We have been mapping for thousands of years and it is second nature. GIS technology just automated the process and made it easier.

Admittedly, this is a bit of an overstatement, but it does set the stage for GIS's largest hurdle— educating the masses of potential users on what GIS is (and isn't) and developing spatial reasoning skills. In many respects, GIS technology is not mapping as usual. The rights, privileges and responsibilities of interacting with mapped variables are much more demanding than interactions with traditional maps and spatial records.

At least as much attention (and ultimately, direct investment) should go into geospatial application development and training as is given to hardware, software and database development. Like the automobile and indoor plumbing, GIS won't be an important technology until it becomes second nature for both accessing mapped data and translating it into information for decisions. Much more attention needs to be focused beyond mapping to that of spatial reasoning, the "softer," less traditional side of geotechnology.

GIS's development has been more evolutionary, than revolutionary. It responds to contemporary needs as much as it responds to technical breakthroughs. Planning and management have always required information as the cornerstone. Early information systems relied on physical storage of data and manual processing. With the advent of the computer, most of these data and procedures have been automated. As a result, the focus of GIS has expanded from descriptive inventories to entirely new applications involving prescriptive analysis. In this transition, map analysis has become more quantitative. This wealth of new processing capabilities provides an opportunity to address complex spatial issues in entirely new ways.

It is clear that GIS technology has greatly changed our perspective of a map. It has moved mapping from a historical role of provider of input, to an active and vital ingredient in the "thruput" process of decision-making. Today's professional is challenged to understand this new environment and formulate innovative applications that meet the complexity and accelerating needs of the twenty-first century.

Author's Note*: The July 1992 issue of GIS World was a special issue for the URISA conference. This supplemental white paper is a distillation of several keynotes, presentations and papers presented by the author.*

Learning Computer-Assisted Map Analysis

(GIS World Supplement, June 1993)

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"…old-fashioned math and statistics can go a long way toward helping us understand GIS"

Geographic Information Systems (GIS) technology is expanding the computer revolution by integrating spatial information with the research, planning and management of forestlands. The teaching of GIS technology poses problems in the classroom, and innovative ways of learning to apply the technology are being developed.

Unlike most other disciplines, GIS technology was born from specialized applications. A comprehensive theory tying these applications together is only now emerging. In one sense GIS technology is similar to conventional map processing, involving traditional maps and drafting aids—pens, run-on shading, rulers, planimeters, dot grids, and acetate sheets for light-table overlays. In another sense these systems provide advanced analytical capabilities, enabling land managers to address complex issues in entirely new ways.

As essential as computer-assisted map analysis has become, the technology is difficult to teach. Practical experience is required as well as theory, yet very few classrooms can provide extensive hands-on learning. Most GIS require expensive and specialized hardware, and even where equipment is available, instructors are faced with teaching the procedures of a system not designed for classroom use. Consequently, the approach most commonly used relies on case studies and selected literature.

Mathematics provides a useful starting point. In fact, GIS theory is based on a mathematical framework of primitive map-analysis operations analogous to those of traditional statistics and algebra. The teacher presents basic data characteristics and map processing as in a math course. Lectures and exercises provide a general toolbox for map analysis that embodies fundamental concepts and stimulates creative application. This toolbox is as flexible as conventional mathematics in expression relationships among variables—but with GIS, the variables are entire maps.

The map analysis toolbox helps resource define and evaluate spatial considerations in land management. For example, forest managers can characterize timber supply by considering the relative skidding and log-hauling accessibility of harvest parcels. Wildlife can consider such factors as proximity to roads and relative housing density in order to map human activity and incorporate this information into conventional habitat maps. Forest planners can assess the visual aesthetics of alternative sites for a facility or clearcut.

THE FUNDAMENTALS

The main purpose of a geographic information system is to process spatial information. The data structure can be conceptualized as a set of "floating maps" with common registration, allowing the user to "look" down and across a stack of maps. The spatial relationships of the data can be summarized (database inquiries) or manipulated (analytic processing). Such systems can be formally characterized as "internally referenced, automated, spatial information systems …designed for data mapping, management, and analysis."

All GIS contain hardware and software for data input, storage, processing, and display of computerized maps. The processing functions of these systems can be grouped into four categories: computer mapping, spatial database management, spatial statistics, and cartographic modeling.

Computer mapping— Also termed automated cartography, computer mapping involves the preparation of map products. The focus of these operations is the input and display of computerized maps.

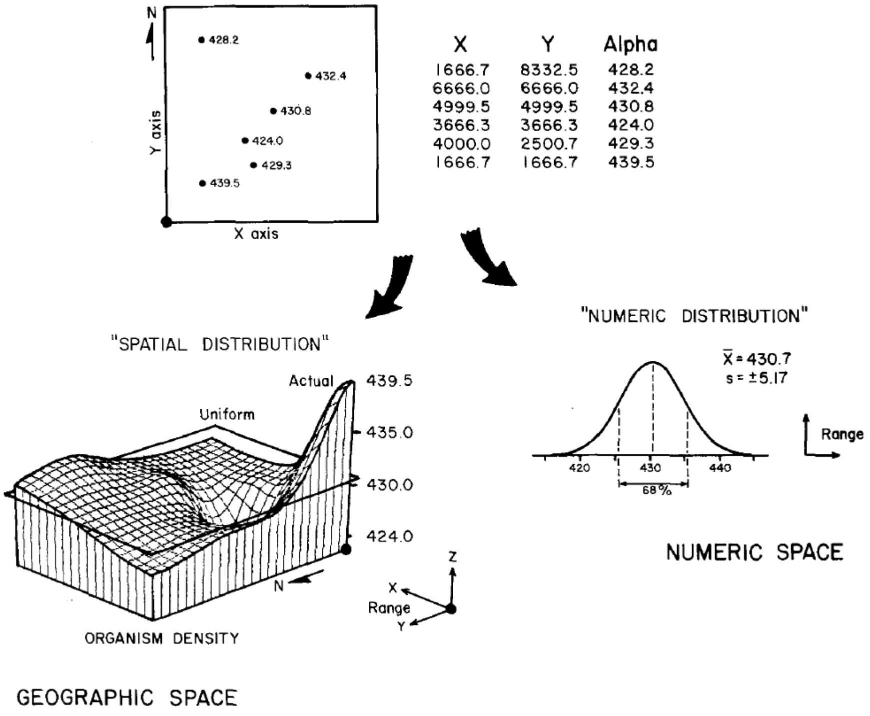
Spatial Database Management— These procedures focus on the storage component of GIS, efficiently organizing and searching large set of data for frequency statistics and coincidence among variables. The database allows rapid updating and examining of mapped information. For example, a spatial database can be searched for areas of silty-loam soil, moderate slope, and ponderosa pine forest cover. A summary table or a map of the results can then be produced.

These mapping and database capabilities have proven to be the backbone of current GIS applications. Aside from the significant advantages of processing speed and ability to handle tremendous volumes of data, such uses are similar to those of manual techniques. Here is where the parallels to mathematics and traditional statistics may be drawn.

