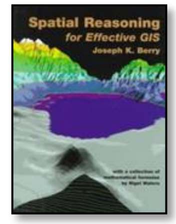
Beyond Mapping II



***Spatial Reasoning* book**

Topic 2 – From Field Samples to Mapped Data

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Averages Are Mean

(GeoWorld, January 1994)

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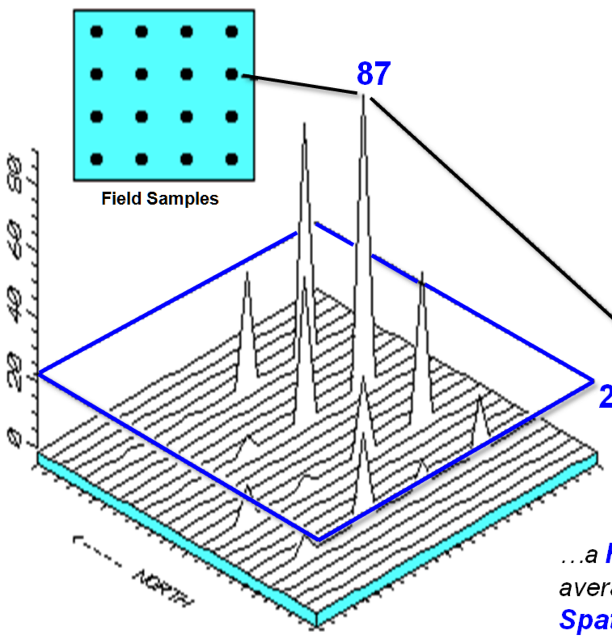
Remember your first brush with statistics? The thought likely conjures up a prune-faced mathematics teacher who determined the average weight of the students in your class. You added the students' individual weights, then divided by the number of students. T e average weight, or more formally stated as the *arithmetic mean* (aptly named for the mathematically impaired), was augmented by another measure termed the *standard deviation*. Simply stated, the mean tells you the typical value in a set of data and the standard deviation tells you how typical that typical is.

So what does that have to do with GIS? It's just a bunch of maps accurately locating physical features on handy, fold-up sheets of paper or colorful wall-hangings. Right? Actually, GIS is taking us beyond mapping— from images to mapped data that are ripe for old pruneface's techniques.

Imagine your old classroom with the bulky football team in the back, the diminutive cheerleaders in front and the rest caught in between. Two things (at least, depending on your nostalgic memory) should come to mind. First, all of the students didn't have the same typical weight; some were heavier and some were lighter. Second, the differences from the typical weight might have followed a geographic pattern; heavy in the back (football players), light in the front (cheerleaders). The second point is the focus of this topic— *spatial statistics*, which "map the variation in geographic data sets."

Figure 1 illustrates the spatial and nonspatial character of a set of animal activity data. The right side of the figure lists the data collected at sixteen locations (Samples #l-16) for two 24-hour periods (P1 in June and P2 in August). Note the varying levels of activity— 0 to 42 for period I and 0 to 87 for Period 2. Because humans can't handle more than a couple of numbers at a time, we reduce the long data listings to their average value— 19 for Period 1 and 23 for Period 2. We quickly assimilate these findings, then determine whether the implied activity change from Period I to Period 2 is too little, too much, or just right (like Goldilocks' assessments of Mama, Papa, and Baby Bears' things). Armed with that knowledge, we make a management decision, such as "blow 'em away," or, in politically correct wildlife-speak, "hold a special hunt to 'cull' the herd for its own good." It's obvious that animal activity is increasing at an alarming rate throughout the project area.



Animal Activity Data

Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
9	2000	1000	10	6
10	2000	1500	17	22
11	2000	2000	34	42
12	2000	2500	22	33
13	2500	1000	20	16
14	2500	1500	28	43
15	2500	2000	42	87
16	2500	2500	34	68
		TOTAL	300	361
		AVERAGE	19	23
		STDEV	12	26

*…a **Horizontal Plane** at **23** characterizes the average activity for Period 2 as a **Uniform Spatial Distribution**— everywhere the same.*

Figure 1. Spatial comparison of Field Samples and their arithmetic mean.

