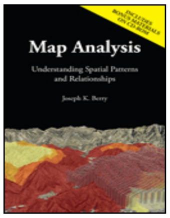
Beyond Mapping III

Topic 9 – Basic Techniques in Spatial Statistics (Further Reading)



Map Analysis book

(***Modeling Error Propagation***)

Move Beyond a Map Full of Errors — *discusses a technique for generating a "shadow map" of error (March 1997)*

Comparing Map Errors — *describes how normalized maps of error can be used to visualize the differences in error surfaces (April 1997)*

(***Point Sampling Considerations***)

What's the Point? — *discusses the general considerations in point sampling design (December 1996)*

Designer Samples — *describes different sampling patterns and their relative advantages (January 1997)*

(***Advanced Concepts in Spatial Dependency***)

Depending on the Data — *discusses the fundamental concepts of spatial dependency (May 1997)*

Uncovering the Mysteries of Spatial Autocorrelation — *describes approaches used in assessing spatial autocorrelation (July 1997)*

Unlocking the Keystone Concept of Spatial Dependency — *discusses spatial dependency and illustrates the effects of different spatial arrangements of the same set of data (November 1998)*

Measuring Spatial Dependency — *describes the basic measures of autocorrelation (December 1998)*

Extending Spatial Dependency to Maps — *describes a technique for generating a map of spatial autocorrelation (January 1999)*

Use Polar Variograms to Assess Distance and Direction Dependencies — *discuses a procedure to incorporate direction as well as distance for assessing spatial dependency (September 2001)*

<*Click here*> *for a printer-friendly version of this topic (.pdf).*

(*Back to the Table of Contents*)

(***Advanced Concepts in Spatial Dependency***)

Move Beyond a Map Full of Errors

(GeoWorld, March 1997)

(*return to top of Topic*)

Previous discussion (February 1997 BM column) described a procedure, termed "Residual Analysis," for checking the reliability of maps generated from field samples, such as a map of phosphorous levels derived from a set of soil samples. The data could have as easily been lead concentrations in an aquifer from a set of sample wells, or any other variable that forms a gradient in geographic space for that matter. The evaluation procedure suggested holding back some of the samples as a test set to "empirically verify" the estimated value at each location. Keep in mind, if you don't attempt to verify, then you are simply accepting the derived map as a perfect rendering of reality (and blind faith isn't a particularly a good basis for visual analysis of mapped data). The difference between a mapped "guess" (interpolated estimate) and "what is" (actual measurement) is termed a residual.

A table of all the residuals for a test set is summarized to determine the overall accuracy of an interpolated map. The residual analysis described last time determined that the Kriging interpolation procedure had the closest predictions to those of the example test set. Its predictions, on average, were only 3.3 units off, whereas the other techniques of Inverse Distance and Minimum Curvature were considerably less accurate (5.1 and 10.5, respectively). The use of an arithmetic average for spatial prediction was by far the worst with an average boo-boo of 22.2 for each guess. The residual table might have identified the relative performance of the various techniques, but it failed to identify where a map is likely predicting too high or too low— that's the role of a residual map.

The table in Figure 1 shows the actual and estimated values from Kriging for each test location, with their residuals shown in parentheses. The posted values on the 3-D surface locate the positioning of the residual values.

Sample	Actual	Kriging	Error
#17	0	2	(2)
18	48	46	(-2)
19	64	65	(1)
20	65	56	(-9)
21	34	30	(-4)
22	0	-1	(-1)
23	6	1	(-5)
24	79	70	(-9)
25	64	68	(4)
26	8	7	(-1)
27	19	19	(0)
28	6	3	(-3)
29	12	14	(2)
30	17	20	(3)
31	9	6	(-3)
32	14	11	(-3)

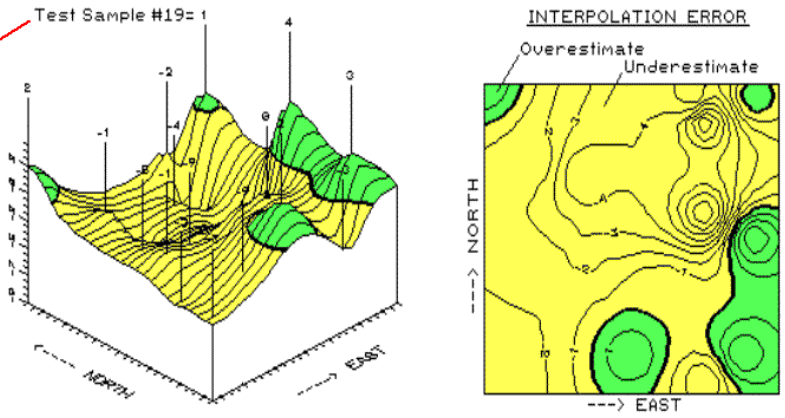


Figure 1. Map of errors from the Kriging interpolation technique.

For example, the value 1 posted in the northeast corner identifies the residual for test sample #19; the 4 to its right identifies the residual for test sample #25 along the eastern edge of the field. The map of the residuals was derived by spatially interpolating the set of residuals. The three-dimensional rendering shows the residual surface, while the 2-D map shows 1-unit contours of error from -9 (under guessed by nine) to 4 (over guessed by four). The thick, dark lines in both plots locates the zero residual contour— locations believed to be right on target. The darker tones identify areas of overestimates, while the lighter tones identify areas of underestimates.

As explained earlier, the sum of the residuals (-28) indicates a overall tendency of the Kriging technique to underestimate the data used in this example. The predominance of lighter tones in the residual map spatially supports this overall conclusion. While the underestimated area occurs throughout the center of the field, it appears that Kriging's overestimates occur along the right (east), bottom (south) and the extreme upper left (northwest) and upper right (northeast) corners.

The upper right portion (northeast) of the error map is particularly interesting. The two -9 "holes" for test samples #20 and #24 quickly rise to the 1 and 4 "peaks" for samples #19 and #25. Such wide disparities within close proximity of each other (steep slopes in 3-D and tight contour lines in 2-D) indicate areas of wildly differing interpolation performance. It's possible that something interesting is going on in the northeast and we ought to explore a bit further. Take a look at the data in the table. The four points represent areas with the highest responses (65, 56, 70 and 78). Come to think of it, a residual of 1 on an estimate of 64 for sample #19 really isn't off by much (only 1.5%). And its underestimated neighbor's of 9 on 65 (sample #20) isn't too bad either (14%).

Maybe a map of "normalized" residuals might display the distribution of error in a more useful form. What do you think? Are you willing to "go the distance" with the numbers? Or simply willing to "click" on an icon in your GIS, sit back, and blindly enjoy the colorful renderings of automated mapping. At very least, the discussions on sampling and assessing interpolation results should have taken your thoughts "beyond pretty pictures" toward "pretty useful mapped data"… remember, all GIS maps are organized sets of numbers and those numbers might be trying to tell you something.

Comparing Map Errors

(GeoWorld, April 1997)

(return to top of Topic)

The previous section challenged you to consider what advantage a normalized map of residuals might have. Recall that an un-normalized map was generated from the boo-boos (more formally termed "residuals") uncovered by comparing interpolated estimates with a test set of known measurements. Numerical summaries of the residuals provided insight into the overall interpolation performance, whereas the map of residuals showed where the guesses were likely high and where they were likely low.

The map on the left side of figure 1 is the "plain vanilla" version of the error map discussed in the previous section. The one on the right is the normalized version. See any differences or similarities? At first glance, the sets of lines seem to form radically different patterns. A closer look reveals that the patterns of the darker tones are identical. So what gives?

First of all, let's consider how the residuals were normalized. The arithmetic mean of the test set (28) was used as the common reference. For example, test location #17 estimated 2 while its actual value was 0, resulting in an overestimate of 2 (2-0= 2). This simple residual is translated into a normalized value of 7.1 by computing (0-2)/28)*100= 7.1, a signed (+ or -) percentage of the "typical" test value. Similar calculations for the remaining residuals brings the entire test set in line with its typical, then a residual map is generated.

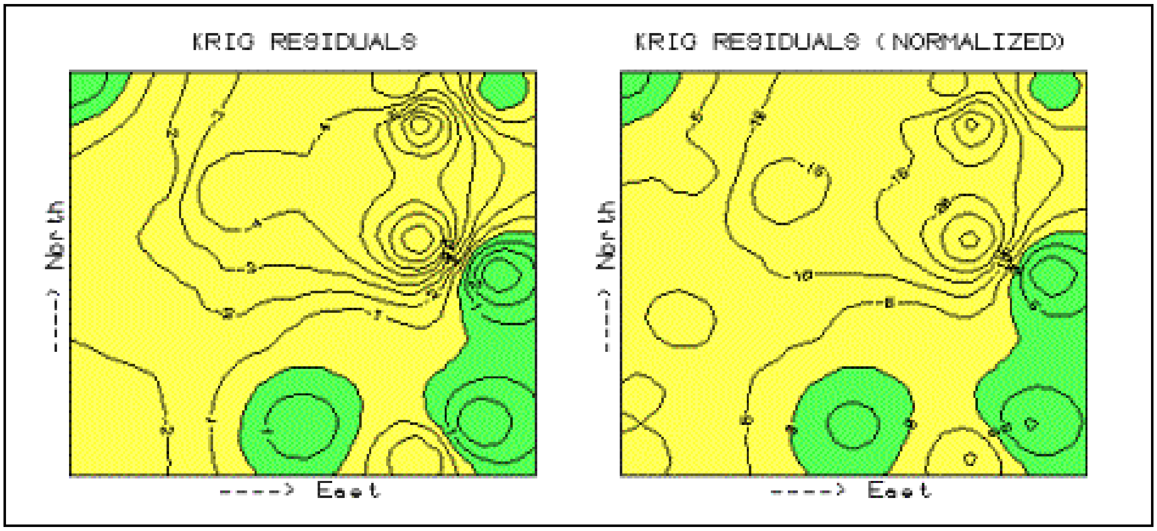


Figure 1. Comparison of error maps (2-D) using Absolute and Normalized Kriging residual values.

Now let's turn our attention back to the maps. As the techy-types among you guessed, the spatial pattern of interpolation error is not effected by normalization (nor is its numerical distribution)— all normalizing did was re-scale the map surface. The differences you detect in the line patterns are simply artifacts of different horizontal "slices" through the two related map surfaces. Whereas a 5% contour interval is used in the normalized version, a contour interval of 1 is used in the absolute version. The common "zero contour" (break between the two tones) in both maps have an identical pattern, as would be the case for any common slice (relative contour step).