Because of those parallels, the generalized GIS structure provides a framework for discussing the various data types and storage procedures involved in computer mapping and data management. It also provides a foundation for advanced analytic operations.

Spatial statistics—The dominant feature of GIS technology is that spatial information is represented numerically rather than in an analog fashion, as in the inked lines of a map. Because of the analog nature of the map sheets, manual analytic techniques are limited to nonquantitative processing. Digital representation, on the other hand, has the potential for quantitative as well as qualitative processing.

GIS have stimulated the development of spatial statistics, a discipline that seeks to characterize the geographic distribution or pattern of mapped data. Spatial statistics differs from traditional statistics by describing the more refined spatial variation in the data, rather than producing typical responses assumed to be uniformally distributed in space.

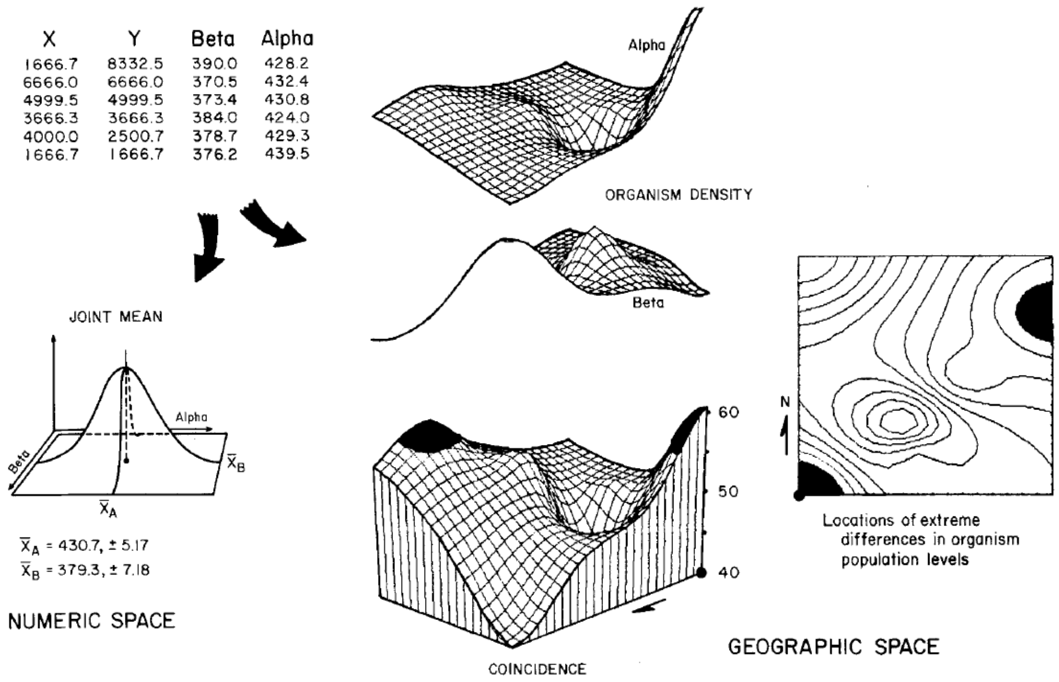


X	Y	Alpha
1666.7	8332.5	428.2
6666.0	6666.0	432.4
4999.5	4999.5	430.8
3666.3	3666.3	424.0
4000.0	2500.7	429.3
1666.7	1666.7	439.5

Figure 1. Spatial Statistics. Whereas traditional statistics identifies the typical response and assumes this estimate to be uniformally distributed in space, spatial statistics seeks to characterize the geographic distribution (pattern) of mapped data. The tabular data identify the location and population density of microorganisms sampled in a lake. Traditional statistical analysis shows an average density of about 430, assumed to be uniformally distributed throughout the lake. Spatial statistics incorporates locational information in mapping variations in the data. The pattern contains considerable variation from the average (shown as a plane) in the northwest portion of the lake.

An example of spatial analysis is shown in figures 1 and 2. Figure 1 depicts density mapping of a microorganism determined from laboratory analysis of surface water samples. Figure 2 depicts the natural extension to multivariate statistics for spatial coincidence between two microorganisms in the samples.

Traditional and spatial statistics complement each other for decision-making. In both statistical approaches, the nature of the data is critical. For analyses similar to those in Figures 1 and 2, thematic values ("what" information") must identify variables that form continuous gradients in both numeric and geographic space.



X	Y	Beta	Alpha
1666.7	8332.5	390.0	428.2
6666.0	6666.0	370.5	432.4
4999.5	4999.5	373.4	430.8
3666.3	3666.3	384.0	424.0
4000.0	2500.7	378.7	429.3
1666.7	1666.7	376.2	439.5

Figure 2. Multivariate Statistics. Maps characterizing spatial variation among two or more variables can be compared and locations of unusual coincidence identified. Traditional statistical analysis identifies the typical paired responses of two microorganism populations. This information assumes that the joint condition is uniformally distributed in space and does not identify where atypical joint occurrences might be found. Spatial statistics compares the two maps of population densities to identify two areas of unusually large differences in the populations.

Many forms of mapped data exist, including digital maps which, coupled with traditional mapping considerations—scale, projection, registration, resolution—can help foresters understand the potential and pitfalls of GIS applications. The quantitative nature of digital maps provides the foundation for a "mathematics of maps."

Just as spatial statistics has been developed by extending concepts of conventional statistics, a spatial mathematics has evolved. This "map algebra" uses sequential processing of mathematical primitives to perform complex map analyses. It is similar to traditional algebra, in which primitive operations (add, subtract, exponentiation) are logically sequenced on variables to form equations; but in map algebra, entire maps composed of thousands of number represent the variables.

Most traditional mathematical capabilities plus an extensive set of advanced map-processing primitives emerge. Transpose, inverse, and diagonalize are examples of new primitives based on the nature of matrix algebra. Within map analysis, the spatial coincidence and juxtapositioning of values among and within maps create new operators such as masking, proximity, and optimal paths.

This set of map-analysis operators can be flexibly combined through a processing structure similar to conventional mathematics. The logical sequence involves retrieval of one or maps from the database, processing that data as specified by the user, creation of a new map containing the processing results, and storage of the new map for subsequent processing.

The cyclical processing is similar to “evaluating nested parentheticals” in traditional algebra. Values for the “known” variables are first defined, then they are manipulated by performing the primitive operations in the order prescribed by the equation.

For example, in the equation $A = (B + C) / D$ the variables B and C are first defined and then added, with the sum stored as an intermediate solution. This intermediate value, in turn, is retrieved and divided by the variable D to derive the value of the unknown variable A. This same processing structure provides the framework for computer-assisted map analysis, but the variables are represented as spatially registered maps. The numbers contained in the solution map (in effect solving for A) are a function of the input maps and the primitive operations performed.