Whoa! You can't say that, and the nonspatial statistics tell you so. That's the role of the standard deviation. A general rule (termed the *coefficient of variation* for the techy-types) tells us, "If the standard deviation is relatively large compared to the arithmetic average, you can't use the average to make decisions." Heck, it's bigger for Period 2! These numbers are screaming "warning, hazardous to your (professional) health" if you use them in a management decision. There is too much variation in the data; therefore, the computed typical isn't very typical.

So what's a wildlife manager to do? A simple solution is to avoid pressing the calculator's standard deviation button, because all it seems to do is trash your day. And we all know that complicated statistics stuff is just smoke and mirrors, with weasel-words like "likelihood" and "probability." Bah, humbug. Go with your gut feeling (and hope you're not asked to explain it in court).

Another approach is to take things a step further. Maybe some of the variation in animal activity forms a pattern in geographic space. What do you think? That's where the left side of figure 1 comes into play. We use the X,Y coordinates of the Field Samples to locate them in geographic space. The 3-D plot shows the geographic positioning (X=East, Y=North) and the measured activity levels (Z=Activity) for Period 2. I'll bet your eye is "mapping the variation in the data"— high activity in the Northeast, low in the Northwest, and moderate elsewhere. That's real information for an on-the-ground manager.

The thick line in the plot outlines the plane (at 23) that spatially characterizes the "typical" animal activity for Period 2. The techy-types might note that we often split hairs in characterizing this estimate (fitting Gaussian, binomial or nonparametric density functions), but in the end, all nonspatial techniques assume that the typical response is distributed uniformly (or randomly) in geographic space. For non-techy types, that means the different techniques might shift the average more or less, but whatever it is, it's assumed to be the same throughout the project area.

But your eye tells you that guessing 23 around Sample #15, where you measured 87 and its neighbors are all above 40, is likely an understatement. Similarly, a guess of 23 around Sample #4, where both it and its neighbors are 0, is likely an overstatement. That's what the relatively large standard deviation was telling you: Guess 23, but expect to be way off (+26) a lot of the time. However, it didn't give you any hints as to where you might be guessing low and where you might be guessing high. It couldn't, because the analysis is performed in numeric space, not in the geographic space of a GIS.

That's the main difference between classical statistics and spatial statistics: classical statistics seeks the central tendency (average) of data in numeric space; spatial statistics seeks to map the variation (standard deviation) of data in geographic space. That is an oversimplification, but it sets the stage for further discussions of spatial interpolation techniques, characterizing uncertainty and map-ematics. Yes, averages are mean, and it's time for kinder, gentler statistics for real-world applications.

Surf's Up

(GeoWorld, February 1994)

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The previous section introduced the fundamental concepts behind the emerging field of spatial statistics. It discussed how a lot of information about the variability in field-collected data is lost using conventional data analysis procedures. Nonspatial statistics accurately reports the typical measurement (arithmetic mean) in a data set and assesses how typical that typical is (standard deviation). However, it fails to provide guidance as to where the typical is likely too low and where it's likely too high. That's the jurisdiction of spatial statistics and its surface modeling capabilities.

The discussion compared a map of data collected on animal activity to its arithmetic mean, similar to inset (a) in figure 1. The sixteen measurements of animal activity are depicted as floating balls, with their relative heights indicating the number of animals encountered in a 24-hour period. Several sample locations in the northwest recorded zero animals, while the highest readings of 87 and 68 are in the northeast (see the previous section for the data set listing).

The average of 23 animals is depicted as the floating plane, which balances the balls above it (dark) and the balls below it (light). In techy-speak, and using much poetic license, "It is the best-fitted horizontal surface that minimizes the squared deviations (from the plane to each floating ball)." In conceptual terms, imagine sliding a window pane between the balls so you split the group— half above and half below. OK, so much for the spatial characterization of the arithmetic mean (that's the easy stuff).