Ok… but if normalizing doesn't change a map surface, why would anyone go to all the extra effort? Because normalizing provides the consistent referencing and scaling needed for comparison among different data sets. You can't just take a couple of maps, plop them on a light table, and start making comparative comments. Everyone knows you have to adjust the map scales so they will precisely overlay (spatial registration). In an analogous manner, you have to adjust their "thematic" scales as well. That's what normalization does.

Now visually compare magnitude and pattern of error between the Kriging and the Average surfaces in figure 2. A horizontal plane aligning at zero on the Z-axis would indicate a "perfect" residual surface (all estimates were exactly the same as their corresponding test set measurements). The Kriging plot on the left is relatively close to this ideal which confirms that the technique is pretty good at spatially predicting the sampled variable. The surface on the right identifying the "whole field" Average technique shows a much larger magnitude of error (surface deflection from Z= 0). Now note the patterns formed by the light and dark blobs on both map surfaces. The Kriging overestimates (dark areas) are less pervasive and scattered along the edges of the field. The Average overestimates occur as a single large blob in the southwestern half of the field.

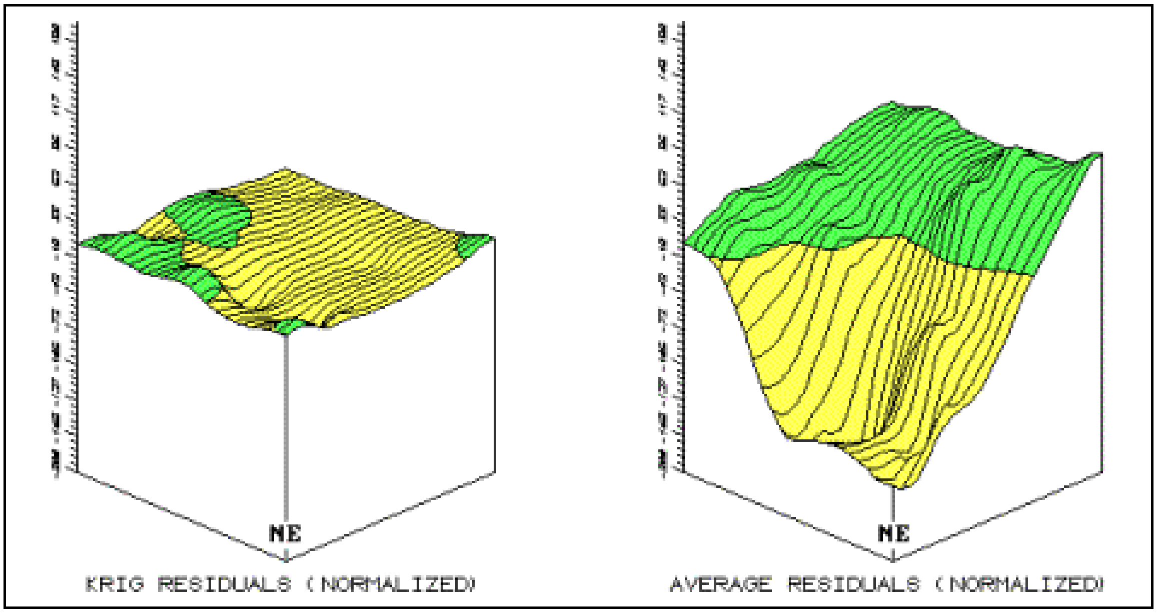


Figure 2. Comparison of Residual Map Surfaces (3-D) Using Residual Values Derived by Kriging, and Average Interpolation Techniques.

What do you think you would get if you were to calculate the volumes contained within the light and dark regions? Would their volumetric difference have anything to do with their Average Unsigned Residual values in the residual table discussed a couple of articles ago? What relationship does the Normalized Residual Index have with the residual surfaces? …bah! This map-ematical side of GIS really muddles its comfortable cartographic side— bring on the colorful maps at megahertz speed and damn the details.

(***Point Sampling Considerations***)

What's the Point?

(GeoWorld, December 1996)

(***return to top of Topic***)

GIS's roots lie in computer mapping and spatial database management; however, for better or worse, more map-ematical applications are rapidly emerging. One such extension involves an excursion into the surrealistic realm of point sampling. For example, a soil nutrient map of the relative amounts of phosphorous throughout a farmer's field can be derived from a set of soil cores (a.k.a., point samples). Whereas the bulk of digital maps arise by the direct encoding of point, line and areal features, point sampling uses a series of dispersed discrete samples to characterize a continuous distribution of a mapped variable. The nature of the maps derived by the two approaches is radically different— a set of "discrete objects" for encoded maps versus a "continuous statistical estimate" for a spatially interpolated map.

The reliability of an encoded map primarily depends on the accuracy of the source document and the fidelity of the digitizer— which in turn is a function of the caffeine level of the prefrontal-lobotomized hockey puck pusher (sic). It's at its highest when GPS is used for "feet-down" digitizing as you stroll throughout a field and trip over a physical object. The reliability of a map based on point sampling, however, depends on the existence of spatial dependency within the data, the sampling design employed, and the interpolation algorithm applied. Several previous columns discussed interpolation algorithms (January through March, 1994 BM columns), so now it's time to turn our attention to spatial dependency and sampling design.

Spatial dependency within a data set simply means that "what happens at one location depends on what is happening around it" (formally termed positive *spatial autocorrelation*). It's this idea that forms the basis of statistical tests for spatial dependency. The Geary Index calculates the squared difference between neighboring sample values, then compares their summary to the overall variance for the entire data set. If the neighboring variance is a lot less than the overall, then considerable dependency is indicated. The Moran Index is similar; however it uses the products of neighboring values instead of the differences. A *variogram* plots the similarity among locations as a function of distance.

Although these calculations vary and arguments abound about the best approach, all of them are reporting the degree of similarity among point samples. I f there is a lot, then you can generate maps from the data; if there isn't much, then you are more than wasting your time. A pretty map can be generated regardless of the degree of dependency, but if dependency is minimal the map is just colorful gibberish… so don't bet the farm on it.

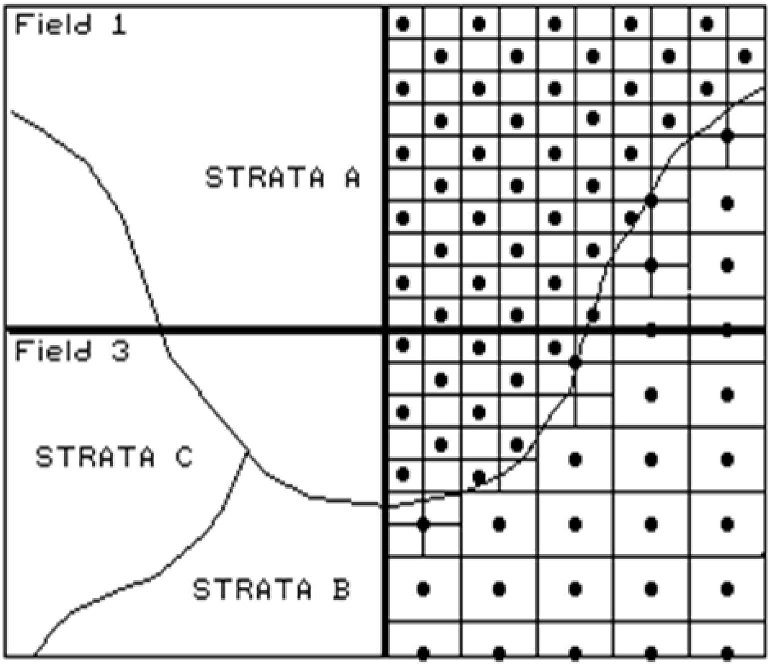


Figure 1. Variable sampling frequencies by soil strata for two fields.

OK, let's say the data set you intend to map exhibits ample spatial autocorrelation. Your next concern is establishing a sampling frequency and pattern that will capture the variable's spatial distribution— sampling design issues. There are four distinct considerations in sampling design: 1) *stratification*, 2) *sample size*, 3) *sampling grid*, and 4) *sampling pattern*.

The first three considerations determine the appropriate groupings for sampling (stratification), the sampling intensity for each group (sample size), and a suitable reference grid (sampling grid) for expressing the sampling intensity for each group (see figure 1). All three are closely tied to the spatial variation of the data to be mapped. Let's consider mapping phosphorous levels within a farmer's field. If the field contains a couple of soil types, you might divide it into two "strata." If previous sampling has shown one soil strata to be fairly consistent (small variance), you might allocate fewer samples than another more variable soil unit, as depicted in the accompanying figure.

Also, you might decide to generate a third stratum for even more intensive sampling around the soil boundary itself. Or, another approach might utilize mapped data on crop yield. If you believe the variation in yield is primarily "driven" by soil nutrient levels, then the yield map would be a good surrogate for subdividing the field into strata of high and low yield variability. This approach might respond to localized soil conditions that are not reflected in the traditional (encoded) soil map.

Historically, a single soil sampling frequency has been used throughout a region, without regard for varying local conditions. In part, the traditional single frequency was chosen for ease and consistency of field implementation and simply reflects a uniform spacing intensity based on how much farmers are willing to pay for soil sampling.

Within GIS/GPS technology, variable sample frequency is a modern alternative. Whether you are sampling soil nutrients, water pollution or people's opinions with the intent of making a map, the sampling frequency should consider the spatial variation in the data, not just the automation of a traditional non-spatial sampling intensity. In addition, the sampling pattern should be spatially based… but that's another story.

Designer Samples

(GeoWorld, January 1997)

(***return to top of Topic***)

The previous section briefly discussed spatial dependency and the first three steps in point sampling design— stratification, sample size and sampling grid. These considerations determine the appropriate areas, or groupings, for sampling (stratification), the sampling intensity for each group (sample size) and a suitable reference grid (sampling grid) for locating the samples. The fourth and final step "puts the sample points on the ground" by choosing a *sampling pattern* to identify individual sample locations.

Traditional, non-spatial statistics tends to emphasize randomized patterns as they insure maximum independence among samples… a critical element in calculating the central tendency of a data set (average for an entire field). However, "the random thing" can actually hinder spatial statistics' ability to map field variability. Arguments supporting such statistical heresy involve a detailed discussion of spatial dependency and autocorrelation, which (mercifully) is postponed to another issue. For current discussion, let's assume sampling patterns other than random are viable candidates.

