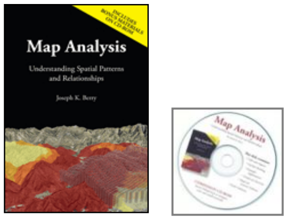
Beyond Mapping III

Topic 9 – Basic Techniques in Spatial Statistics


Map Analysis book/CD

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GIS Data Are Rarely Normal

(GeoWorld, October 1998)

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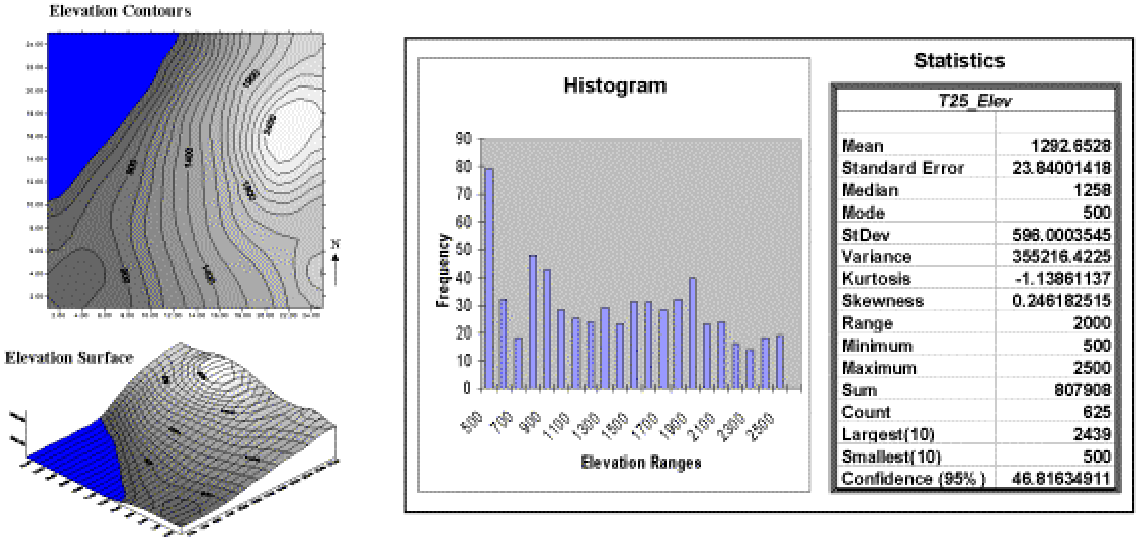
Most of us are familiar with the old "bell-curve" for school grades. You know, with lots of C's, fewer B's and D's, and a truly select set of A's and F's. Its shape is a perfect bell, symmetrical about the center with the tails smoothly falling off toward less frequent conditions.

Although the distribution is familiar and easy to visualize, the ***normal distribution*** (bell-shaped) isn't as normal (typical) as you might think. For example, *Newsweek* recently noted that the average grade at a major ivy-league university isn't a solid C with a few A's and F's sprinkled about as you might imagine, but an A- with a lot of A's trailing off to lesser amounts of B's, C's and (heaven forbid) the rare D's and F's.

The frequency distributions of mapped data also tend toward the *ab*-normal (formally termed ***asymmetrical***). For example, consider the elevation data shown in figure 1. The contour map and 3-D surface on the left depicts the geographic distribution of the data. Note the distinct pattern of the terrain with higher elevations in the northeast and lower ones along the western portion. As is normally the case with mapped data, the elevation values are neither uniformly nor randomly distributed in geographic space. The unique pattern is the result complex physical processes driven by a host of factors—not spurious, arbitrary, constant or even “normal” events.



Statistics

T25_Elev	
Mean	1292.6528
Standard Error	23.84001418
Median	1258
Mode	500
StDev	596.0003545
Variance	355216.4225
Kurtosis	-1.13861137
Skewness	0.246182515
Range	2000
Minimum	500
Maximum	2500
Sum	807908
Count	625
Largest(10)	2439
Smallest(10)	500
Confidence (95%)	46.81634911

Figure 1. Mapped data are characterized by their geographic distribution (maps on the left) and their numeric distribution (histogram and statistics on the right).

Now turn your attention to the numeric distribution of the data depicted in the right side of the figure. The ***data view*** was generated by simply transferring the gridded elevation values to Excel, then applying the *Histogram* and *Descriptive Statistics* options of the Data Analysis add-in tools. The mechanics used to plot the histogram and generate the statistics were a piece-of-cake, but the real challenge is to make some sense of it all. Note that the data aren’t distributed as a normal bell-curve, but appear flattened and slightly shifted to the left. The tall spike at the lowest elevation range (500-600 feet) is due to the lake in the northwest corner. If the lake was drained (or its bathometry considered) some of the spike’s values would be assigned smaller elevations and the distribution would broaden and flatten even more.

If the terrain contained a plateau or mesa instead of the smooth hill in the northeast, there would be a spike at the high end of the histogram. What do you think the histogram would look like if the area contained several chimney-rocks or “whoodoos” scattered about a flat plain? Or if the area were centered on an escarpment?

The mental exercise linking geographic space with data space is a good one, and some general points ought to be noted. First, there isn’t a fixed relationship between the two views of the data’s distribution (geographic and data). A myriad of geographic patterns can result in the same histogram. That’s because spatial data contains additional information—*where*, as well as *what*—and the same data summary of the “what’s” can reflect a multitude of spatial arrangements (“where’s).

But is the reverse true? Can a given geographic arrangement result in different data views? Nope, and it’s this relationship that catapults mapping and geo-query into the arena of mapped data analysis. Traditional analysis techniques assume a functional form for the frequency distribution (histogram shape), with the standard normal (bell-shaped) being the most prevalent. Last June’s column described the basic descriptive statistics Excel’s summary table— *maximum, minimum, range, mode, median, mean (average), variance, standard deviation* and an additional one termed *coefficient of variation*. The discussion described how these statistics portray the central tendency (typical condition) of a data set. In effect, they reduce the complexity of a large number of measurements to just a handful of numbers and provide a foothold for further analysis.