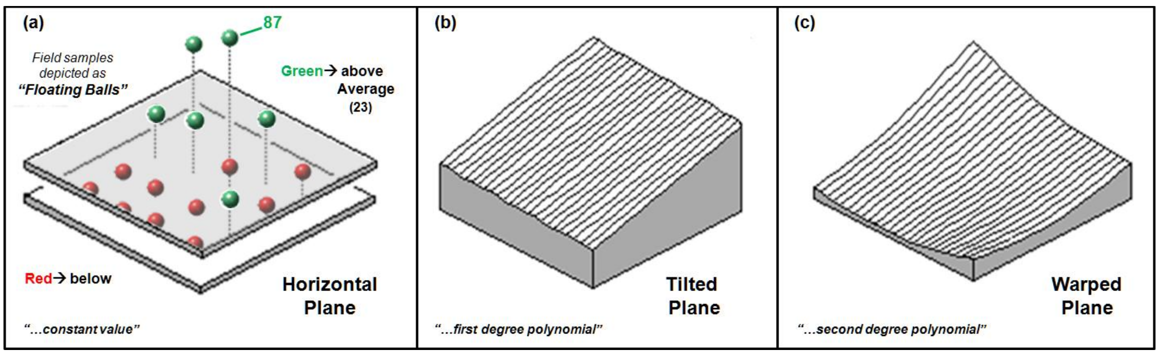

Figure 1. Surface-fitted approximations of the geographic distribution of a mapped data set.

Now relax the assumption that the plane has to remain horizontal (inset b). Tilt the plane every which way until you think it fits the floating balls even better (the squared deviations should be the smallest possible). Being a higher life form, you might be able to conceptually fit a tilted plane to a bunch of floating balls, but how does the computer mathematically fit a plane to the pile of numbers it sees? In techy-speak, "It fits a first degree polynomial to two independent variables." To the rest of us, it solves an ugly equation that doesn't have any exponents. How it actually performs that mathematical feat is best left as one of life's great mysteries, such as how they get the ships inside those tiny bottles (Google search "polynomial surface fitting" if you really have to know).

Inset c in figure l relaxes yet another assumption: The surface has to be a flat plane. In that instance, in techy-speak, "it fits a second degree polynomial to two independent variables" by solving for another ugly equation with squared terms-definitely not for the mathematically faint-of-heart. That allows our plane to become pliable and pulled down in the center. If we allowed the equation to get really ugly N^{th} degree equations), our pliable plane could get as warped as a flag in the wind.

Figure 2 shows a different approach to fitting a surface to our data *iteratively smoothed*. Consider the top series of insets. Imagine replacing the floating balls (insert a) with columns of modeler's clay rising to the same height as each ball (insert b). In effect you have made a first order guess of the animal activity throughout the project area by assigning the map value of the closest field sample. In techy terms, you generated Thessian polygons, with sharp boundaries locating the perpendicular bisectors among neighboring samples.

Now for the fun stuff. Imagine cutting away some of the clay at the top of the columns and filling in at the bottom. However, your computer can't get in there and whack-away like you, so it mimics your fun by moving an averaging window around the matrix of numbers forming the nearest neighbor map. When the window is centered over one of the sharp boundaries, it has a mixture of big and small map values, resulting in an average somewhere in between-a whack off the top and a fill in at the bottom.

Inset (c) in figure shows the results of one complete pass of the smoothing window. The lower set of insets (d) through (e) show repeated passes of the smoothing window. Like erosion, the mountains (high animal activity) are pulled down and the valleys (low animal activity) are pulled up. If everything goes according to theory, eventually the process approximates a horizontal plane floating at the arithmetic mean. That brings you back to where you started— 23 animals assumed everywhere.