Cartographic modeling—This mathematical structure forms a conceptual framework easily adapted to a variety of applications in a familiar and intuitive manner. For example:

$$\%\text{Change} = \frac{(\text{new value} - \text{old value})}{\text{old value}} * 100$$

This equation the percent change in value for a parcel of land. In a similar manner, a map of percent change in land value for an entire town may be expressed in such GIS commands as:

```
COMPUTE NEWVALUE.MAP MINUS OLDVALUE.MAPFOR DIFFERENCE.MAP
COMPUTE DIFFERENCE.MAP TIMES 100 DIVIDED BY OLDVALE.MAP
   FOR PERCENTCHANGE.MAP
```

Within this model, data for current and past land values are collected and encoded as computerized maps. These data are evaluated as shown above to form a solution map of percent change. The simple model might be extended to provide coincidence statistics, such as

```
CROSSTAB ZONING.MAPWITH PERCENTCHANGE.MAP
```

for a table summarizing the spatial relationship between the zoning map and change in land value. Such a table would show which zones experienced the greatest increase in market value.

The basic model might also be extended to include such geographic searches as

```
  RENUMBER PERCENTCHANGE.MAP FOR BIG_CHANGES.MAP
    ASSIGNING 0 TO -20 THRU 20 ASSIGNING 1 TO 20 THRU 100
```

for a map search that isolates those areas that experienced more than a +20 percent change in market value.

ANALYTIC TOOLBOX

In traditional statistics and mathematics, pencil and paper are all this is needed to complete exercises. Use of a pocket calculator or computer enhances this experience by considering larger, more realistic sets of numbers. However, the tremendous volume of data involved in even the simplest map-processing task requires a computer for solution. Also, many of the advanced operations, such as effective distance measures and optimal paths, are so analytically complex that computer processing is essential.

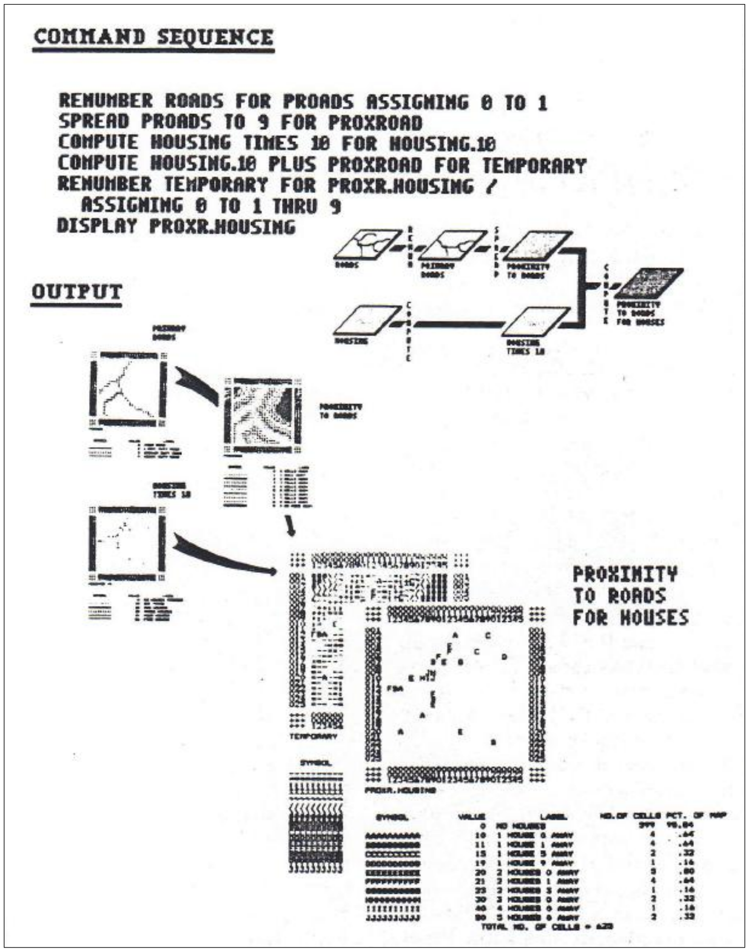
Classroom needs are not being ignored. Software and materials supporting instruction in computer-assisted map analysis are appearing on the scene. Among these is the *Map Analysis Package (MAP)*, a widely distributed system developed at Yale in 1976 for mainframe computers. Over 90 universities, many with natural-resource programs, have acquired these materials.

The *Professional Map Analysis Package (pMAP)* is a commercially developed implementation of MAP for PCs. The *Academic Map Analysis Package (aMAP)* is a special educational version of pMAP available from Yale for classroom use. All of the student exercises can be performed on a standard IBM PC or similar system.

As an example of a student exercise, consider the model outlined in figure 3. The exercise proposed that students "generate a map characterizing both the number of houses and their simple proximity to major roads, given maps of housing locations (HOUSING) and the road network (ROADS)." A conceptual flowchart and the actual sequence of pMAP commands forming the solution is shown at the top. Some of the intermediate maps and the final map are shown at the bottom.

Optional assignments in the exercise required students to "produce a similar map identifying both the cover type and the distance to the nearest house for all locations in the study area" and "extend the technique to characterize each map location as to its general housing density within a tenth-of-a-mile radius, as well as it proximity to the nearest major road." Both of these options require minimal modification of the basic technique—redefinition of the input maps in the first case and slight editing of two others in the second (see Author's Note 2 for more discussion of class exercises and course organization).

GIS technology is revolutionizing how we handle maps. Beginning in the 1970s, an ever-increasing portion of mapped data has been collected and processed in digital format. Currently, this computer processing emphasizes computer mapping and database management. These techniques allow users to update maps quickly, generate descriptive statistics, make geographic searches for areas of specified coincidence, and display the results as a variety of colorful and useful products.



COMMAND SEQUENCE

```
RENUMBER ROADS FOR PROADS ASSIGNING 0 TO 1
SPREAD PROADS TO 9 FOR PROXROAD
COMPUTE HOUSING TIMES 10 FOR HOUSING.10
COMPUTE HOUSING.10 PLUS PROXROAD FOR TEMPORARY
RENUMBER TEMPORARY FOR PROXR.HOUSING /
  ASSIGNING 0 TO 1 THRU 9
DISPLAY PROXR.HOUSING
```

Figure 3. Student Exercise. Proximity to major roads is first determined and then combined with housing information. Two-digit codes identify the number of houses (first digit; tens) and the distance to the nearest major road (second digit; ones).

Emerging capabilities are extending the revolution to how we analyze spatial information. These operations provide an analytic toolbox for expressing spatial relationships within and among maps, analogous to traditional statistics and algebra. This quantitative approach is changing basic concepts of map content, structure, and use.

Maps are moving from images that describe the location of features to mapped data quantifying a physical or abstract system in prescriptive terms. Students of GIS technology are developing a proficiency in a sort of “spatial spreadsheet” that allows them to integrate spatial information with research, planning, and management in natural-resource fields.