A brief discussion of the additional indices in Excel’s table is warranted. The ***sum*** and the ***count*** should be obvious—the total of all the measurements (sum= 807,908 “total” feet above sea level doesn’t mean much in this context) and the number of measurements (count= 625 data values indicates a fairly big data set as traditional statistics go, but fairly small for spatial statistics). The ***largest/smallest*** statistic in the table identifies the average of a user-specified number of values (10 in this case) at the extreme ends of the data set. It is interesting to note that the average of the 10 smallest elevation values (500) is the same as the minimum value, while the average of the 10 largest values (2439) is well below the maximum value of 2500.

The ***standard error*** calculates the average difference between the individual data values and the mean (StdError= sum [[x-mean]**2] / [n*[n-1]]). If the average deviation is fairly small, then the mean is fairly close to each of the sample measurements. The standard error for the elevation data is 23.84001418 (whoa Excel, way too many decimals— nothing in statistics is that precise). The statistic means that the mean is on the average (got that?) about 24 feet above or below the 625 individual elevation values comprising the map. Useful information, but often the attention of most GIS applications is focused on the areas of “unusually” high or low areas (the *outliers*), not how well the average “fits” the entire data set.

The ***confidence level*** is a range on either side of a sample mean that you are fairly sure contains the population (true) average. For example, if you have some data on mail order delivery times, you can determine, with a particular level of confidence (usually 95%), the earliest and latest a product will likely arrive.

The elevation data’s confidence value of 46.81634911 suggests that we can be fairly sure that the “true” average elevation is between 1245 and 1340. But this has a couple of important assumptions—that the data represents a good sample and that the normal curve is a good representation of the actual distribution.

But what if the distribution isn’t normal? What if it is just a little *ab*-normal? What if it is a lot?

That's the stuff of doctoral theses, but there are some general considerations that ought to be noted. First, there are some important statistics that provide insight into how normal a data set is. ***Skewness*** tells us if the data is lop-sided. Formally speaking, it "characterizes the degree of asymmetry of a distribution around its mean." Positive skewness indicates a distribution shifted to left, while negative skewness indicates a shift to the right and zero skewness is indicates perfectly symmetrical data. The larger the value, the more pronounced is the lop-sided shift. In the elevation data, a skewness value of .246182515 indicates a slight shift to the right.

Another measure of *ab*-normality is termed ***kurtosis***. It characterizes the relative "peakedness or flatness" of a distribution compared with the "ideal" bell-shaped distribution. A positive kurtosis indicates a relatively peaked distribution, while a negative kurtosis indicates a relatively flat one and 0 is just the right amount (sounds like Goldilock's "papa, mamma and baby bear" sizing of distribution shape). Its magnitude reports the degree of distortion from a perfect bell-shape. The –1.13861137 kurtosis value for the elevation data denotes a substantial flattening.

All in all, the skewness and kurtosis values don't bode well for the elevation data being normally distributed. In fact, a lot of spatial data isn't very normal…some might say most. So what do you do? Throw away the Excel-type descriptive statistics? Punt on statistical analysis and simply generate really cool graphics for visceral visions of the relationships? Do you blindly go ahead and impose assumptions of normalcy just to force-fit normal analysis procedures? Good questions, but they will have to wait for the next section's discussions.

The Average Is Hardly Anywhere

(GeoWorld, May 2006)

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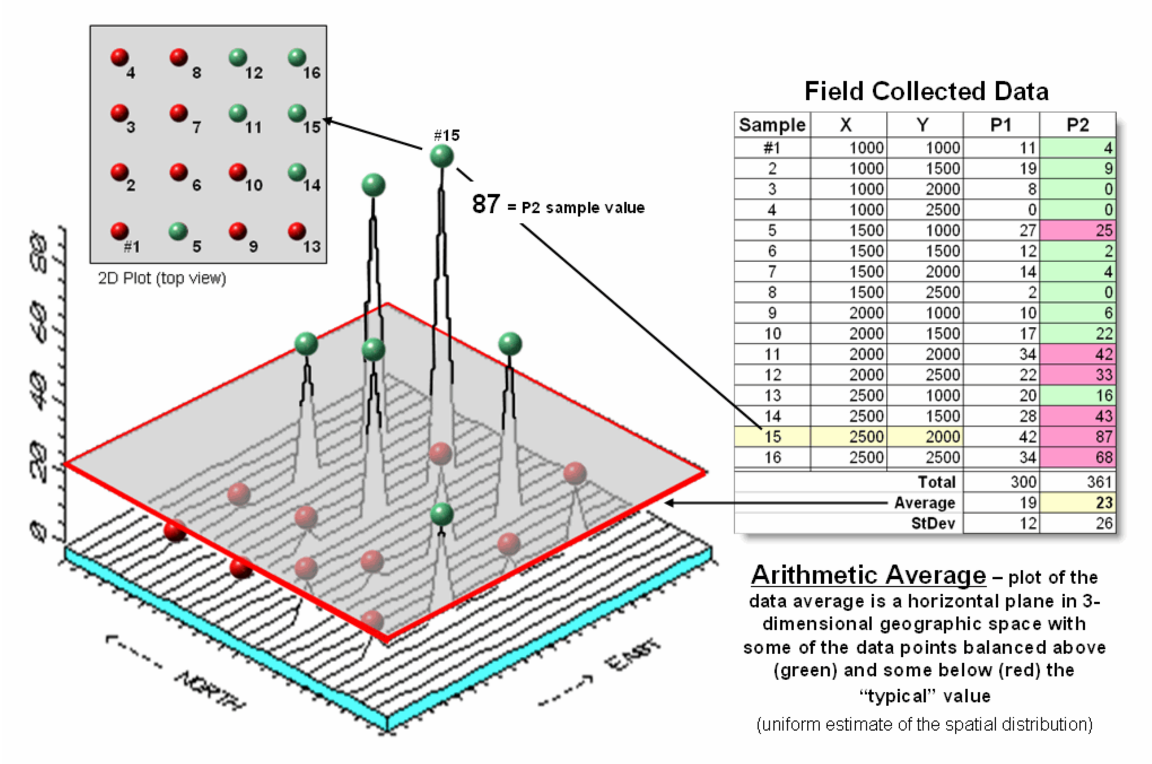
Remember your first encounter with statistics? In the days of old before politically correct examples, you might have calculated the average weight of the students in the class by adding up all of the students' weights and then dividing by the number of students. The *average* weight gives you an estimate of how heavy a "typical student" is and the *standard deviation* tells you how typical that typical is.