Figure 1 identifies five systematic patterns, as well as a completely random one. Note that the *regular pattern* exhibits a uniform distribution in geographic space. The *staggered start* does so as well, except the equally spaced Y-axis samples alternate the starting position at one half the sampling grid spacing. The result is a "diamond" pattern rather than a "rectangular" one. The diamond pattern is generally considered better suited for generating maps as it provides more inter-sample distances for spatial interpolation. The *random start* pattern begins each column "transect" at a randomly chosen Y coordinate within the first grid cell, thereby creating even more inter-sample distances. The result is a fairly regularly spaced pattern, with "just a tasteful hint of randomness."

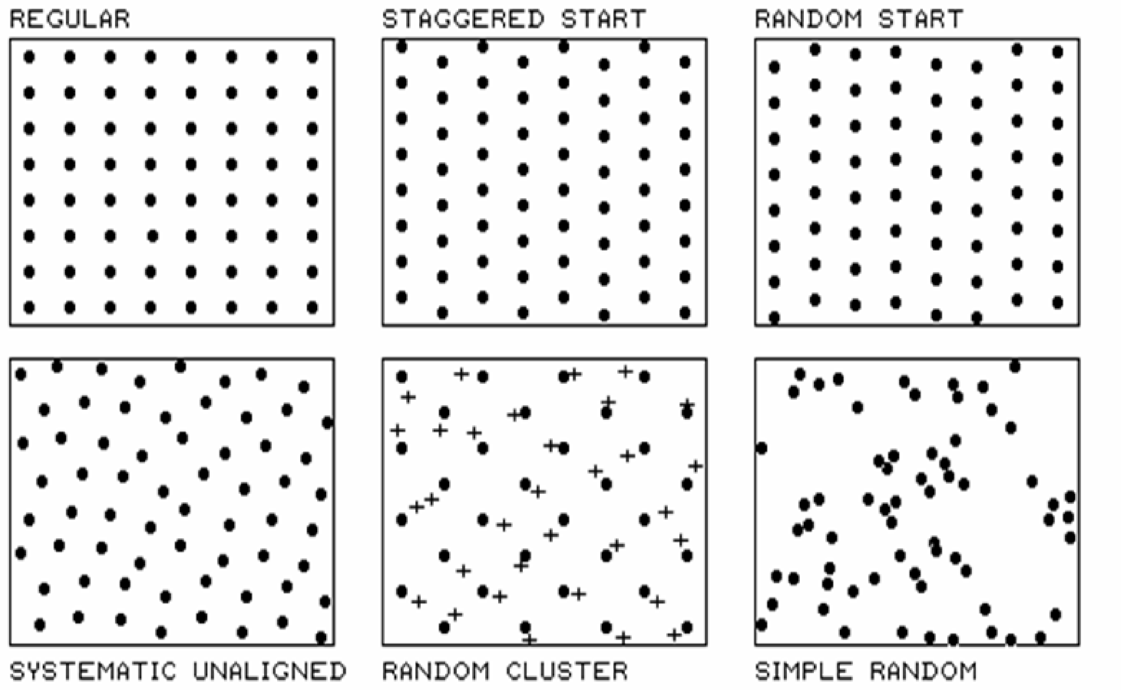


Figure 1. Basic spatial sampling patterns.

The *systematic unaligned* pattern also results in a somewhat regularly spaced pattern, but exhibits even more randomness as it is not aligned in either the X or Y direction. A study area (i.e., farmer's field) is divided into a sampling grid of cells equal to the sample size. The pattern is formed by first placing a random point in the cell in the lower-left corner of the grid to establish a pair of X and Y offsets. Random numbers are used to specify the distance separating the initial point from the left border (termed the X-offset) and from the bottom border (termed the Y-offset).

For the bottom row of the sampling grid, the X-offset is held constant while Y is randomly varied. For the left-most column, the Y-offset is held constant while X is varied. Sample points are then placed in the remaining grid cells insuring that the X-offsets are the same along each row and the Y-offsets are the same along each column. The result is a set of sample points that are roughly equally spaced, but out of alignment.

The “dots” in the random cluster pattern establish an underlying uniform pattern (every other staggered start sample point in this example). The “crosses” locate a set of related samples that are randomly chosen (both distance and direction) within the enlarged grid space surrounding each regularly placed each dot. Note that the pattern is not as regularly spaced as the previous techniques, as half of the points are randomly set, however, it has other advantages. The random subset of points provides a foothold for a degree of unbiased statistical inference, such as a t-test of significance differences among population means.

The *simple random* pattern uses random numbers to establish X and Y coordinates within the entire study area. It allows full use of statistical inference (whole field non-spatial statistics), but the “clumping” of the samples results in large “gaps” thereby limiting its application for mapping (site-specific spatial statistics).

So which pattern should be used? Generally speaking, the Regular and Simple Random patterns are the worst for spatial analysis. If you have trouble locating yourself in space (haven’t bought into GPS yet?), then Random Start might get the nod. If you can freely navigate in space, then Systematic Unaligned might be best. But if you want to perform statistical inference, then Random Cluster might be considered. Non-generally speaking, you should use whichever pattern works best with your data and your objectives… how to tell which is best is left for another time.

*(**Advanced Concepts in Spatial Dependency**)*

Depending on the Data

(GeoWorld, May 1997)

(return to top of Topic)

Historically, maps have reported the precise position of physical features for the purpose of navigation. Not long after emerging from the cave, early man grabbed a stick and drew in the sand a route connecting the current location to the best woolly mammoth hunting grounds, neighboring villages ripe for pillaging, the silk route to the orient and the flight plan for the first solo around the world. The basis for the navigational foundation of mapping lies in referencing systems and the expression of map features as organized sets of coordinates. The basis for modern GIS, however, lies in the concept of spatial dependency.

GIS technology embraces the traditional aspects of mapping. In fact it greatly extends the access and ease of use in generating custom views of the mosaic of elements comprising our real world. But its revolutionary impacts involve how we perceive, conceptualize and communicate thoughts on the linkages and relative importance of spatially explicit relationships. The emphasis has moved from the mapping question of "*where is what*" to the social, economic, environmental (etc.) questions of "*so what*" and "*what should we do where?*"

The technical focus has been enlarged to include a growing set of procedures for discovering and expressing the dependencies within and among mapped data. Spatial dependency identifies relationships based on relative positioning. Certain trees tend to occur on certain soil types, slopes and climatic zones. Animals tend to prefer specific biological and contextual conditions. Particularly good sales prospects for luxury cars tend to cluster in a few distinct parts of a city. In fact, is anything randomly placed in geographic space? A rock, a bird, a person, a molecule?

There are two broad types of spatial dependency: 1) spatial variable dependence and 2) spatial relations dependence. *Spatial variable dependency* stipulates that what occurs at a map location is related to:

1) the <u>conditions of that variable at nearby locations</u> (termed spatial autocorrelation); and/or
2) the <u>conditions of other variables at or around that location</u> (termed spatial correlation).

Spatial autocorrelation forms the backbone of all interpolation techniques. They relate neighboring sample points to predict a variable response at unmeasured locations. If the neighboring points are spatially independent, there is little justification for generating a map surface. For example, if the elevation is 100 feet here and you note that it is 50 over there, there has to be at least one location of 75 feet in between. That's because elevation forms a highly auto-correlated gradient in geographic space. True, it might be that the 75 foot location is part way down the precipice a foot in front of you, but it has to exist. The continuum is there, it's just that the functional form isn't a simple linear transition between the points.

Contrast an elevation surface with a map of roads. If you're standing on a heavy duty, road-type 4 (watch out for buses) and note a light duty road-type 1 over there, it is absurd to assume that there is a road-type 2.5 somewhere in between. Two basic spatial autocorrelation factors are not at play— formation of a spatial gradient and existence of partial states. Neither of the assumptions makes sense for the occurrence of the discrete map objects forming a road map (termed a *choropleth map*).

However, both factors make sense for an elevation surface (termed an *isopleth map*). But spatial autocorrelation isn't black and white, present or not present; it occurs in varying degrees for different map types and spatial variables. The degree to which a map exhibits intra-variable dependency determines the nature and strength of the relationships one can derive about its geographic distribution.

In a similar manner, inter-variable dependency affects our ability to track spatial relationships. Spatial correlation forms the basis for mapping relationships among maps. For example, in the moisture limited ecosystems of Colorado, spruce and fir forests most often occur on northern slopes, while sparse pine forests tend to occur on the dry southern exposures. Soil conditions and depth play a big part in forest vitality, as well the frequency of catastrophic events, such as fire. Animal, bird, insect and micro-organism populations are strongly dependent on terrain and forest conditions. Boy and girl scouts as well as resource managers have known these general "rules" of spatial coincidence for years. Scientists have written about them for years. What has changed are the "tools" for deriving, verifying and applying more detailed and spatially explicit relationships.

Historically, scientists have used sets of discrete samples to investigate relationships among field plots on the landscape, in a manner similar to Petri dishes on a laboratory table— each sample is assumed to be spatially independent. GIS technology provides the geo-referencing necessary to align sets of map variables and the analytic tools to investigate correlation among their spatial patterns. For example, a map of animal activity can be statistically related to a set of habitat determinant maps, such as terrain form and vegetative cover, on a pure coincidence basis (point-by-point).

Introduction of neighboring conditions, such as proximity to water and cover type diversity, expands the simple alignment analysis to one of spatial context. The derived relationship can be empirically verified by generating a map predicting animal activity for another area and comparing it to known animal activity within that area. Once established, the verified spatial relationship can be used directly by managers in their operational GIS. In fact the cultural alignment of science and management activities brought about by GIS technology (i.e., same "tool" used by both scientists and managers) might prove to be as significant as the technical capability to align map layers for analysis. At minimum, it should expedite the linkages among basic science, applied research and technology transfer.

The other broad type of spatial dependency involves the nature of the relationship itself. *Spatial relations dependency* stipulates that relationships among mapped variables can be:

1) constant throughout space and time (termed *spatial homogeneity*); or
2) variable as a function of space and/or time (termed *spatial heterogeneity*).