***Author's Notes**: 1) The June 1993 issue of GIS World was a special issue for the URISA conference. This supplemental white paper is based on a Journal of Forestry paper of the same title appearing, in the October 1986 issue. 2) See a 1979 paper entitled "An Academic Approach to Cartographic Modeling in Management of Natural Resources" describing a graduate-level course in map analysis/modeling using a "Map Algebra" instructional approach posted at http://www.innovativegis.com/basis/Papers/Other/Harvard1979/HarvardPaper.pdf.*

A Mathematical Structure for Analyzing Maps

(GIS World Supplement, June 1993)

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ABSTRACT— the growing use of computers in environmental management is profoundly changing data collection procedures, analytic processes, and even the decision-making environment itself. The emerging technology of geographic information systems (GIS) is expanding this revolution to integrate spatial information fully into research, planning, and management of land. In one sense, this technology is similar to conventional map processing involving traditional maps and drafting aids, such as pens, rub-on shading, rulers, planimeters, dot grids, and acetate sheets for light-table overlays. In another sense, these systems provide advanced analytic capabilities, enabling managers to address complex issues in entirely new ways. This report discusses a fundamental approach to computer-assisted map analysis that treats entire maps as variables. The set of analytic procedures for processing mapped data forms a mathematical structure analogous to traditional statistics and algebra. All of the procedures discussed are available for personal computer environments.

The historical use of maps has been for navigation through unfamiliar terrain and seas. Within this context, preparation of maps that accurately locate special features became the primary focus of attention. More recently, analysis of mapped data has become an important part of resource and environmental planning. During the 1960s, manual analytic procedures for overlaying maps were popularized by the work of McHarg (1969) and others. These techniques mark an important turning point in the use of maps—from one emphasizing physical descriptors of geographic space, to one spatially characterizing appropriate land management actions. This movement from descriptive to prescriptive mapping has set the stage for revolutionary concepts of map structure, content, and use.

Spatial analysis involves tremendous volumes of data. Manual cartographic techniques allow manipulation of these data, but they are fundamentally limited by their nonquantitative nature. Traditional statistics, on the other hand, enable quantitative treatment of the data, but, until recently, the sheer magnitude of mapped data made such processing prohibitive. Recognition of this limitation led to "stratified sampling" techniques developed in the early part of this century. These techniques treat spatial considerations at the onset of analysis by dividing geographic space into homogeneous response parcels. Most often, these parcels are manually delineated on an appropriate map, and the "typical" value for each parcel determined. The results of any analysis is then assumed to be uniformly distributed throughout each parcel. The area-weighted average of a set of parcels statistically characterizes the typical response for an extended area. Mathematical modeling of spatial systems has followed a similar approach of spatially aggregating variation in model variables. Most ecosystem models, for example, identify "level variables" and "flow rates" presumed to be typical for vast geographic expanses.

A comprehensive spatial statistics and mathematics, for many years, has been variously conceptualized by both theorist and practitioner. Until modern computers and the computerized map, these concepts were without practical implementation. The computer has provided the means for both efficient handling of voluminous data and the quantitative analysis required by these concepts. From this perspective, the current revolution in mapping is rooted in the digital nature of the computerized map. Increasingly, geographic information system (GIS) technology is being viewed as providing new capabilities for expressing spatial relationships, as well as efficient mapping and management of spatial data. This report discusses the fundamental consideration of the emerging "toolbox" of analytic capabilities.

Geographic Information Systems (GIS)

The main purpose of a geographic information system (GIS) is to process spatial information. These systems can be defined as internally referenced, automated, spatial information systems--designed for data mapping, management, and analysis. These systems have an automated linkage between the type of data, termed the thematic attribute, and the whereabouts of that data, termed the locational attribute. This structure can be conceptualized as a stack of "floating maps," with common spatial registration, allowing the user to "look" down and across the stack. The spatial relationships of the data can be examined (that is, inventory queries) or manipulated (that is, analytic processing). The locational information may be organized as a collection of line segments identifying the boundaries of point, linear, and areal features. An alternative organization establishes an imaginary grid pattern over a study area and stores numbers identifying the characteristic at each grid space. Although there are significant practical differences in these data structures, the primary conceptual difference is that the grid format stores information on the interior of areal features, and implies boundaries; whereas, the segment format stores information about boundaries, and implies interiors. Generally, the line segment structure is best for inventory-oriented processing, while the grid structure is best for analysis-oriented processing. The difficulty of line segments in characterizing spatial gradients, such as elevation or housing density, coupled with the frequent necessity to compute interior characteristics limit the use of this data type for many of the advanced analytic operations. As a result, most modern GIS contain programs for converting between the two data structures.

Regardless of the data storage structure, all GIS contain hardware and software for data input, storage, processing, and display of digital maps. The processing functions of these systems can be grouped into four broad categories:

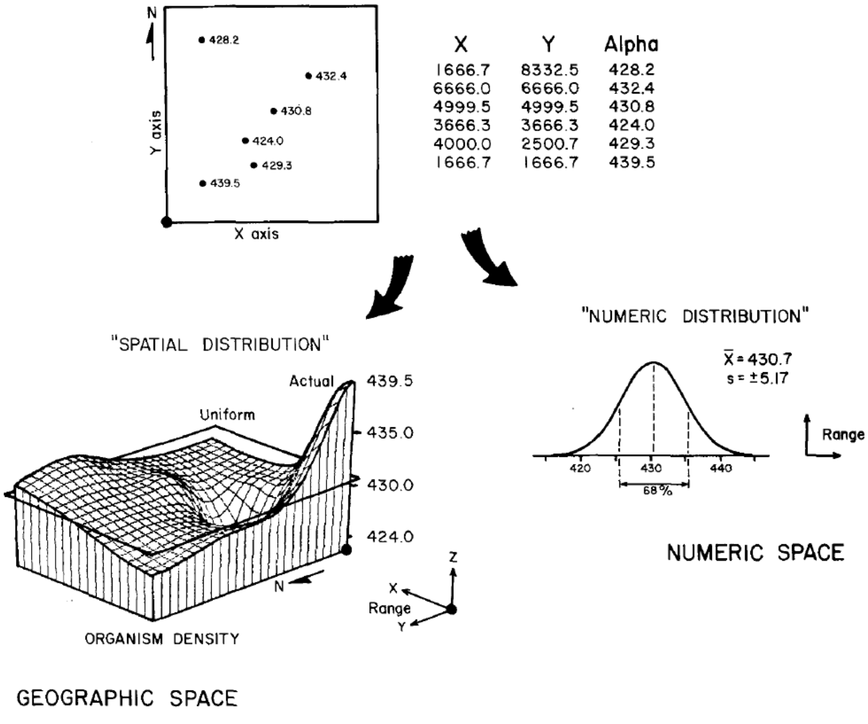
- Computer mapping
- Spatial data base management
- Spatial statistics
- Cartographic modeling

Most GIS contain some capabilities from each of these categories. An inventory-oriented system will emphasize mapping and management functions, whereas, an analysis-oriented system will focus on statistics and modeling functions.