Now imagine your old classroom with the bulky jocks in the back, the diminutive bookworms in front, and the rest caught in between. Two things should come to mind—1) not all of the students had the same typical weight (some were heavier and some were lighter) and 2) the differences from the typical weight might have followed a geographic pattern (heavy in the back, light in the front). The second point forms the foundation of surface modeling that "maps the variation in geographic data sets."



Field Collected Data

Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
9	2000	1000	10	6
10	2000	1500	17	22
11	2000	2000	34	42
12	2000	2500	22	33
13	2500	1000	20	16
14	2500	1500	28	43
15	2500	2000	42	87
16	2500	2500	34	68
		Total	300	361
		Average	19	23
		StDev	12	26

Arithmetic Average – plot of the data average is a horizontal plane in 3-dimensional geographic space with some of the data points balanced above (green) and some below (red) the "typical" value

(uniform estimate of the spatial distribution)

Figure 1. Comparison of the spatial distributions of field samples (floating balls) and their arithmetic average (horizontal plane).

Figure 1 illustrates the spatial and non-spatial character of a set of animal activity data. The right side of the figure lists the number of sightings at sixteen locations for two 24-hour periods (P1 in June; P2 in August). Note the varying levels of activity— 0 to 42 for P1 and 0 to 87 for P2 that can be reduced to their average values of 19 and 23, respectively. A wildlife manager might ponder these findings, and then determine whether the implied typical activity is too little, too much or just right for a particular management action.

But the average does not address the variation in the data set— that's the role of the standard deviation. As a general rule (termed the Coefficient of Variation) "…if the standard deviation is relatively large compared to the arithmetic average, the average cannot be used to make decisions" as there is too much unexplained variation in the data (i.e., the computed average isn't very typical).

Possibly some of the variation in animal activity forms a pattern in geographic space that might help extend the explanation. That is where the left side of figure 1 comes into play with a three-dimensional plot used to show the geographic location (X, Y) and the measured activity levels (Z). I'll bet your eye is "mapping the variation in the data" as high activity in the Northeast, low in the Northwest and moderate elsewhere.

The thick line in the plot outlines a horizontal plane at 23 (arithmetic average) that spatially characterizes the typical animal activity as uniformly distributed in geographic space (horizontal plane). But your eye tells you that guessing 23 around Sample #15 with a measured value of 87, is likely an understatement. Similarly, a guess of 23 around Sample #4 with a value of 0 is likely an overstatement. That is what the relatively large standard deviation indicated— guess 23 but expect to be way off (+ 26) a lot of the time.

The non-spatial procedure, however, doesn't provide a hint as to where the average might be guessing too low and where it might be guessing high. That's the main difference between traditional statistics and spatial statistics— traditional statistics characterizes the central tendency (average) of data in numeric space; spatial statistics seeks to map the variation (standard deviation) of data in geographic space.

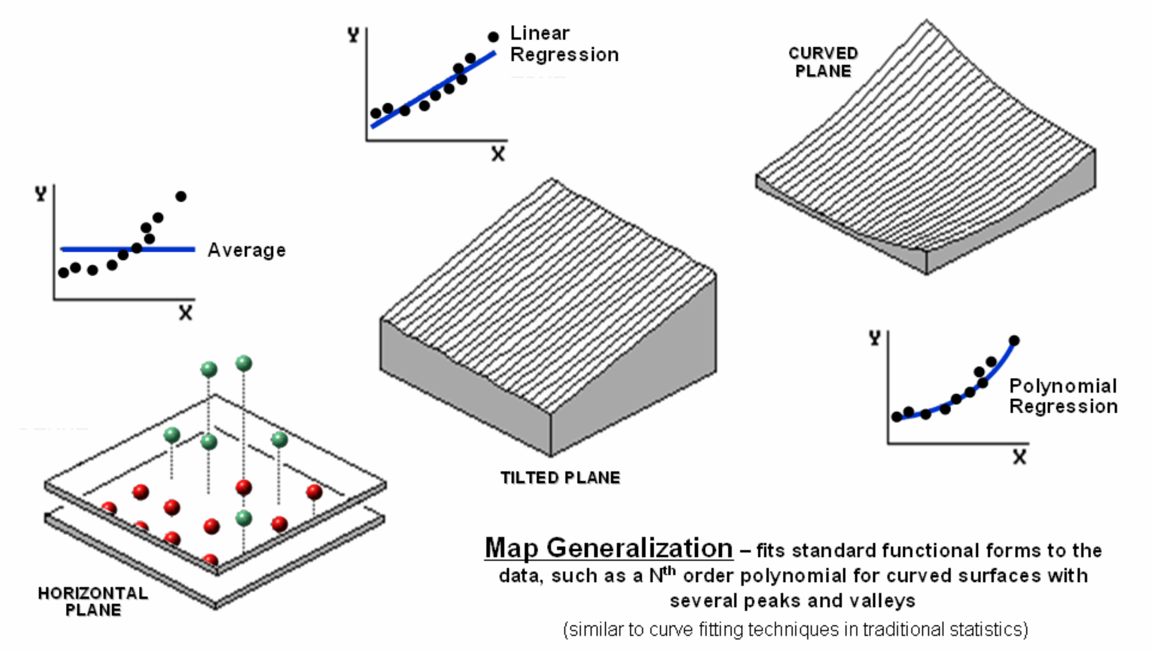

Y
Linear Regression
X
CURVED PLANE
Y
Average
X
Y
Polynomial Regression
X
TILTED PLANE
HORIZONTAL PLANE
Map Generalization – fits standard functional forms to the data, such as a Nth order polynomial for curved surfaces with several peaks and valleys
(similar to curve fitting techniques in traditional statistics)

Figure 2. Map Generalization can be used to approximate geographic trends.