Very few relationships exhibit pure spatial homogeneity by remaining constant over space and time. Even the "laws" of thermal dynamics have conditional boundaries— water freezes at zero degrees centigrade… as long as it is pure and at sea level. Pour in some salt and carry it to top of a mountain and it will freeze at a different kinetic temperature. Similarly, most spatial relationships exhibit spatial heterogeneity and vary to some degree with space and time.

For example, a habitat unit across a river might be considered disjoint and inaccessible to a non-swimming and flightless animal. However, if the river freezes in the winter, then the spatial relationships defining habitat needs to change with the seasons. Similarly, a forest growth model developed for Colorado might be inappropriate for application in Oregon. It's likely the basic map layers and logic structuring the model are identical, but the relative weights assigned to the growth functions vary with geographic regions and the model must be "tuned" for local variants.

The complexities of spatial dependence are not unique to resource models. GIS modeling of product sales, traffic flows, and voter patterns exhibit varying degrees of spatial variable and relations dependencies. Awareness of these concepts and their effects are as important to modern GIS as understanding data structures and geo-referencing procedures— possibly even more important?

Uncovering the Mysteries of Spatial Autocorrelation

(GeoWorld, July 1997)

(*return to top of Topic*)

The following discussion violates all norms of journalism, as well as common sense. It attempts to describe an admittedly complex technical subject without the prerequisite discussion of the theoretical linkages, provisional statements, and enigmatic equations. I apologize in advance to the statistical community for the important points left out of the discussion… and to the rest of you for not leaving out more.

The last section identified spatial autocorrelation as the backbone of all interpolation techniques used to generate maps from point sampled data. The term refers to the degree of similarity among neighboring points. If they exhibit a lot similarity, or spatial dependence, then they ought to derive a good map; if they are spatially independent, then expect pure, dense gibberish. So how do we measure whether "what happens at one location depends on what is happening around it?"

Previous discussion (December 1996 BM column) introduced two simple measures to determine whether a data set has what it takes to make a map— the Geary and Moran indices. The *Geary Index* looks at the differences in the values between each sample point and its closest neighbor. If the differences in neighboring values tend to be less than the differences among all values in the data set, then spatial autocorrelation exists. The mathematical mechanics are easy (at least for a tireless computer)— 1) add up all of the differences between each location's value and the average for the entire data set (overall variation), 2) add up all of the differences between values for each location and its closest neighbor (neighbors variation), and then 3) compare the two summaries using an appropriately ugly equation to account for "degrees of freedom and normalization."

If the differences among the neighbors are a lot smaller than the overall variation, then a high degree of positive spatial dependency is indicated. If they are about the same, or if the neighbors variation is larger (a rare "checkerboard-like" condition), then the assumption that "close things are more similar" fails… and, if the dependency test fails, so will the interpolation of the data. The *Moran Index* simply uses the products between the values, rather than the differences to test the dependency within a data set. Both approaches are limited, however, as they merely assess the closest neighbor, regardless of its distance.

That's where a *variogram* comes in. I t is a plot (neither devious nor spiteful) of the similarity among values based on the distance between them. Instead of simply testing whether close things are related, it shows how the degree of dependency relates to varying distances between locations. Most data exhibits a lot of similarity when distances are small, then progressively less similarity as the distances become larger.

In figure 1, you would expect more similarity among the neighboring points (shown by the lines), than sample points farther away. Geary and Moran consider just the closest neighbors (orthogonal distances of above, below, right and left for the regular grid sampling design). A variogram shows the dependencies for other distances, or spatial frequencies, contained in the data set (such as the diagonal distances).

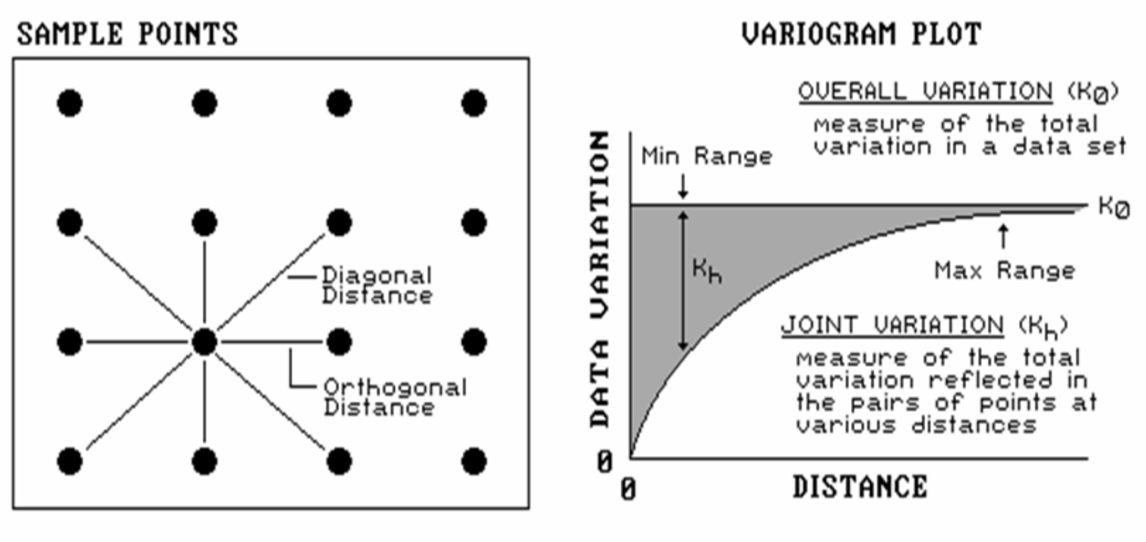


Figure 1. Plot of the similarity among sample points as a function of distance (shaded portion) shows whether interpolation of the data is warranted.

If you keep track of the multitude of distances connecting all locations and their respective differences, you end up with a huge table of data relating distance to similarity. In this case, the overall variation in a data set (termed the variance) is compared to the joint variation (termed the covariance) for each set of distances. For example, there is a lot of points that are “one orthogonal step away” (four for the example point). If we compute the difference between the values for all the “one-steppers,” we have a measure reminiscent of Moran’s “neighbors variation”— differences among pairs of values.

A bit more “mathematical conditioning” translates this measure into the covariance for that distance. If we focus our attention on all of the points “a diagonal step away” (four around the example point), we will compute a second similarity measure for points a little farther away. Repeating the joint variation calculations for all of the other spatial frequencies (two orthogonal steps, two diagonal steps, etc.), results in enough information to plot the variogram shown in the figure.

Note the extremes in the plot. The top horizontal line indicates the total variation within the data set (overall variation; variance). The origin (0,0) is the unique case for distance= 0 where the overall variation in the data set is identical to the joint variation as both calculations use essentially the same points. As the distance between points is increased, subsets of the data are scrutinized for their dependency (joint variation; covariance). The shaded portion in the plot shows how quickly the spatial dependency among points deteriorates with distance.

The maximum range position identifies the distance between points beyond which the data values are considered to be independent of one another. This tells us that using data values beyond this distance for interpolation is dysfunctional (actually messes-up the interpolation). The minimum range position identifies the smallest distance (one orthogonal step) contained in the data set. If most of the shaded area falls below this distance, it tells you there is insufficient spatial dependency in the data set to warrant interpolation.

True, if you proceed with the interpolation a nifty colorful map will be generated, but it'll be less than worthless. Also true, if we proceed with more technical detail (like determining optimal sampling frequency and assessing directional bias in spatial dependency), most this column's readership will disappear (any of you still out there?).

Unlocking the Keystone Concept of Spatial Dependency

(GeoWorld, November 1998)

(***return to top of Topic***)

Previous discussion (October 1998 BM column) investigated the numerical character of a gridded elevation surface. Keystone to the discussion was the degree of "normality" exhibited in the data as measured by commonly used ***descriptive statistics***— *min, max, range, median, mode, mean (or average), variance, standard deviation, standard error, confidence level, skewness and kurtosis*. All in all, it appeared that the elevation data didn't fit the old "bell-shaped" curve very well.

So, how useful is "normal" statistics in GIS applications? That's an interesting debate set off by a letter to the editor (GIS World, Vol. 11, No. 9), with several individuals contributing their thoughts to the "Column Supplements" page at *www.innovativegis.com/basis*. I f you're a techy-type and wired to the internet you might want to check out the extended discussion.

At the risk of overstepping my bounds of expertise, let me suggest that using the average to represent the central tendency of a data set is usually OK. However, when the data isn't normally distributed the average might not be a good estimator of the "typical" condition. Similarly, the standard deviation can be an ineffective measure of dispersion for *ab*-normally distributed data… *it all depends*.

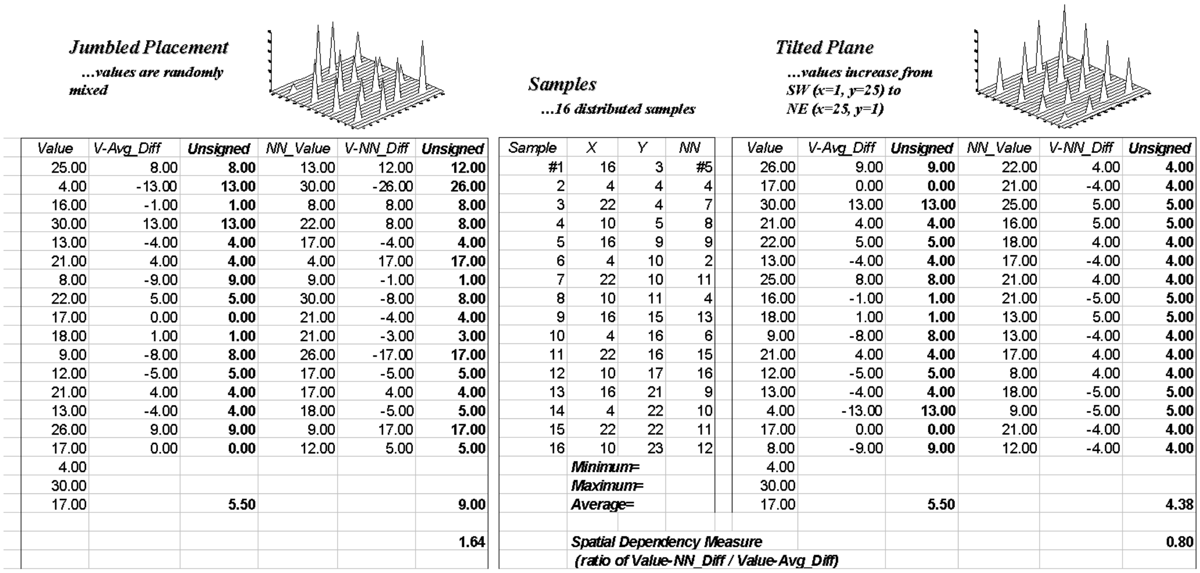
So, what's a GIS'er to do, short of getting an advanced degree in statistics or abducting a statistician? The correct response is to enter the murky realm of non-parametric statistics. The easy response is to forget all of the past columns with statistical leanings and blissfully apply the average and standard deviation to all of the data falling within a polygon.