Computer mapping, also termed automated cartography, involves the preparation of map products. The focus of these operations is on the input and display of computerized maps. Spatial data base management, on the other hand, focuses on the storage component of GIS technology. Like nonspatial data base systems, these procedures efficiently organize and search large sets of data for frequency statistics and/or coincidence among variables. For example, a spatial data base may be searched to generate a map of areas of silty-loam soil, moderate slope, and ponderosa pine forest cover. These mapping and data base capabilities have proven to be the backbone of current GIS applications. Once a data base is compiled, they allow rapid updating and examining of mapped data. However, other than the significant advantage of speed, these capabilities are similar to those of manual techniques. The remainder of this paper investigates the emerging analytic concepts of spatial statistics and cartographic modeling.

Spatial Statistics

Spatial statistics seeks to characterize the geographic pattern or distribution of mapped data. This approach differs from traditional statistics as it describes the spatial variation in the data, rather than distilling the data for typical responses that are assumed to be uniformly distributed in space. For example, consider the hypothetical data presented in Figure 1.



X	Y	Alpha
1666.7	8332.5	428.2
6666.0	6666.0	432.4
4999.5	4999.5	430.8
3666.3	3666.3	424.0
4000.0	2500.7	429.3
1666.7	1666.7	439.5

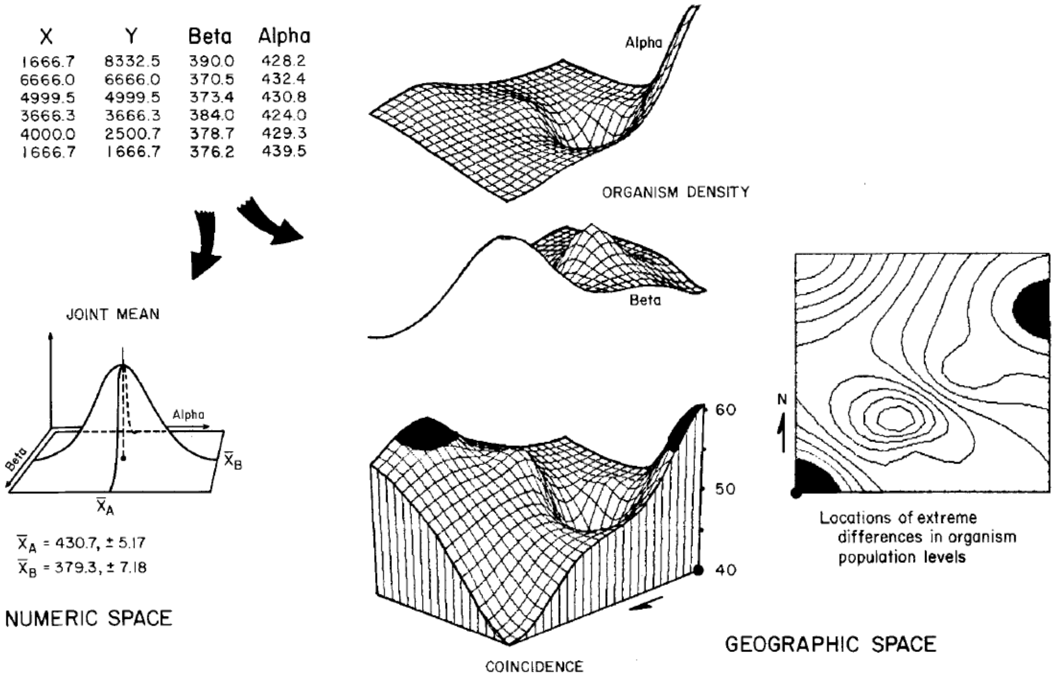
Figure 1. Spatially characterizing data variation. Traditional statistics identifies the typical response and assumes this estimate to be distributed uniformly in space. Spatial statistics seeks to characterize the geographic distribution, or pattern, of mapped data.

The large square at the top is a map of sample locations for a portion of a lake. The tabular data to the right identify both the location and microorganism density determined from laboratory analysis of the surface water samples. Traditional statistics would analyze the data by fitting a numerical distribution to the data to determine the typical response. Such density functions as standard normal, binomial, and Poisson could be tried, and the best-fitting functional form chosen. The lower-right inset characterizes the fitting of a "standard normal curve" indicating an average organism density of 430.7 ± 5.17 units. These parameters describing the central tendency of the data in numerical space are assumed to be uniformly distributed in geographic space. As shown in the lower-left plot, this assumption implies a horizontal plane at 430.7 units over the entire area. The geographic distribution of the standard deviation would form two planes (analogous to "error bars") above and below the average. This traditional approach concentrates on characterizing the typical thematic response, and disregards the locational information in the sampled data.

Spatial statistics, on the other hand, incorporates locational information in mapping the variation in thematic values. Analogous to traditional statistics, a density function is fitted to the data. In this instance, the distribution is characterized in geographic space, rather than numerical space. To conceptualize this process, visualize a pillar at each sample location rising above the lake in Figure 1 to the height of its thematic value (that is, measured organism density). The lower-left inset shows a continuous surface that responds to the peaks and valleys implied by the pillars of the sampled data. This surface was fitted by "inverse-distance-squared weighted averaging" of the sample values using an inexpensive personal computer software package (Golden 1985). The distribution shows considerable variation in the data in the southwest portion, tending to conflict with the assumption that variation is uniformly distributed in space. Analogous to traditional statistics, other surface fitting techniques, such as Kriging, spline, or polynomial functions, could be tried and the best-fitting functional form chosen. Analysis of the residuals, from comparisons of the various surfaces with the measured values, provides an assessment of fit.

Figure 2 depicts multivariate statistical analysis. If the level of another microorganism was also determined for each water sample, its joint occurrence with the previously described organism could be assessed. In traditional statistics, this involves fitting of a density function in multivariate statistical space. For the example, a standard normal surface was fitted, with its “joint mean" and "covariance matrix" describing the typical paired occurrence. Generally speaking, the second organism occurs less often (379.3 vs. 430.7) with a negative correlation (-.78) between the two populations.

The right portion of Figure 2 depicts an analysis of the spatial distributions of the two populations. For this analysis the two maps of population density are compared (that is, one subtracted from the other) to generate a coincidence surface. A planimetric map of the surface, registered to the original map of the lake, is on the right side of the figure. The darkened areas locate areas of large differences (that is, more than average difference plus average standard deviation) in the two organism populations. The combined information provided by traditional and spatial statistics complements each other for decision making. Characterizing the typical response develops a general impression of the data. The mapping of the variance in the data refines this impression by locating areas of abnormal response. In the example, the approximately 50 units difference in average population densities may not warrant overall concern. However, the pockets of larger differences may direct further investigation, or special management action, within those areas of the lake.