Figure 2 illustrates *Map Generalization* as an approach to mapping the spatial trend in the data using polynomial surface fitting. The average of 23 is depicted as a horizontal plane that does its best to balance half of the balls above it and half below it by minimizing the set of squared deviations from the plane to each floating ball (similar to curve-fitting regression techniques in traditional statistics).

Now relax the assumption that the plane has to remain horizontal. Tilt it every-which-way until it better fits the floating balls (Tilted Plane). Or even better, the assumption of a flat plane can be relaxed and the surface can curve to better fit the ups and downs of the data points. The right portion of the figure fits a 2nd degree polynomial (Curved Plane).

Figure 3 shows an entirely different approach to fitting a surface to the data by using the *Spatial Interpolation* technique of iterative smoothing. Imagine replacing the floating balls with columns of modeler's clay rising to the same height as each ball. In effect this is a blocky first-order estimate of the animal activity throughout the project area derived by simply assigning the closest field sample to each map location.

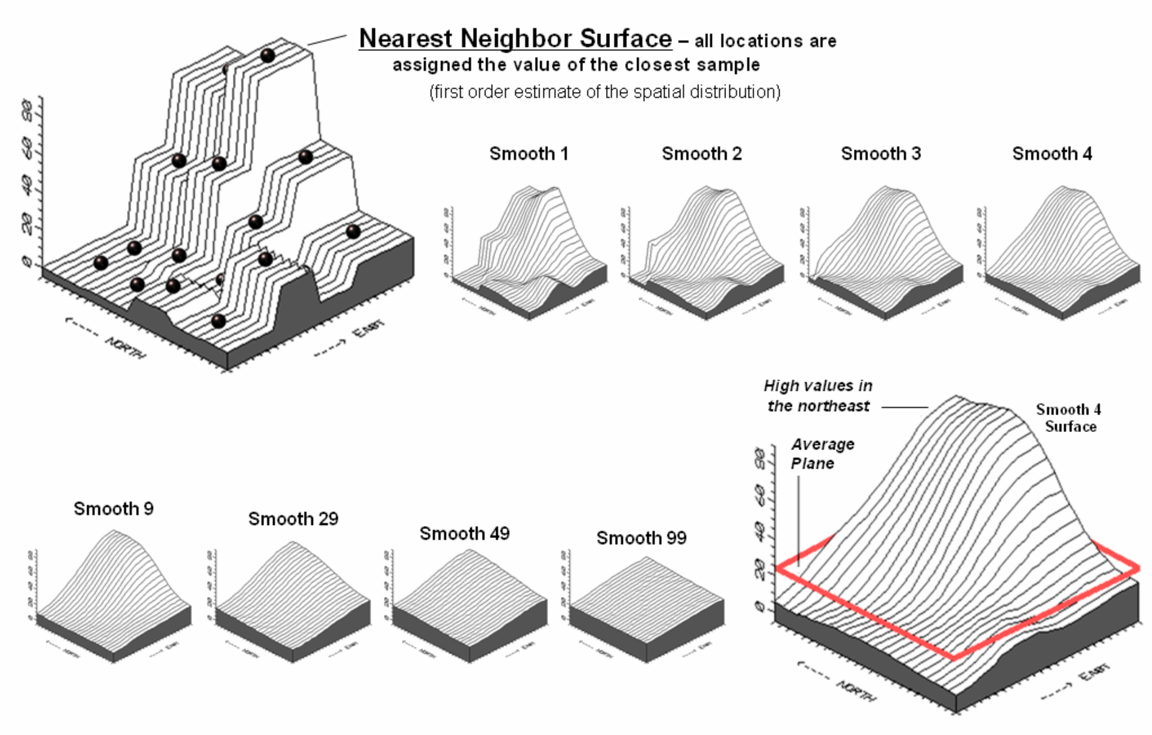


Figure 3. Iteratively Smoothed approximations of localized geographic distribution.
<Click here to download an animated slide set of Interactive Smoothing>

Now imagine whacking away some of the clay at the top of the columns and filling-in at the bottom. In the example, a 3x3 averaging window was moved around the matrix of values to systematically smooth the blocky surface. When the window is centered over one of the sharp boundaries, it has a mixture of small and larger map values, resulting in an average somewhere in between... a localized whack off the top and a fill-in at the bottom.

The series of plots in the figure show repeated passes of the smoothing window from once through ninety-nine times. Like erosion, the mountains (high animal activity) are pulled down and the valleys (low animal activity) are pulled up. According to theory, the process eventually approximates a horizontal plane floating at the arithmetic average.

The bottom line is that field collected data with geographic coordinates holds a lot more information than simply a reduction to a typical value. The nest section investigates other more powerful methods for estimating the spatial, as well as numerical distribution inherent in mapped data.