Yet another response is to use the "poor man's" answer to asymmetric data— use the median to represent the "typical" condition and the quartile range to estimate the data's dispersion (the quartile range corresponds to the middle 50% of the frequency distribution). Suggesting such a "seat-of-the-pants" statistical procedure should provide the last whack to my extended neck and set-off another round of discussions in the Column Supplements.

Even more disturbing, however, is the realization that while descriptive statistics might provide insight into the numerical distribution of the data, they provide no information what-so-ever into the spatial distribution of the data. As noted last month, all sorts of terrain configurations can produce exactly the same set of descriptive statistics. That's because traditional measures are breed to ignore geographic patterns— in fact spatial independence is an underlying assumption.

So how can one tell if there is spatial dependency locked inside a data set? You know, Tobler's first law of geography that "all things are related but nearby things are more related than distant things." Let's use Excel and some common sense to investigate this keystone concept and the approach used in deriving a descriptive statistics that tracks spatial dependency (see Author's Note).

The left side of the figure 1 identifies sixteen sample points in a 25 column by 25 row analysis grid (origin at 1, 1 in the upper left, northwest corner). The positioning of the samples are depicted in the two 3-D plots. Note that the sample positions are the same (horizontal axes), only the measurements at each location vary (vertical axis). The plot on the left depicts sample values that form a plane constantly increasing from the southwest to the northeast. The plot on the right depicts a jumbled arrangement of the same measurement values.



Jumbled Placement
...values are randomly mixed

Samples
...16 distributed samples

Tilted Plane
...values increase from SW (x=1, y=25) to NE (x=25, y=1)

Value	V-Avg_Diff	**Unsigned**	NN_Value	V-NN_Diff	**Unsigned**	Sample	X	Y	NN	Value	V-Avg_Diff	**Unsigned**	NN_Value	V-NN_Diff	**Unsigned**
25.00	8.00	**8.00**	13.00	12.00	**12.00**	#1	16	3	#5	26.00	9.00	**9.00**	22.00	4.00	**4.00**
4.00	-13.00	**13.00**	30.00	-26.00	**26.00**	2	4	4	4	17.00	0.00	**0.00**	21.00	-4.00	**4.00**
16.00	-1.00	**1.00**	8.00	8.00	**8.00**	3	22	4	7	30.00	13.00	**13.00**	25.00	5.00	**5.00**
30.00	13.00	**13.00**	22.00	8.00	**8.00**	4	10	5	8	21.00	4.00	**4.00**	16.00	5.00	**5.00**
13.00	-4.00	**4.00**	17.00	-4.00	**4.00**	5	16	9	9	22.00	5.00	**5.00**	18.00	4.00	**4.00**
21.00	4.00	**4.00**	4.00	17.00	**17.00**	6	4	10	2	13.00	-4.00	**4.00**	17.00	-4.00	**4.00**
8.00	-9.00	**9.00**	9.00	-1.00	**1.00**	7	22	10	11	25.00	8.00	**8.00**	21.00	4.00	**4.00**
22.00	5.00	**5.00**	30.00	-8.00	**8.00**	8	10	11	4	16.00	-1.00	**1.00**	21.00	-5.00	**5.00**
17.00	0.00	**0.00**	21.00	-4.00	**4.00**	9	16	15	13	18.00	1.00	**1.00**	13.00	5.00	**5.00**
18.00	1.00	**1.00**	21.00	-3.00	**3.00**	10	4	16	6	9.00	-8.00	**8.00**	13.00	-4.00	**4.00**
9.00	-8.00	**8.00**	26.00	-17.00	**17.00**	11	22	16	15	21.00	4.00	**4.00**	17.00	4.00	**4.00**
12.00	-5.00	**5.00**	17.00	-5.00	**5.00**	12	10	17	16	12.00	-5.00	**5.00**	8.00	4.00	**4.00**
21.00	4.00	**4.00**	17.00	4.00	**4.00**	13	16	21	9	13.00	-4.00	**4.00**	18.00	-5.00	**5.00**
13.00	-4.00	**4.00**	18.00	-5.00	**5.00**	14	4	22	10	4.00	-13.00	**13.00**	9.00	-5.00	**5.00**
26.00	9.00	**9.00**	9.00	17.00	**17.00**	15	22	22	11	17.00	0.00	**0.00**	21.00	-4.00	**4.00**
17.00	0.00	**0.00**	12.00	5.00	**5.00**	16	10	23	12	8.00	-9.00	**9.00**	12.00	-4.00	**4.00**
4.00							***Minimum=***			4.00					
30.00							***Maximum=***			30.00					
17.00		**5.50**			**9.00**		***Average=***			17.00		**5.50**			**4.38**
					1.64		***Spatial Dependency Measure***								**0.80**
							(ratio of Value-NN_Diff / Value-Avg_Diff)								

Figure 1. The spatial dependency in a data set compares the "typical" and "nearest neighbor" differences— if the nearest neighbor differences are less than the typical differences, then "nearby things are more similar than distant things."

The first column (labeled *Value*) in the Tilted and Jumbled worksheets confirm that the traditional descriptive statistics are identical— derived from same values, just in different positions. The second column calculates the difference between each value and the average of the entire set of samples. The sign of the difference indicates whether the value is above or below the average, or typical value.

The third column (labeled *Unsigned*) identifies the magnitude of the difference by taking its absolute value— **|*Value – Average*|**. The average of all the unsigned differences summarizes the "typical" difference. The relatively large figure of 5.50 for both the Tilted and Jumbled data sets establishes that the individual samples aren't very similar overall.

The next three columns in both worksheets provide insight into the spatial dependency in the two data sets by evaluating Tobler's first law. The *NN_Value* column identifies the value for the nearest neighboring (closest) sample. It is determined by solving for the distance from each sample location to all of the others using the Pythagorean theorem ($c^2= a^2 + b^2$), then assigning the measurement value of the closest sample.

The final two columns calculate the unsigned difference between the value at a location and its nearest neighboring value, then compute the unsigned difference— **|V*alue – NN_Value*|**. Note that the Tilted data's nearest neighbor difference (4.38) is considerably less than that for the Jumbled data (9.00).

Now the stage is set. If the nearest neighbor differences are less than the typical differences, then "…nearby things are more related than distant things." A simple ***Spatial Dependency Measure*** is calculated as the ratio of the two differences. If the measure is 1.0, then minimal spatial dependency exists. As the measure gets smaller, increased positive spatial dependency is indicated; as it gets larger, increased negative spatial dependency is indicated (nearby things are less similar than distant things).

OK, so what if the basic set of descriptive statistics can be extended to include a measure of spatial dependency? What does it tell you? How can you use it? Its basic interpretation is to what degree the data forms a discernible spatial pattern. If spatial dependency is minimal or negative there is little chance that geographic space can be used to explain the variation in the data. In these conditions, assigning the average (or median) to an entire polygon is warranted. On the other hand, if strong positive spatial dependency is indicated, you might consider subdividing the polygon into more homogenous parcels to better "map the variation" locked in a data set. Or better yet, treat the area as a continuous surface (gridded data). But further discussion of refinements in calculating and interpreting spatial dependency must be postponed until next time.

____

Author's Note: *the Excel worksheets supporting the discussions of the Tilted and Jumbled data sets (as well as a Blocked and a Random pattern) can be downloaded from the "Column Supplements" page at* www.innovativegis.com/basis.

Measuring Spatial Dependency

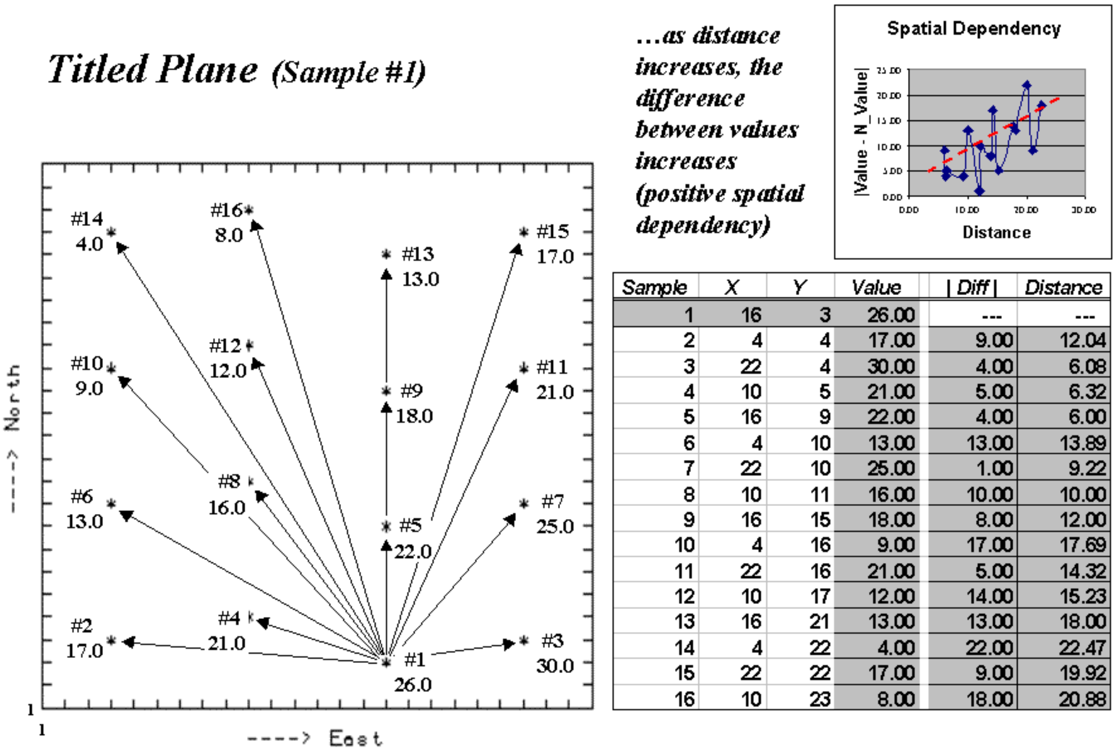
(GeoWorld, December 1998)

(***return to top of Topic***)

Recall the previous section's discussion of "nearest neighbor" spatial dependency to test the assertion that "nearby things are more related than distant things." The procedure was simple—

calculate the difference between each sample value and its closest neighbor (|Value - NN_Value|), and then compare them to the differences based on the typical condition (|Value - Average|). I f the Nearest Neighbor and Average differences are about the same, little spatial dependency exists. If the nearby differences are substantially smaller than the typical differences, then strong positive spatial dependency is indicated and it is safe to assume that nearby things are more related.