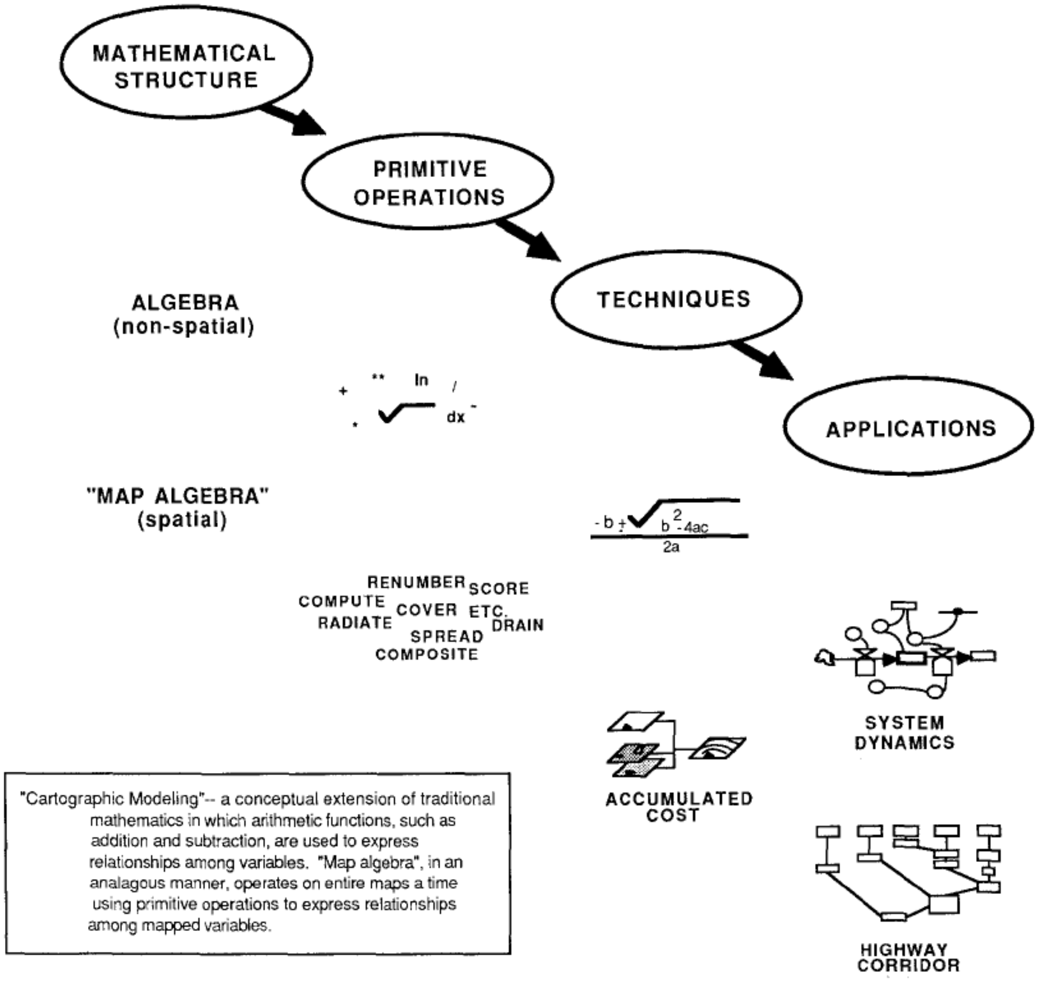
X	Y	Beta	Alpha
1666.7	8332.5	390.0	428.2
6666.0	6666.0	370.5	432.4
4999.5	4999.5	373.4	430.8
3666.3	3666.3	384.0	424.0
4000.0	2500.7	378.7	429.3
1666.7	1666.7	376.2	439.5

Figure 2. Assessing coincidence among mapped data. Maps characterizing spatial variation among two or more variables can be compared and locations of unusual coincidence identified.

In both statistical approaches, the nature of the data is crucial. For spatial analyses similar to the example, the thematic values must be numerically "robust" and the locational attribute "isopleth." These terms identify map variables that form gradients in both numeric and geographic space. By contrast, a variable might contain "non-robust" values (for instance, arbitrary numbers associated with soil types) that are discontinuous, or "choropleth," in geographic space (for instance, abrupt land use boundaries). A complete discussion of the considerations dealing with the various types of data in spatial statistics is beyond the introductory scope of this article. Both Davis (1973) and Ripley (1981) offer good treatise of this subject.

A Mathematical Structure for Map Analysis

Just as a spatial statistics may be identified, a spatial mathematics is also emerging. This approach uses sequential processing of mathematical primitives to perform a wide variety of complex map analyses (Berry 1985). By controlling the order in which the operations are executed, and using a common database to store the intermediate results, a mathematical-like processing structure is developed. This "map algebra" (Figure 3) is similar to traditional algebra in which primitive operations, such as addition, subtraction, and exponentiation, are logically sequenced for specified variables to form equations; however, in map algebra the variables represent entire maps consisting of numerous values. Most of the traditional mathematical capabilities, plus an extensive set of advanced map processing primitives, comprise this map analysis "toolbox." As with matrix algebra (a mathematics operating on groups of numbers defining variables), new primitives emerge that are based on the nature of the data. Matrix algebra's transposition, inversion, and diagonalization are examples of extended operations. Within map analysis, the spatial coincidence and juxtapositioning of values among and within maps create new operators, such as proximity, spatial coincidence, and optimal paths.



"Cartographic Modeling"-- a conceptual extension of traditional mathematics in which arithmetic functions, such as addition and subtraction, are used to express relationships among variables. "Map algebra", in an analagous manner, operates on entire maps a time using primitive operations to express relationships among mapped variables.

Figure 3. Cartographic modeling. In mathematics, primitive operators, such as addition and subtraction, are used to express relationships among variables. "Map algebra," in an analogous manner, operates on entire maps at a time using primitive operators to express relationships within and among mapped variables.

These operators can be accessed through general purpose map analysis packages, similar to the numerous matrix algebra packages. The map analysis package (MAP) (Tomlin 1983) is an example of a comprehensive general purpose system that has been acquired by over 350 computer centers throughout the world. A commercial implementation of this system, the Professional Map Analysis Package (pMAP) (SIS 1986), has recently been released for personal computers. All of the processing discussed in this paper was done with this inexpensive pMAP system in a standard IBM PC environment. The logical sequencing of map processing involves:

- Retrieval of one or more maps from the database
- Processing those data as specified by the user
- Creation of a new map containing the processing results
- Storage of the new map for subsequent processing

Each new map derived as processing continues is spatially registered to the other maps in the database. The values comprising the derived maps are a function of the statistical or mathematical summary of the values on the "input maps."

This cyclical processing provides an extremely flexible structure similar to "evaluating nested parentheticals” in traditional algebra. Within this structure, a mathematician first defines the values for each dependent variable and then solves the equation by performing the primitive mathematical operations on those numbers in the order prescribed by the equation. For example, the simple equation

$$A = (B + C)/D$$

identifies that the variables B and C are first defined, and then added, with the sum being stored as an intermediate solution. The intermediate value, in turn, is divided by the variable D to calculate the solution variable A. This same mathematical structure provides the framework for computer-assisted map analysis.