Under the Hood of Spatial Interpolation

(GeoWorld, June 2006)

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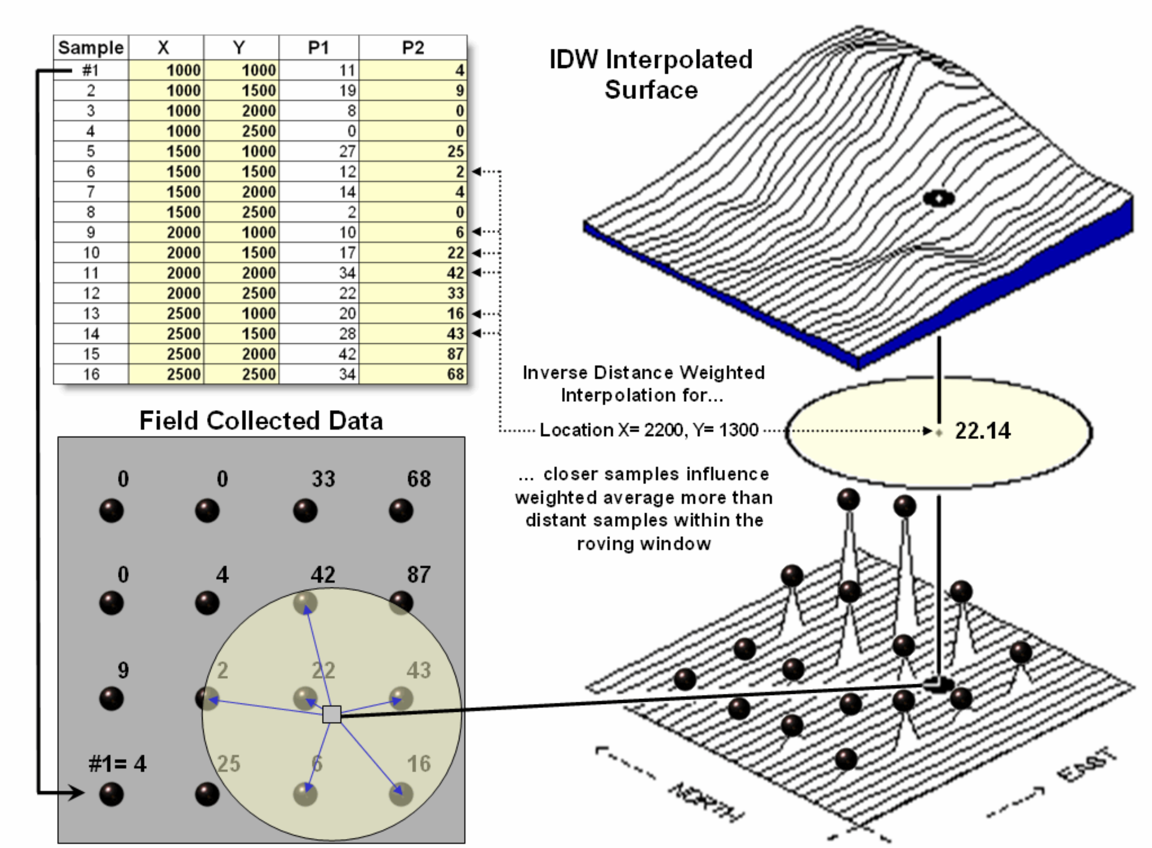
The previous section described how field collected data (discrete points) can be used to generate a map (continuous surface) of the data's spatial patterns. The derived surface extends the familiar concept of central tendency to a map of the geographic distribution of the data. Whereas traditional statistics identifies the typical value in a data set, surface modeling identifies "where" you might expect to find the typical and not so typical responses.

The Iterative Smoothing approach described last time is a simple data-driven procedure. However, all of the interpolation techniques share a similar approach that generates estimates of a mapped variable based on the data values within the vicinity of each map location. In effect, this establishes a "roving window" that moves throughout an area summarizing the field samples it encounters within its reach. The summary estimate is assigned to the center of the window, and then it moves on. The extents of the window (both size and shape) influence the result, regardless of the summary technique. In general, a large window capturing numerous values tends to smooth the data. A smaller window tends to result in a rougher surface.

Three factors affect the window's configuration— its reach, number of samples, and balancing. The *reach*, or search radius, sets a limit on how far the computer will go in collecting data values. The *number of samples* establishes how many data values will be used. The *balancing* of the data attempts to eliminate directional bias by insuring that values are selected from all directions around the window's center.



Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
9	2000	1000	10	6
10	2000	1500	17	22
11	2000	2000	34	42
12	2000	2500	22	33
13	2500	1000	20	16
14	2500	1500	28	43
15	2500	2000	42	87
16	2500	2500	34	68