But just how are they related? And just how far is "nearby?" To answer these questions the procedure needs to be expanded to include the differences at the various distances separating the samples. As with the previous discussions, Excel can be used to investigate these relationships (see Author's Note). The plot on the left side of figure 1 identifies the positioning and sample values for the Tilted Plane data set described last month.



...as distance increases, the difference between values increases (positive spatial dependency)

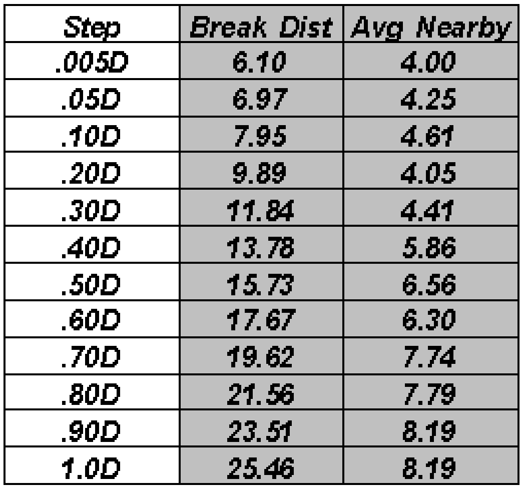
Sample	X	Y	Value	| Diff |	Distance
1	16	3	26.00	---	---
2	4	4	17.00	9.00	12.04
3	22	4	30.00	4.00	6.08
4	10	5	21.00	5.00	6.32
5	16	9	22.00	4.00	6.00
6	4	10	13.00	13.00	13.89
7	22	10	25.00	1.00	9.22
8	10	11	16.00	10.00	10.00
9	16	15	18.00	8.00	12.00
10	4	16	9.00	17.00	17.69
11	22	16	21.00	5.00	14.32
12	10	17	12.00	14.00	15.23
13	16	21	13.00	13.00	18.00
14	4	22	4.00	22.00	22.47
15	22	22	17.00	9.00	19.92
16	10	23	8.00	18.00	20.88

Figure 1. Spatial dependency as a function of distance for sample point #1.

The arrows emanating from sample #1 shows its 15 paired values. The table on the right summarizes the unsigned differences (|**Diff** |) and distances (**Distance**) for each pair. Note that the "nearby" differences (e.g., #3= 4.0, #4= 5.0 and #5= 4.0) tend to be much smaller than the "distant" differences (e.g., #10= 17.0, #14= 22.0, and #16= 18.0). The graph in the upper right portion of the figure plots the relationship of sample differences versus increasing distances. The dotted line shows a trend of increasing differences (a.k.a. dissimilarity) with increasing distances.

Now imagine calculating the differences for all the sample pairs in the data set—the 16 sample points combine for 120 sample pairs—(N*(N-1)/2)= (16*15)/2= 120). Admittedly, these calculations bring humans to their knees, but it's just a microsecond or so for a computer. The result is a table containing the |Diff | and Distance values for all of the sample pairs.

The extended table embodies a lot of information for assessing spatial dependency. The first step is to divide the samples into two groups— close and distant pairs. For consistency across data sets, let's define the "breakpoint" as a proportion of the maximum distance (**D_{max}**) between sample pairs. Figure 2 shows the results of applying a dozen breakpoints to divide the data set into "nearby" and "distant" sample sets. The first row in the table identifies very close neighbors (.005D_{max}= 6.10) and calculates the average nearby differences (|**Avg_Nearby**|) as 4.00. The remaining rows in the table track the differences for increasing distances defining nearby samples. Note that as neighborhood size increases, the average difference between sample values increases. For this data set, the greatest difference occurs for the neighborhood that captures all of the data (1.00D_{max}= 25.46 with an average difference of 8.19).



Step	Break Dist	Avg Nearby
.005D	6.10	4.00
.05D	6.97	4.25
.10D	7.95	4.61
.20D	9.89	4.05
.30D	11.84	4.41
.40D	13.78	5.86
.50D	15.73	6.56
.60D	17.67	6.30
.70D	19.62	7.74
.80D	21.56	7.79
.90D	23.51	8.19
1.0D	25.46	8.19

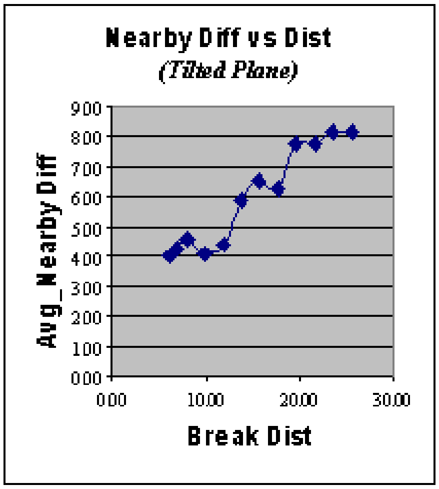


Figure 2. Average "nearby" differences for increasing breakpoint distances used to define neighboring samples.

The techy-types among us will note that the plot of "Nearby Diff versus Dist" in figure 2 is similar to that of a *variogram*. Both assess the difference among sample values as a function of distance. However, the variogram tracks the difference at discrete distances, while the "Nearby Diff versus Dist" plot considers all of the samples within increasingly larger neighborhoods.

This difference in approach allows us to directly assess the essence of spatial dependency— whether "nearby things are more related than distant things." A **"**distance-based spatial dependency measure (SD_D)**"** can be calculated as— SD_D = [|Avg_Distant| - |Avg_Nearby|] / |Avg_Distant|.

The effect of this processing is like passing a donut over the data. When centered on a sample location, the "hole" identifies nearby samples, while the "dough" determines distant ones. The "hole" gets progressively larger with increasing breakpoint distances. If, at a particular step, the nearby samples are more related (smaller |Avg_Nearby| differences) than the distant set of samples (larger |Avg_Distant| differences), positive spatial dependency is indicated.

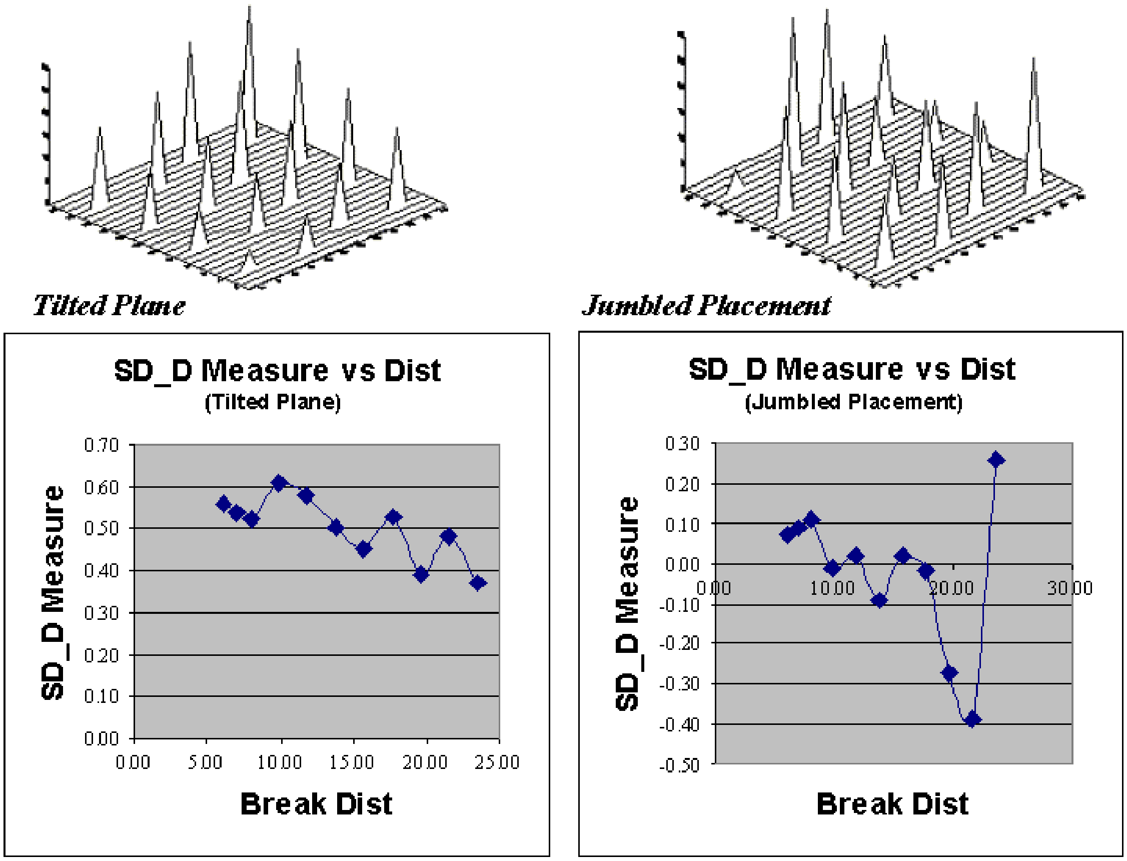


Figure 3. Comparing spatial dependency by directly assessing differences of a sample's value to those within nearby and distant sets.

Now let's put the SD_D measure to use. Figure 3 plots the measure for the Tilted Plane (TP with constantly increasing values) and Jumbled Placement (JP with a jumbled arrangement of the same values) sample sets discussed the previous section. First notice that the measures for TP are positive for all breakpoint distances (nearby things are always more related), whereas they bounce around zero for the JP pattern. Next, notice the magnitudes of the measures— fairly large for TP (big differences between nearby and distant similarities); fairly small for JP. Finally, notice the trend in the plots—downward for TP (declining advantage for nearby neighbors); flat, or unpredictable for JP.