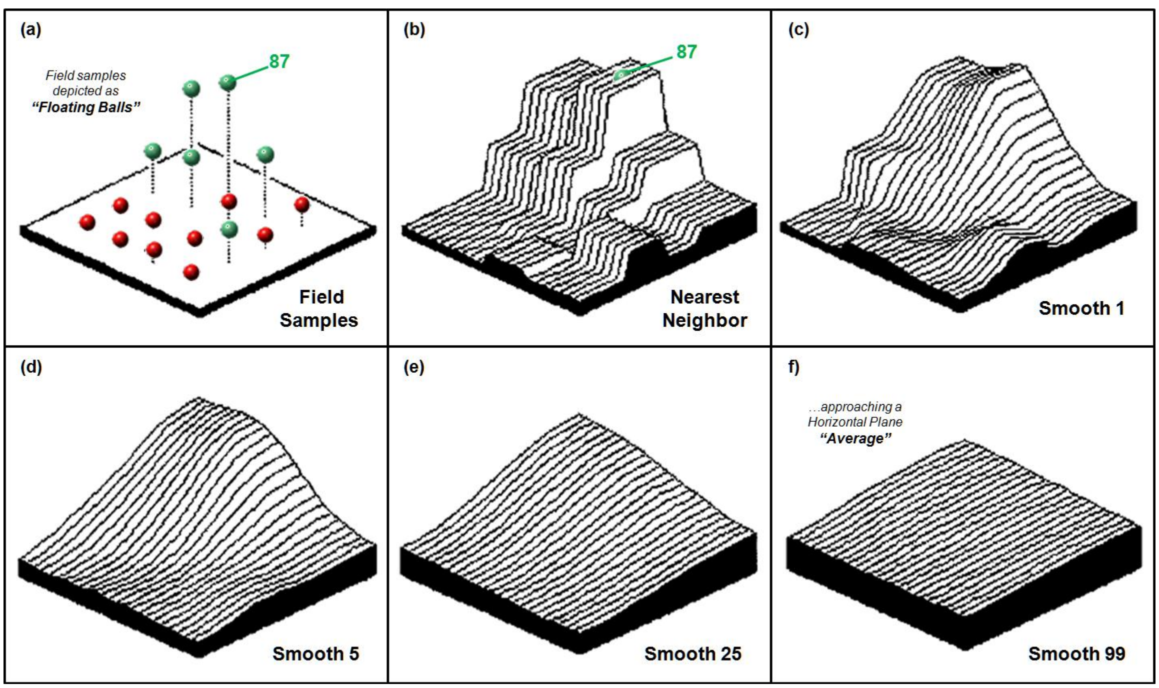

Figure 2. Iteratively smoothed approximations of the spatial distribution of a mapped data set.

So what did all that GIS gibberish accomplish? For one thing, you now have a greater appreciation of the potential and pitfalls in applying classical statistics to problems in geographic space. For another, you should have a better feel for a couple of techniques used to characterize the geographic distribution of a data set. (Topic 6 will take these minutiae to even more lofty heights. ... I bet you can't wait.) More importantly, however, now you know surf's up with your data sets.

Maneuvering on GIS's Sticky Floor

(GeoWorld, March 1994)

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Have you heard of the glass ceiling in organizational structure? In today's workplace there is an even more insidious pitfall holding you back: the sticky floor of technology. You are assaulted by cyber-speak and new ways of doing things. You might be good at what you do, but "they" keep changing what it is you do.

For example, GIS takes the comfortable world of mapping and data management into a surrealistic world of map-ematics and spatial statistics. The previous dicussions described how discrete *field data* can be used to generate a *continuous map surface* of the data. These surfaces extend the familiar (albeit somewhat distasteful) concept of central tendency to a map of the geographic distribution of the data. Whereas classical statistics identifies the typical value in a data set, spatial statistics identifies where you might expect to find unusual responses.

The previous section described *polynomial fitting* and *iterative smoothing* techniques for generating map surfaces. Now let's tackle a few more of the surface modeling techniques. But first, let's look for similarities in computational approaches among the various techniques. They all generate estimates of a mapped variable based on the data values within the vicinity of each map location. In effect, that establishes a roving window that moves about an area summarizing the field data it encounters. The summary estimate is assigned to the center of the window, then the window moves on. The extent of the window (both size and shape) sways the result, regardless of the summary technique. In general, a large window capturing a bunch of values tends to smooth the data. A smaller window tends to result in a rougher surface with abrupt transitions.

Three factors affect the window's extent: its reach, number of samples, and balancing. The *reach*, or search radius, sets a limit on how far the computer will go in collecting data values. The *number of samples* establishes how many data values should be used. If there are more than enough values within a specified reach, the computer uses just the closest ones. If there aren't enough values, it uses all it can find within the reach. *Balancing* consideration attempts to eliminate directional bias by ensuring that the values are selected in all directions around the window's center.

Once a window is established, the summary technique comes into play. *Inverse distance* is easy to conceptualize. It estimates a value for an unsampled location as an average of the data values within its vicinity. The average is weighted, so the influence of the surrounding values decreases with the distance from the location being estimated. Because that is a static averaging method, the estimated values never exceed the range of values in the original field data. Also, it tends to pull down peaks and pull up valleys in the data. Inverse distance is best suited for data sets with random samples that are fairly independent of their surrounding locations (i.e., no regional trend).