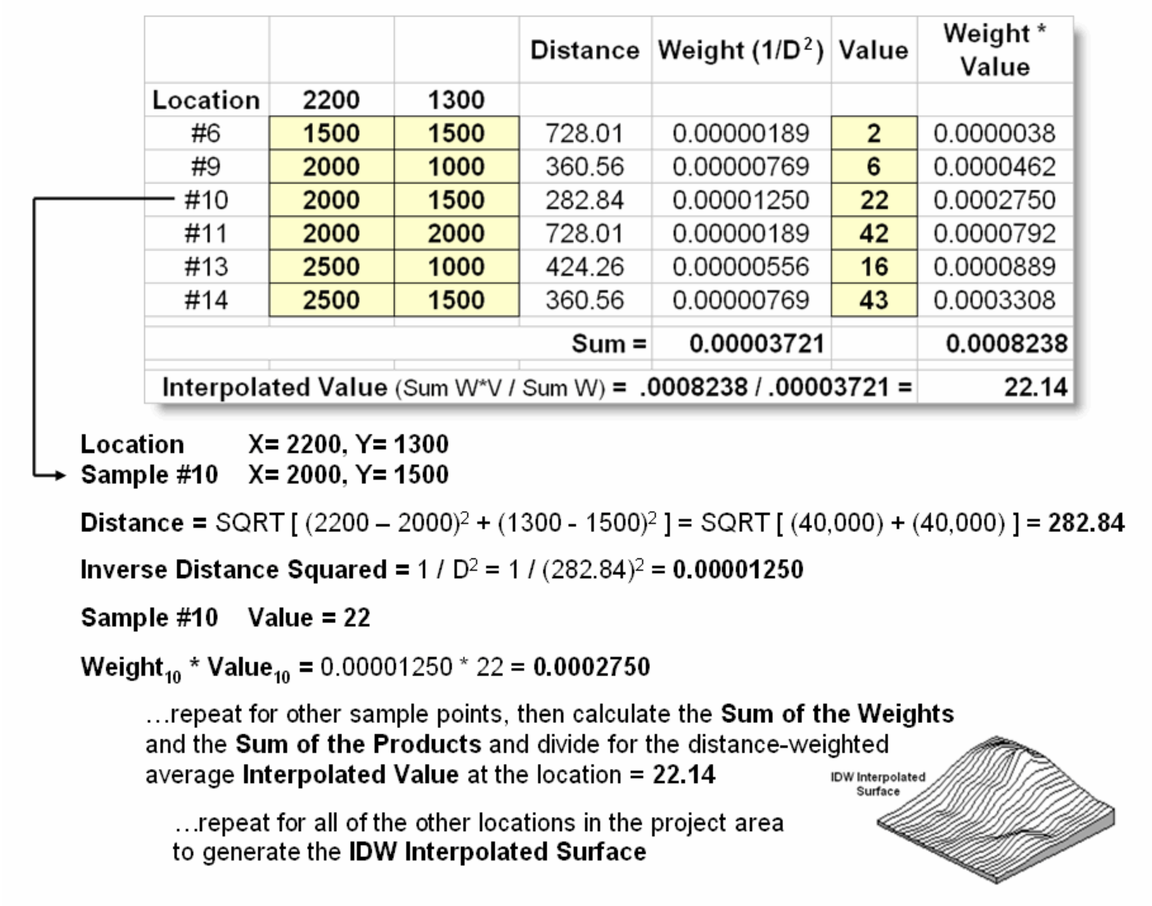
IDW Interpolated Surface
Field Collected Data
0
0
33
68
0
4
42
87
9
2
22
43
#1= 4
25
6
16
Inverse Distance Weighted Interpolation for...
Location X= 2200, Y= 1300
22.14
... closer samples influence weighted average more than distant samples within the roving window
NORTH
EAST

Figure 1. A roving window is used to identify and summarize nearby sample values.

Once a window is established, the summary technique comes into play. The *Inverse Distance Weighted (IDW)* technique is easy to conceptualize (see figure 1). It estimates a value for a location as the weighted-average of the nearby data values within the roving window. The average is weighted so the influence of the surrounding values decrease with increasing distance from the location being estimated.

Figure 2 shows the IDW calculations for the location identified in figure 1. The Pythagorean Theorem is used to calculate the geographic distance between the location and the six sample values in the window. A weight for each sample is determined as $1/D^2$, then the weighted average of the samples is calculated and assigned to the location. The process is repeated for each location in the project area.

Because IDW is a static averaging method, the estimated values can never exceed the range of values in the original sample data. Also, it tends to "pull-down peaks and pull-up valleys" in the data. Inverse distance is best suited for data sets where the samples are fairly independent of their surrounding locations (i.e., no strong regional trend).



			Distance	Weight (1/D^2)	Value	Weight * Value
Location	2200	1300				
#6	1500	1500	728.01	0.00000189	2	0.0000038
#9	2000	1000	360.56	0.00000769	6	0.0000462
#10	2000	1500	282.84	0.00001250	22	0.0002750
#11	2000	2000	728.01	0.00000189	42	0.0000792
#13	2500	1000	424.26	0.00000556	16	0.0000889
#14	2500	1500	360.56	0.00000769	43	0.0003308
			Sum =	0.00003721		0.0008238
Interpolated Value (Sum W*V / Sum W) = .0008238 / .00003721 =						22.14

Location X= 2200, Y= 1300
Sample #10 X= 2000, Y= 1500

Distance = SQRT [$(2200 – 2000)^2$ + $(1300 - 1500)^2$] = SQRT [(40,000) + (40,000)] = **282.84**

Inverse Distance Squared = 1 / D^2 = 1 / $(282.84)^2$ = **0.00001250**

Sample #10 Value = 22

Weight_{10} * Value_{10} = 0.00001250 * 22 = **0.0002750**

…repeat for other sample points, then calculate the **Sum of the Weights** and the **Sum of the Products** and divide for the distance-weighted average **Interpolated Value** at the location = **22.14**

…repeat for all of the other locations in the project area to generate the **IDW Interpolated Surface**

Figure 2. IDW calculations use simple geographic distance to weight the average of samples within a roving window.

Another approach, *Kriging (KRIG)* uses regional variable theory based on an underlying variogram. That's techy-speak implying that there is a lot of math behind the approach. In effect, the technique develops custom weights based on trends in the sample data. The moving average weights that define the data trends can result in estimated values that exceed the field data's range of values. Also, there can be unexpected results in large areas without data values. The technique is most appropriate for fairly dense, systematically sampled data exhibiting discernable regional trends.

The center portion figure 3 illustrates how the Krig weights are derived. The distances between each sample point and all of the other sample points are calculated. At the same time, the differences in the sample values are recorded. Common sense suggests that “nearby things are more alike than distant things” and a plot of Distance versus Difference often looks something like the *variogram* in the center of the figure.