So what does all this tell us? If the sign, magnitude and trend of the SD_D measures are like TP's, then positive spatial dependency is indicated and the data conforms to the underlying assumption of most spatial interpolation techniques. If the data is more like JP, then "beware of flakey interpolation."

Author's Note: *the Excel worksheets supporting the discussion of the Tilted and Jumbled data sets (as well as a Blocked and Random pattern) can be downloaded from the "Column Supplements" page at www.innovativegis.com/basis.*

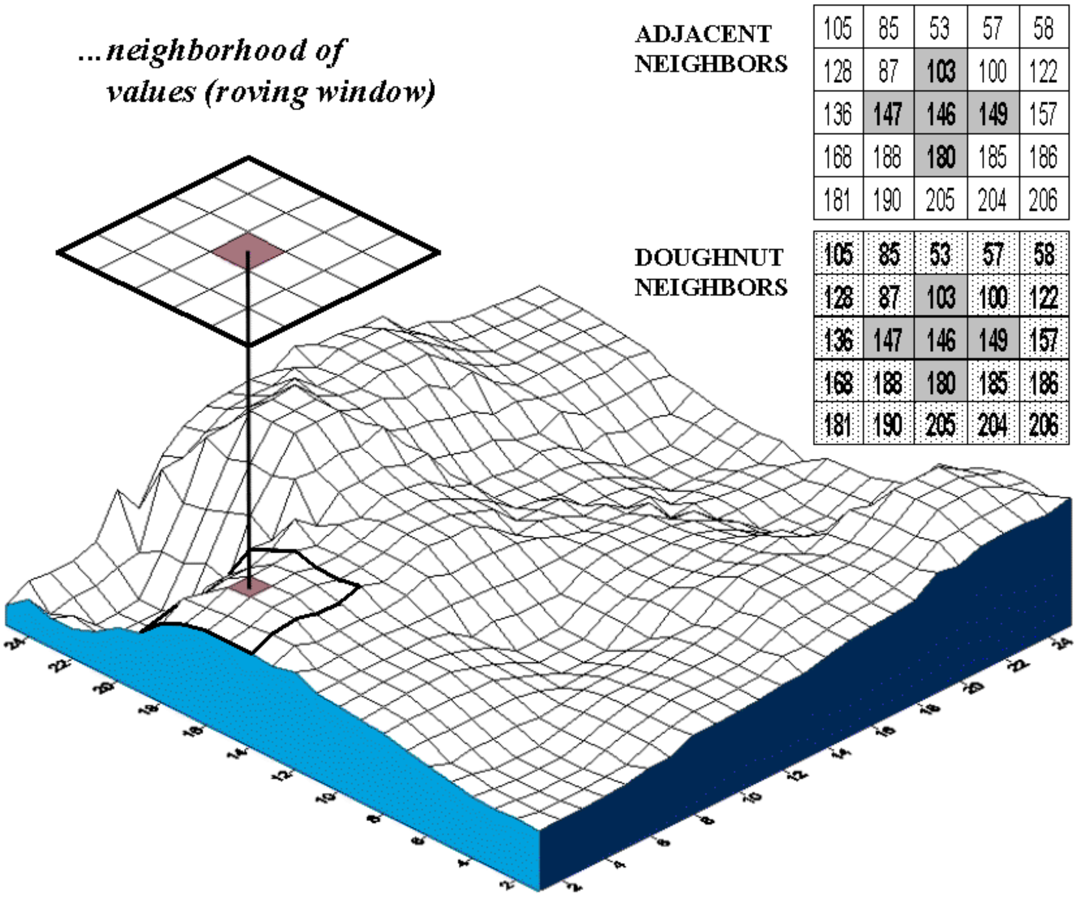
Extending Spatial Dependency to Maps

(GeoWorld, January 1999)

(return to top of Topic)

The past four sections have focused on the important geographical concept of spatial dependency— that nearby things are more related than distant things. The discussion to date has involved sets of discrete sample points taken from a variety of geographic distributions. Several techniques were described to generate indices tracking the degree of spatial dependency in point sampled data.

Now let's turn our attention to continuously mapped data, such as satellite imagery, soil electric conductivity, crop yield or product sales surfaces. In these instances, a grid data structure is used and a value is assigned to each cell based on the condition or character at that location. The result is a set of data that continuously describes a **mapped variable**. These data are radically different from point sampled data as they fully capture the spatial relationships throughout an entire area.



ADJACENT NEIGHBORS

105	85	53	57	58
128	87	103	100	122
136	147	146	149	157
168	188	180	185	186
181	190	205	204	206

DOUGHNUT NEIGHBORS

105	85	53	57	58
128	87	103	100	122
136	147	146	149	157
168	188	180	185	186
181	190	205	204	206

Figure 1. Spatial dependency in continuously mapped data involves summarizing the data values within a "roving window" that is moved throughout a map.

The analysis techniques for spatial dependency in these data involve moving a "roving window" throughout the data grid. As depicted in figure 1, an instantaneous moment in the processing establishes a set of neighboring cells about a map location. The map values for the center cell and its neighbors are retrieved from storage and depending on the technique, the values are summarized. The window is shifted so it centers over the next cell and the process is repeated until all map locations have been evaluated. Various methods are used to deal with incomplete windows occurring along map edges and areas of missing data.

The configuration of the window and the summary technique is what differentiates the various spatial dependency measures. All of them, however, involve assessing differences between map values and their relative geographic positions. In the context of the data grid, if two cells are close together and have similar values they are considered spatially related; if their values are different, they are considered unrelated, or even negatively related.

Geary's C and **Moran's I** introduced in the 1950's are the most frequently used measures for determining spatial autocorrelation in mapped data. Although the equations are a bit intimidating—

Geary's C = $[(n - 1) \text{ SUM } w_{ij} (x_i - x_j)^2] / [(2 \text{ SUM } w_{ij}) \text{ SUM } (x_i - m)^2]$

Moran's I = $[n \text{ SUM } w_{ij} (x_i - m) (x_j - m)] / [(\text{SUM } w_{ij}) \text{ SUM } (x_i - m)^2]$

where, n = number of cells in the grid

m = the mean of the values in the grid

x_i = value of cell in group i and x_j = value of cell in group j

w_{ij} = a switch set to 1 if the cells are adjacent; 0 if not adjacent (diagonal)

—however, the underlying concept is fairly simple.

For example, Geary's C simply compares the squared differences in values between the center cell and its adjacent neighbors (numerator tracking "$x_i - x_j$") to the overall difference based on the mean of all the values (denominator tracking "$x_i - m$"). If the adjacent differences are less, then things are positively related (similar, clustered). If they are more, then things are negatively related (dissimilar, checkerboard). And if the adjacent differences are about the same, then things are unrelated (independent, random). Moran's I is a similar measure, but relates the product of the adjacent differences to the overall difference.

Now let's do some numbers. An *adjacent neighborhood* consists of the four contiguous cells about a center cell, as highlighted in the upper right inset of figure 1. Given that the mean for all of the values across the map is 170, the essence for this piece of Geary's puzzle is

$$C = [(146-147)^2 + (146-103)^2 + (146-149)^2 + (146-180)^2] / [4 * (146-170)^2]$$
$$= [1 + 1849 + 9 + 1156] / [4 * 576] = 3015 / 2304 = 1.309$$

Since the Geary's C ratio is just a bit more than 1.0, a slightly uncorrelated spatial dependency is indicated for this location. As the window completes its pass over all of the other cells, it keeps a running sum of the numerator and denominator terms at each location. The final step applies some aggregation adjustments (the "eye of newt" parts of the nasty equation) to calculate a single measure encapsulating spatial autocorrelation over the whole map— a Geary's C of 0.060

and a Moran’s I of 0.943 for the map surface shown in figure 1. Both measures report strong positive autocorrelation for the mapped data. The general interpretation of the C and I statistics can be summarized as follows.

0 < C < 1	Strong positive autocorrelation	I > 0
C > 1	Strong Negative autocorrelation	I < 0
C = 1	Random distribution of values	I = 0

In the tradition of good science, let me suggest a new, related measure— **Berry’s ID**. This *Intuitive Dependency (ID)* measure simply assigns the calculated ratio from Geary’s formula to each map location. The result is a map indicating the spatial dependency for each location (pieces of the puzzle), instead of a single value summarizing the entire map. In the example, 1.309 is assigned to the center location in the figure. However a value of 0.351 is assigned to the cell directly above it and 4.675 is assigned to the cell directly below it …you do the math.

Although this new measure might be intuitive—adjacent differences (nearby things) versus overall difference (distant things)—it’s much too ugly for statistical canonization. First, the values are too volatile and aren’t constrained to an easily interpreted range. More importantly, the measure doesn’t directly address “localized spatial autocorrelation” because the nearby differences are compared to distant differences represented as the map mean.

That’s where the *doughnut neighborhood* comes in. The roving window is divided into two sets of data—the adjacent values (inside ring of nearby things) and the doughnut values (outside ring of distant things). One could calculate the mean for the doughnut values and substitute it for Geary’s C’s denominator. But since there’s just a few numbers in the outer ring, why not use the actual variation between the center and each doughnut value? That directly assesses whether nearby things are more related than distant things for each map neighborhood. A user can redefine “distant things” simply by changing the size of the window. In fact, if you recall last month’s article, a series of window sizes could be evaluated and differences between the maps at various “doughnut radii” could provide information about the geographic sensitivity of spatial dependency throughout the mapped area (sort of a mapped variogram).

But let’s take the approach one step further for a new measure we might call **Berry IT** (yep, you got it… "bury it" for tracking the *Intimidating Territorial* autocorrelation). Such a measure is reserved for the statistically adept as it performs an F-test for significant difference between the adjacent and doughnut data groups for each neighborhood.