Within this processing structure, four fundamental classes of map analysis operations may be identified. These include:

- *Reclassifying maps*— involving the reassignment of the values of an existing map as a function of the initial value, position, size, shape, or contiguity of the spatial configuration associated with each category.
- *Overlaying maps*— resulting in the creation of a new map where the value assigned to every location is computed as a function of the independent values associated with that location on two or more existing maps.
- *Measuring distance and connectivity*— involving the creation of a new map expressing the distance and route between locations as simple Euclidean length, or as a function of absolute or relative barriers.
- *Characterizing and summarizing neighborhoods*— resulting in the creation of a new map based on the consideration of values within the general vicinity of target locations.

These major groupings can be further classified as to the basic approaches used by the various processing algorithms. This mathematical structure forms a conceptual framework that is easily adapted to modeling the spatial relationships in both physical and abstract systems. Detailed discussion of the various analytic procedures is beyond the introductory scope of this article. Reference to the papers by Berry and Tomlin (1982a and b) provides comprehensive discussion and examples of each of the classes of operations.

Cartographic Modeling

The cyclical processing structure of map analysis enables primitive spatial operations to be combined to form equations, or "cartographic models," in a familiar and intuitive manner. For example, in traditional algebra, an equation for the percent change in value of a parcel of land may be expressed as

$$\%Change = \frac{(\text{new value} - \text{old value})}{\text{old value}} * 100$$

In a similar manner, a map of percent change in market values for an entire town may be expressed, in pMAP command language, as:

```
 COMPUTE NEWVALUE.MAP MINUS OLDVALUE.MAP TIMES 100
     DIVIDEDBY OLDVALUE.MAP FOR PERCENTCHANGE.MAP
```

Within this model, data for current and past land values are collected and encoded as computerized maps. These data are evaluated, as shown above, to form a solution map of percent change. The simple model might be extended to provide coincidence statistics, such as…

```
CROSSTABULATE ZONING.MAP WITH PERCENTCHANGE.MAP
```

…for a table summarizing the spatial relationship between type of zoning and change in land value (to answer the question— which zones have experienced the greatest decline in market value?). The basic model might also be extended to include geographic searches, such as…

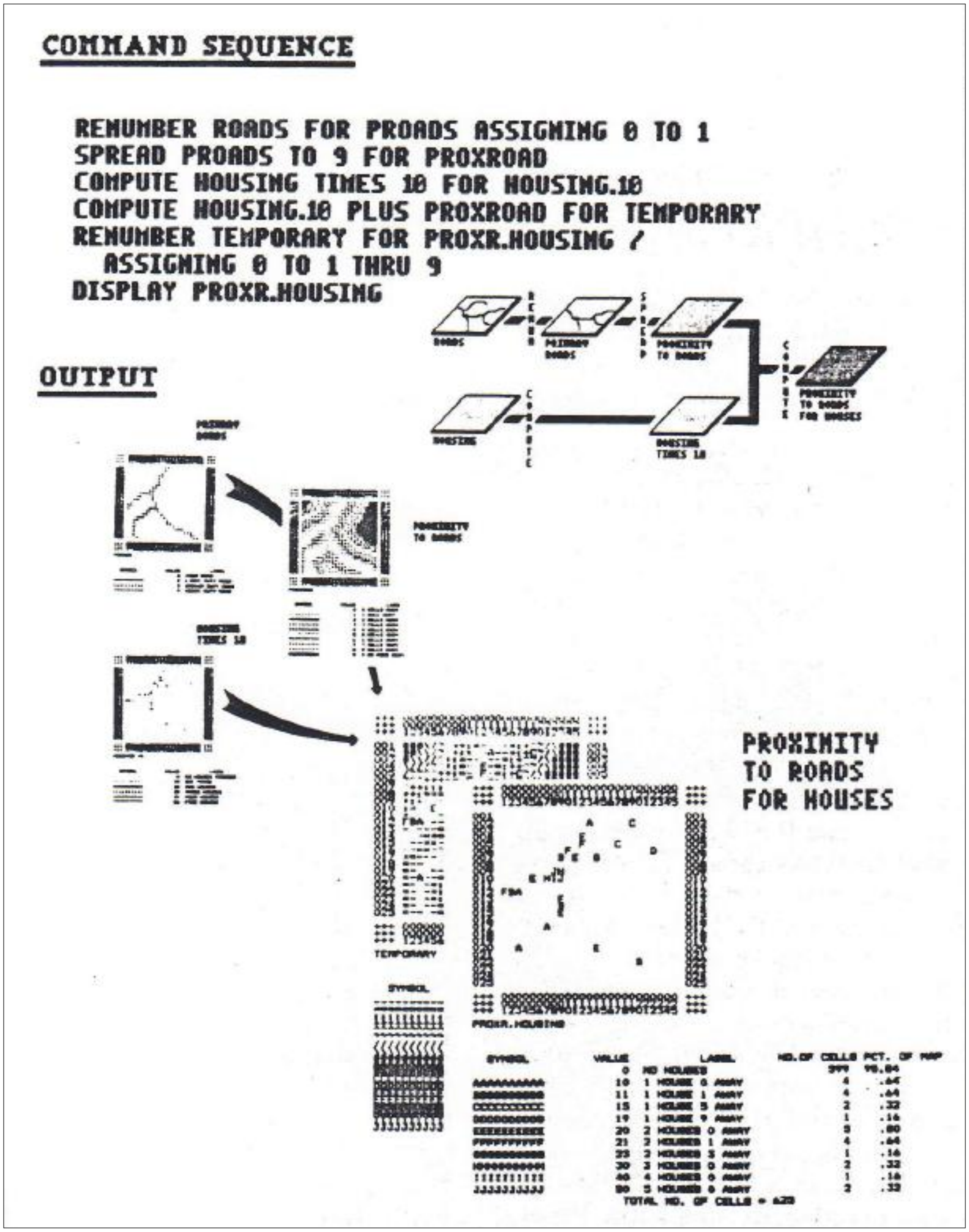
```
RENUMBER PERCENTCHANGE.MAP ASSIGNING 0 TO -- 20 THRU 20
   FOR BIGCHANGES.MAP
```

…for a map isolating those areas which have experienced more than a + 20% or -20% change in market value.

Another simple model is outlined in Figure 4. It creates a map characterizing the number of houses and their proximity to major roads, given maps of housing locations (HOUSING) and the road network (ROADS). The model incorporates reclassifying, distance measuring, and overlaying operations. An extension to the model (not shown) uses a neighborhood operation to characterize each map location as to its general housing density within a tenth of a mile radius, as well as its proximity to roads. Incorporation of this modification requires the addition of one line of code and slight editing of two others. A working-copy display of the results and tabular summary is shown at the bottom of Figure 4. The pMAP command sequence for this analysis is shown in the upper portion.