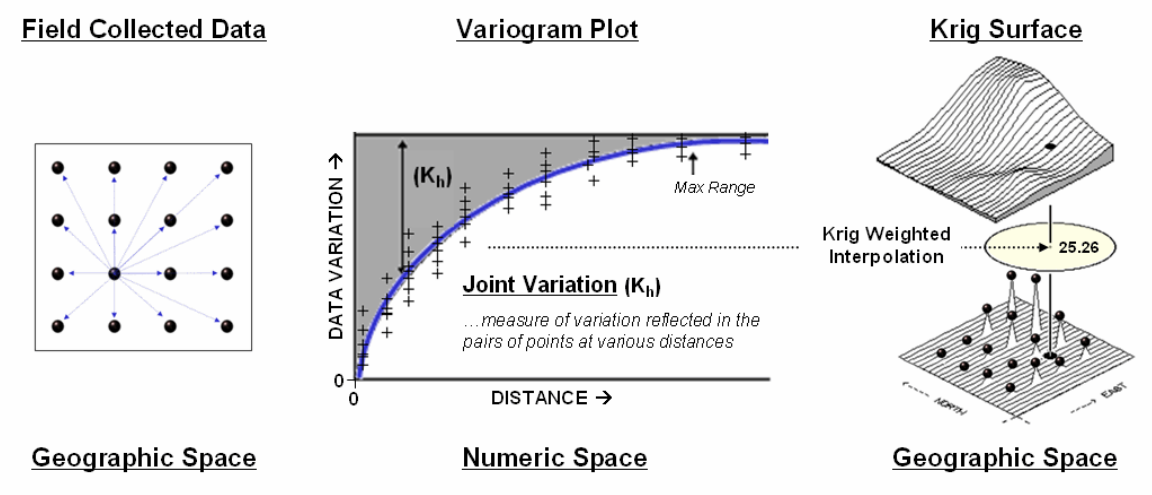


Figure 3. A variogram plot depicts the relationship between distance and similarity between sample values.

The origin of the plot at *0,0* is a unique case. The distance between samples is zero; therefore, there shouldn't be any dissimilarity (data variation = zero) as the location is exactly the same as itself. As the distance between points increase, subsets of the data are scrutinized for their dependency. The shaded portion in the idealized plot shows a rapid deterioration of the spatial dependency among the sample points. The maximum range (*Max Range*) position identifies the distance between points beyond which the data values are considered independent. This suggests that using data values beyond this distance for interpolation isn't useful.

An equation is fit to the Distance-Difference data and used to determine the weights and reach used in averaging the sample values within the roving window. Whereas IDW uses a fixed geometric function, Krig derives the weights by investigating the spatial dependency in the sample data (see Author's Note). Keep in mind that most analytical GIS systems have a dozen or more spatial interpolation techniques— but they all share the common basics of a window configuration and a summary procedure.

Author's Note: *Let me apologize in advance for such a terse treatment of a complex subject. See "Advanced Concepts in Spatial Dependency" section of the Further Reading links for a more in depth discussion the various measures of spatial autocorrelation, their interpretation and use in interpolation.*

Justifiable Interpolation

(GeoWorld, February 1997)

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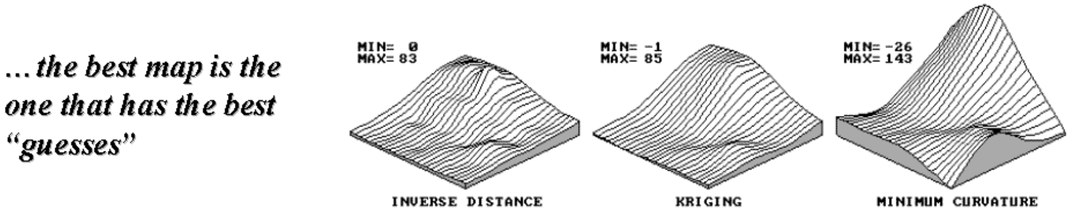
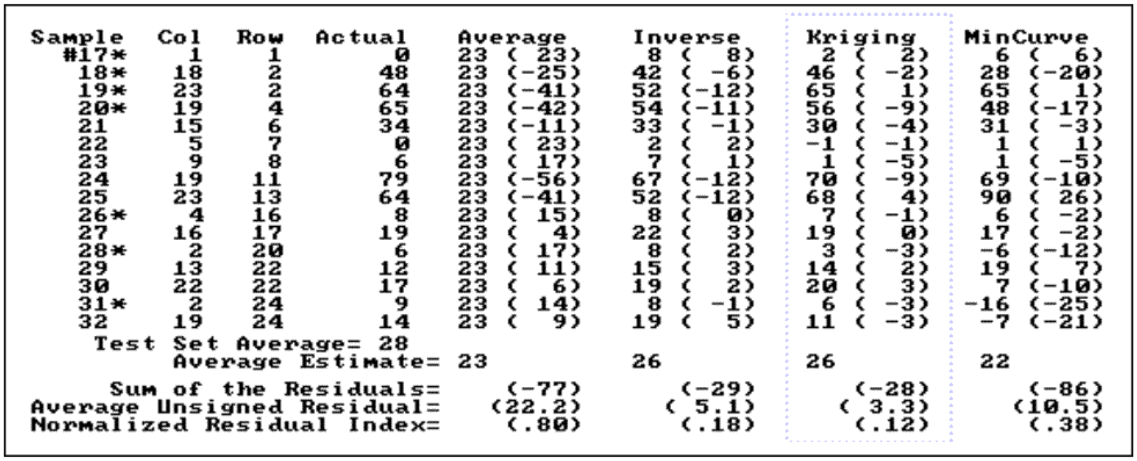
The previous sections and references discuss the basic considerations in generating maps from point data. Recall that both sampling design and interpolation technique greatly affect the results when converting discrete point samples (e.g., soil samples) to continuous map surfaces (e.g., map of phosphorous levels throughout a field). The only thing for certain is that there isn't a single best procedure that covers all situations. Some rules of thumb and general considerations have been discussed, but the only good way to determine the "best" map is to empirically verify the results. This involves generating several maps under different sampling/interpolation procedures, then testing the results against a set of known measurements, aptly termed a "test set." As in horseshoes and bocce ball, the closest one wins.