Author’s Note: *check out this month’s Column Supplement at www.innovativegis.com for more info and an Excel worksheet applying several concepts for mapping spatial dependency.*

Use Polar Variograms to Assess Distance and Direction Dependencies

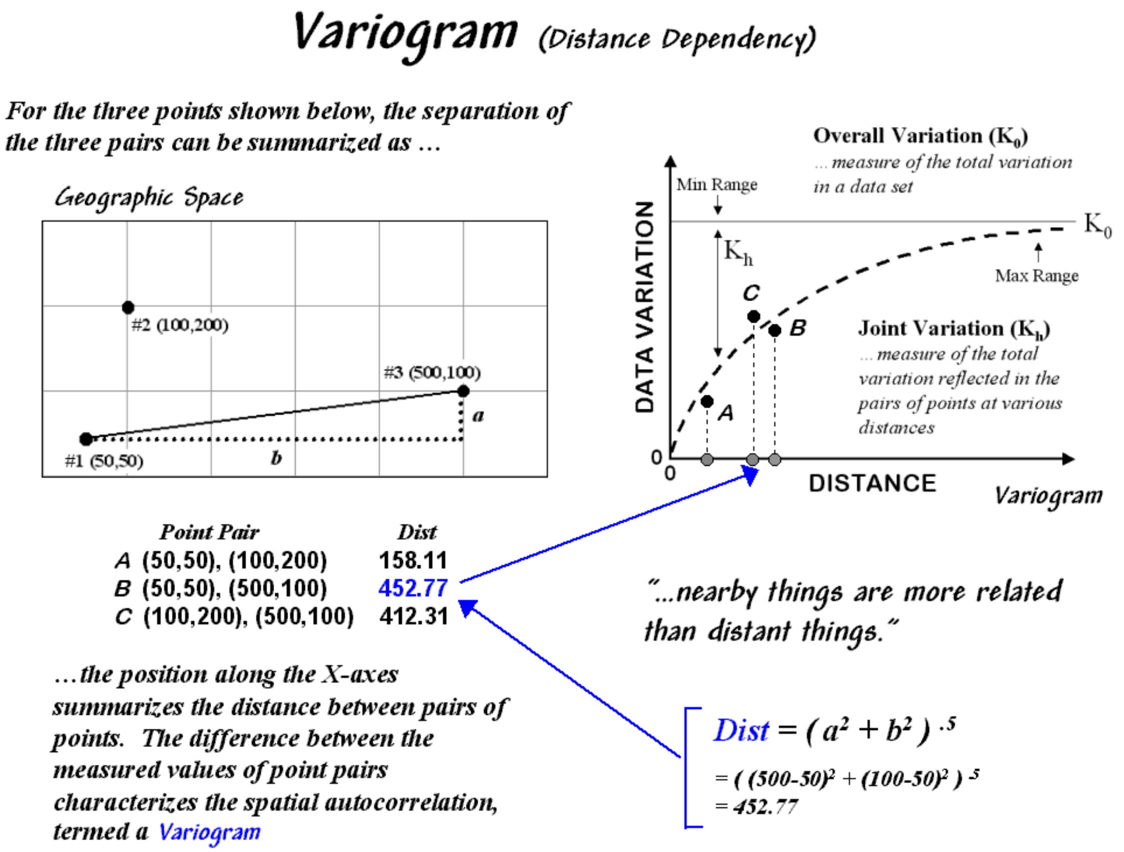
(GeoWorld, September 2001)

(return to top of Topic)

The previous sections have investigated *spatial dependency*—the assumption that “nearby things are more related than distant things.” This autocorrelation forms the basic concept behind spatial interpolation and the ability to generate maps from point sampled data. If there is a lot of spatial autocorrelation in a set of samples, expect a good map; if not, expect a map of pure, dense gibberish.

An *index of spatial autocorrelation* compares the differences between nearby sample pairs with those from the average of the entire data set. One would expect a sample point to be more like its neighbor than it is to the overall average. The larger the disparity between the nearby and average figures the greater the spatial dependency and the likelihood of a good interpolated map.

A *variogram plot* takes the investigation a bit farther by relating the similarity among samples to the array of distances between them. Figure 1 outlines the mechanics and important aspects of the relationship. The distance between a pair of points is calculated by the Pythagorean Theorem and plotted along the X-axis. A normalized difference between sample values (termed *semi-variance*) is calculated and plotted along the Y-axis. Each point-pair is plotted and the pattern of the points analyzed.


Variogram (Distance Dependency)
For the three points shown below, the separation of the three pairs can be summarized as …
Geographic Space
#2 (100,200)
#3 (500,100)
#1 (50,50)
a
b
Overall Variation (K_0)
… measure of the total variation in a data set
Min Range
K_0
K_h
Max Range
C
B
A
Joint Variation (K_h)
… measure of the total variation reflected in the pairs of points at various distances
DATA VARIATION
0
0
DISTANCE
Variogram
“…nearby things are more related than distant things.”
…the position along the X-axes summarizes the distance between pairs of points. The difference between the measured values of point pairs characterizes the spatial autocorrelation, termed a Variogram

	Point Pair	Dist
A	(50,50), (100,200)	158.11
B	(50,50), (500,100)	452.77
C	(100,200), (500,100)	412.31

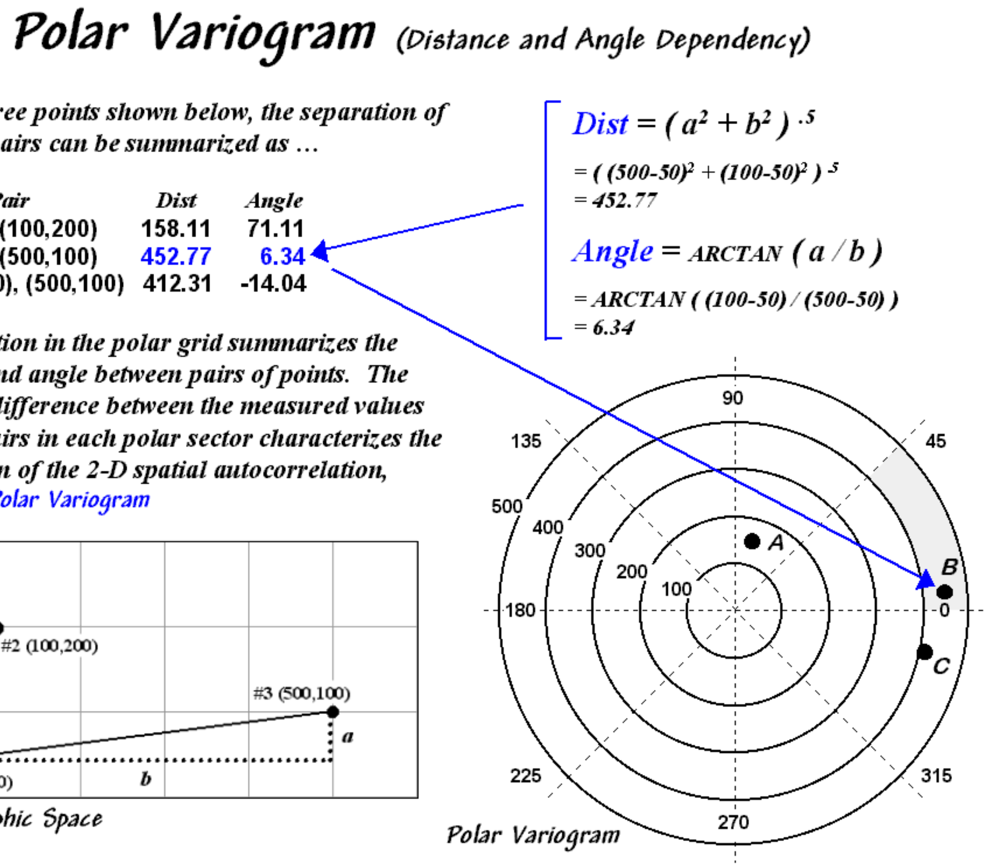
$$Dist = (a^2 + b^2)^{.5}$$
$$= ((500-50)^2 + (100-50)^2)^{.5}$$
$$= 452.77$$

Figure 1. A variogram relates the difference between sample values and their distance.

Spatial autocorrelation exists if the differences between sample values systematically increase as the distances between sample points becomes larger. The shape and consistency of the pattern of points in the plot characterize the degree of similarity. In the figure, an idealized upward curve is indicated. If the remaining point-pairs continue to be tightly clustered about the curve considerable spatial autocorrelation is indicated. If they are scattered throughout the plot without forming a recognizable pattern, minimal autocorrelation is present.

The "goodness of fit" of the points to the curve serves as an index of the spatial dependency— a good fit indicates strong spatial autocorrelation. The curve itself provides relative weights for the samples surrounding a location as it is interpolated— the weights are calculated from the equation of the curve.

A *polar variogram* takes the concept a step further by considering directional bias as well as distance. In addition to calculating distance, the direction between point-pairs is determined using the "opposite-over-adjacent (tangent)" geometry rule. A polar plot of the results is constructed with rings of increasing distance divided into sectors of different angular relationships (figure 2).



Polar Variogram (Distance and Angle Dependency)

For the three points shown below, the separation of the three pairs can be summarized as …

	Point Pair	Dist	Angle
A	(50,50), (100,200)	158.11	71.11
B	(50,50), (500,100)	452.77	6.34
C	(100,200), (500,100)	412.31	-14.04

…the position in the polar grid summarizes the distance and angle between pairs of points. The "typical" difference between the measured values of point pairs in each polar sector characterizes the distribution of the 2-D spatial autocorrelation, termed a Polar Variogram

$$Dist = (a^2 + b^2)^{.5}$$
$$= ((500-50)^2 + (100-50)^2)^{.5}$$
$$= 452.77$$

$$Angle = \text{ARCTAN}(a / b)$$
$$= \text{ARCTAN}((100-50) / (500-50))$$
$$= 6.34$$

Figure 2. A polar variogram relates the difference between sample values to both distance and direction.

Each point-pair plots within one of the sectors (shaded portion in figure 2). The difference between the sample values within each sector forms a third axis analogous to the "data variation" (Y-axis) in a simple variogram.

The relative differences for the sectors serve as the weights for interpolation. During interpolation, the distance and angle for a location to its surrounding sample points are computed and the weights for the corresponding sectors are used. If there is a directional bias in the data, the weights along that axis will be larger and the matching sample points in that direction will receive more importance.

The shape and pattern of the polar variogram surface characterizes the distance and directional dependencies in a set of data— the X and Y axes depict distance and direction between points while the Z-axis depicts the differences between sample values.

An idealized surface is lowest at the center and progressively increases. If the shape is a perfect bowl, there is no directional bias. However, as ridges and valleys are formed directional dependencies are indicated. Like a simple variogram, a polar variogram provides a graphical representation of spatial dependency in a data set— it just adds direction to the mix of spatial dependency assessments.

(return to top of Topic)

(Back to the Table of Contents)