The procedure used in the example may be generalized to identify "type-distance" combinations among any set of features within a mapped area. Consider the following generalization:

onemap → X
anothermap → Y
XDIST = distance as a fn(x)
z = (y * 10) + XDIST
compositemap ← Z



COMMAND SEQUENCE

```
RENUMBER ROADS FOR PROADS ASSIGNING 0 TO 1
SPREAD PROADS TO 9 FOR PROXROAD
COMPUTE HOUSING TIMES 10 FOR HOUSING.10
COMPUTE HOUSING.10 PLUS PROXROAD FOR TEMPORARY
RENUMBER TEMPORARY FOR PROXR.HOUSING /
  ASSIGNING 0 TO 1 THRU 9
DISPLAY PROXR.HOUSING
```

OUTPUT

PROXIMITY TO ROADS FOR HOUSES

```
   SYMBOL            VALUE          LABEL       NO.OF CELLS PCT. OF MAP
                       0   NO HOUSES                  599    95.84
 AAAAAAAAAAA          10   1 HOUSE 0 AWAY               4      .64
 BBBBBBBBBBB          11   1 HOUSE 1 AWAY               4      .64
 CCCCCCCCCCC          15   1 HOUSE 5 AWAY               2      .32
 DDDDDDDDDDD          19   1 HOUSE 9 AWAY               1      .16
 EEEEEEEEEEE          20   2 HOUSES 0 AWAY              5      .80
 FFFFFFFFFFF          21   2 HOUSES 1 AWAY              4      .64
 GGGGGGGGGGG          23   2 HOUSES 3 AWAY              1      .16
 HHHHHHHHHHH          30   3 HOUSES 0 AWAY              2      .32
 IIIIIIIIIII          40   4 HOUSES 0 AWAY              1      .16
 JJJJJJJJJJJ          50   5 HOUSES 0 AWAY              2      .32
                        TOTAL NO. OF CELLS = 625
```

Figure 4. *A simple cartographic model. This analysis combines the information on housing locations (HOUSING) and the road network (ROADS) to derive a map characterizing the number of houses and their proximity to the nearest major road.*

This "macro" technique may be stored as a command file and accessed at anytime. For example, the following command sequence can be entered to assess the covertype (Y) and distance to the nearest house (x) for all map locations.

Keyboard Entry

```
ASSOCIATE X WITH HOUSING
ASSOCIATE Y WITH COVERTYPE
READ c:macro.cmd
```

Stored Command File

```
SPREAD X TO 9 FOR PROX_X
COMPUTE Y TIMES l0 FOR Y_10
COMPUTE PROX_X PLUS Y_10 FOR Z
```

```
DISPLAY Z
```

Note: the Z map contains a two-digit code with the "tens digit" indicating the Covertype and the "ones digit" indicating the proximity to the nearest house. For example a 21 value indicates locations that are classified as cover type 2 and close to a house (only 1 cell away).

The ASSOCIATE command temporarily defines specified maps as generalized variables. The READ command transfers input control to the designated file, and the stored set of generalized commands are processed as if they were being entered from the keyboard. When the model is finished executing, input control is returned to the keyboard for user interactive processing.

Generalized Structure for Map Analysis

The development of a generalized analytic structure for map processing is similar to those of many other nonspatial systems. For example, the popular dBASE III package contains less than 20 analytic operations, yet they may be flexibly combined to create "models" for such diverse applications as address lists, inventory control, and commitment accounting. Once developed, these logical sequences of dBASE sentences can be "fixed" into menus for easy end-user operations. A flexible analytic structure provides for dynamic simulation as well as database management. For example, the Multiplan "spreadsheet" package allows users to define the interrelationships among variables. By specifying a logical sequence of interrelationships and variables, a financial model of a company's production process may be established. By changing specific values of the model, the impact of uncertainty in fiscal assumptions can be simulated. The advent of database management and spreadsheet packages has revolutionized the handling of nonspatial data.

Computer-assisted map analysis promises a similar revolution for handling spatial data. For example, a model for siting a new highway could be developed. The analysis would likely consider economic and social concerns (for example, proximity to high housing density, visual exposure to houses), as well as purely engineering ones (for example, steep slopes, water bodies). The combined expression of both physical and nonphysical concerns, in a quantified spatial context, is a major benefit. However, the ability to simulate various scenarios (for instance, steepness is twice as important as visual exposure; proximity to housing four times as important as all other considerations) provides an opportunity to fully integrate spatial information into the decision-making process. By noting how often and where the optimal route changes as successive runs are made, information on the unique sensitivity to siting a highway in a particular locale is described.

In addition to flexibility, there are several other advantages in developing a generalized analytic structure for map analysis. The systematic rigor of a mathematical approach forces both theorist and user to consider carefully the nature of the data being processed, it also provides a comprehensive format for instruction which is independent of specific disciplines or applications (Berry 1986). In addition, the flowchart of processing succinctly describes the components of an analysis. This communication enables decision makers to understand more fully the analytic process, and actually comment on model weightings, incomplete considerations, or erroneous assumptions. These comments, in most cases, can be easily incorporated and new results generated in a timely manner. From a decision-maker's point of view, traditional manual techniques of map analysis are separate from the decision itself. They require considerable time to perform and many of the considerations are subjective in their evaluation. From this perspective, the decision maker attempts to interpret results, bounded by the often vague assumptions and system expression of the technician. Computer-assisted map analysis, on the other hand, encourages the involvement of the decision maker in the analytic process. From this perspective, spatial information becomes an active and integral part of the decision process itself.

Conclusion

Geographic information system (GIS) technology is revolutionizing how maps are handled. Since the 1970s, an ever-increasing portion of mapped data is being collected and processed in digital format. Currently, this processing emphasizes computer mapping and database management capabilities. These techniques allow users to update maps quickly, generate descriptive statistics, make geographic searches for areas of specified coincidence, and display the results as a variety of colorful and useful products. Newly developing capabilities extend this revolution to how mapped data are analyzed. These techniques provide an analytic "toolbox" for expressing the spatial interrelationships of maps. Analogous to traditional algebra and statistics, primitive operations are logically sequenced on variables to form spatial models; however, the variables are represented as entire maps.

This quantitative approach to map analysis is changing basic concepts of map structure, content, and use. From this perspective, maps move from images describing the location of features to mapped data quantifying a physical or abstract system in prescriptive terms. This radical change in map analysis has promoted a more complete integration of spatial information into the decision-making process.

***Author's Notes**: 1) This supplement is based on an article published in the Journal of Environmental Management, Springer-Verlag, Vol 11, No. 3, pp. 317-325, 1986. J.K. Berry. 2) The professional map analysis package (pMAP) described in this article was developed and written by the author and Dr. Kenneth L. Reed of Spatial Information Systems, Inc. Several of the concepts and algorithms are based on the widely distributed mainframe map analysis package (MAP) developed by C. Dana Tomlin (1983) and distributed by Yale University. 3) References:*

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