The accompanying table show large differences among three interpolated surfaces generated from the same set of point sampled data, termed the "interpolation set." The algorithmic nuances and relative advantages and disadvantages of the Inverse Distance, Kriging and Minimum Curvature techniques were discussed earlier. What should spring to mind at this point is "OK, they're different, so how can I tell which is best?" An empirical verification technique, termed residual analysis, summarizes the differences between interpolation estimates and actual measurements for each test location.





Sample	Col	Row	Actual	Average	Inverse	Kriging	MinCurve
#17*	1	1	0	23 (23)	8 (8)	2 (2)	6 (6)
18*	18	2	48	23 (-25)	42 (-6)	46 (-2)	28 (-20)
19*	23	2	64	23 (-41)	52 (-12)	65 (1)	65 (1)
20*	19	4	65	23 (-42)	54 (-11)	56 (-9)	48 (-17)
21	15	6	34	23 (-11)	33 (-1)	30 (-4)	31 (-3)
22	5	7	0	23 (23)	2 (2)	-1 (-1)	1 (1)
23	9	8	6	23 (17)	7 (1)	1 (-5)	1 (-5)
24	19	11	79	23 (-56)	67 (-12)	70 (-9)	69 (-10)
25	23	13	64	23 (-41)	52 (-12)	68 (4)	90 (26)
26*	4	16	8	23 (15)	8 (0)	7 (-1)	6 (-2)
27	16	17	19	23 (4)	22 (3)	19 (0)	17 (-2)
28*	2	20	6	23 (17)	8 (2)	3 (-3)	-6 (-12)
29	13	22	12	23 (11)	15 (3)	14 (2)	19 (7)
30	22	22	17	23 (6)	19 (2)	20 (3)	7 (-10)
31*	2	24	9	23 (14)	8 (-1)	6 (-3)	-16 (-25)
32	19	24	14	23 (9)	19 (5)	11 (-3)	-7 (-21)
Test Set Average=			28				
Average Estimate=				23	26	26	22
Sum of the Residuals=				(-77)	(-29)	(-28)	(-86)
Average Unsigned Residual=				(22.2)	(5.1)	(3.3)	(10.5)
Normalized Residual Index=				(.80)	(.18)	(.12)	(.38)

Figure 1. Residual table identifying actual and estimated values.

The table in figure 1 contains a test set of sixteen random samples (#17-32) used to evaluate three maps. The “Actual” column lists the measured values for the test locations identified by “Col, Row” coordinates. The difference between these values and those predicted by the three interpolation techniques form the residuals shown in parentheses. The “Average” column compares the whole field arithmetic mean of 23 (guess 23 everywhere) for each test location.

For example, the first sample (#17) guessed 23 but was actually 0, therefore it’s off by 23. Note the sign of the residual indicates whether the guess was too high (positive) or too low (negative). If it was a perfect data world, you would expect the Sum of the Residuals to be 0 (just as many high as low misses— balanced boo-boo). However, in this case there is a net residual of -77 indicating a strong bias to under estimate.

The Average Unsigned Residual is simply the average magnitude of mistakes, regardless whether they’re over or under. For the “Average” estimate expect to be off by about 22.2 each time you guess. The Normalized Residual Index is simply the ratio of the Average Unsigned Residual to the Test Set Average. An index of .80 (22.2/28) indicates that the “Average” technique isn’t very good at spatial estimates.

Now take a look at the residual analysis for the three interpolation techniques. All three are considerably better than the whole field average (Normalized Residual Indices of .18, .12 and .38, respectively, compared to the whopping .80 for the simple whole field average). The “Kriging” technique is best, having a 3.3 Average Unsigned Residual with a tendency to under estimate (a Sum of the Residuals of -28). The “Inverse” technique is a close second with an Average Unsigned Residual of 5.1 and a nearly identical under estimate bias of -29. The “MinCurve” technique indices indicate that it is a distant third, but still much better than using the whole field average for spatial estimates.

However, we haven’t considered other potential affects. The asterisks identify test measurements outside the set of samples used for generating the map surfaces. Estimating these values is termed spatial extrapolation and some techniques are better than others. I wonder if there are significant differences among the three techniques in their spatial interpolation and extrapolation estimates (get out your calculator). Would you expect the conclusions to change if the “test” and “interpolation” data samples were swapped and another residual analysis performed? What if different sampling designs were tried? What about different interpolation parameters (window reach, number of samples and balancing)? At least residual analysis gives us a place to start looking for answers.

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*Further Online Reading: (Chronological listing posted at **www.innovativegis.com/basis/BeyondMappingSeries/**)*

(Modeling Error Propagation)

Move Beyond a Map Full of Errors — *discusses a technique for generating a "shadow map" of error (March 1997)*

Comparing Map Errors — *describes how normalized maps of error can be used to visualize the differences in error surfaces (April 1997)*

(Point Sampling Considerations)

What's the Point? — *discusses the general considerations in point sampling design (December 1996)*

Designer Samples — *describes different sampling patterns and their relative advantages (January 1997)*

(Advanced Concepts in Spatial Dependency)

Depending on the Data — *discusses the fundamental concepts of spatial dependency (May 1997)*

Uncovering the Mysteries of Spatial Autocorrelation — *describes approaches used in assessing spatial autocorrelation (July 1997)*

Unlocking the Keystone Concept of Spatial Dependency — *discusses spatial dependency and illustrates the effects of different spatial arrangements of the same set of data (November 1998)*

Measuring Spatial Dependency — *describes the basic measures of autocorrelation (December 1998)*

Extending Spatial Dependency to Maps — *describes a technique for generating a map of spatial autocorrelation (January 1999)*

Use Polar Variograms to Assess Distance and Direction Dependencies — *discuses a procedure to incorporate direction as well as distance for assessing spatial dependency (September 2001)*

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