Inset (a) of figure 1 contains contour and 3-D plots of the inverse distance (squared) surface generated from the animal activity data described in the previous sections (Period 2 with 16 evenly spaced sampled values from 0 to 87). Note that the inverse distance technique is sensitive to sampled locations and tends to put bumps and pock-marks around these areas.

Opaquely speaking, kriging uses regional variable theory based on an underlying linear variogram. That's techy-speak implying that there is a lot of math behind this one. In effect, the technique develops a custom window based on the trend in the data. Within the window, data values along the trend's direction have more influence than values opposing the trend. The moving average that defines the trend in the data can result in estimated values that exceed the field data's range of values. Also, there can be unexpected results in large areas without data values. The technique is most appropriate for systematically sampled data exhibiting discernible trends.

The center portion of figure 1 depicts the kriging surface of the animal activity data. In general, it appears somewhat smoother than the inverse distance method's plot. Note that the high points in the same region of the map tend to be connected as ridges, and the low points are connected as valleys.

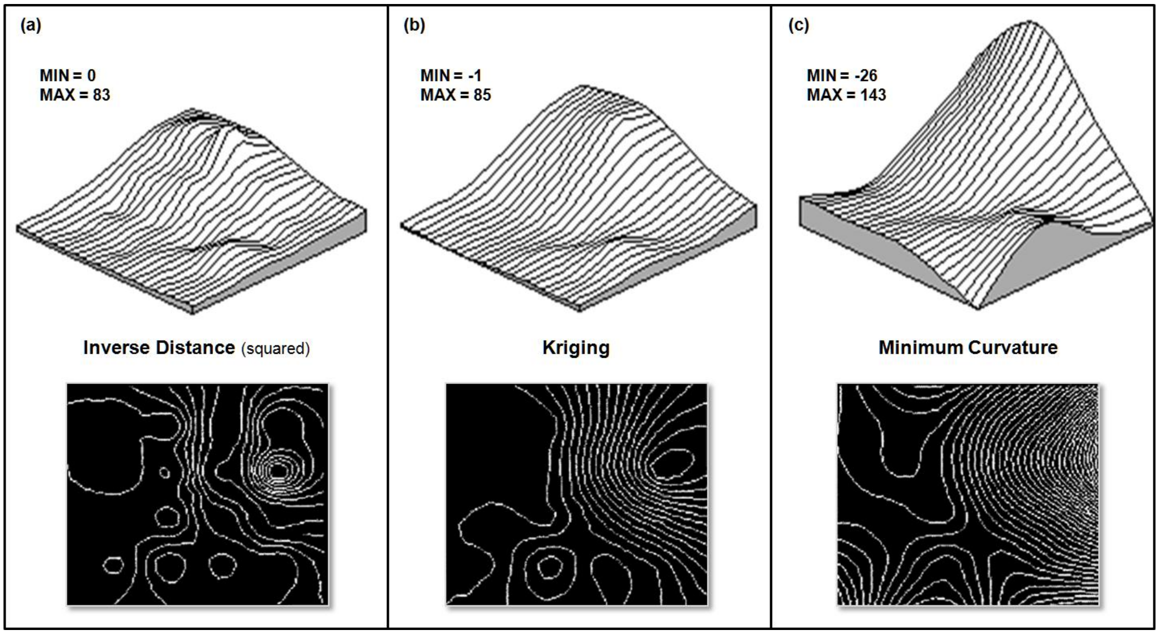

Figure 1. Comparison of Inverse Distance, Kriging and Minimum Curvature interpolation results. (Interpolation and 3D Surface and 2D Contour plots generated by SURFER, Golden Software)

Minimum curvature first calculates a set of initial estimates for all map locations based on the sampled data values. Similar to the iterative smoothing technique discussed in the previous section, minimum curvature, repeatedly applies a smoothing equation to the surface. The smoothing continues until successive changes at each map location are less than a specified maximum absolute deviation, or a maximum number of iterations has been reached. In practice, the process is done on a coarse map grid and repeated for finer and finer grid spacing steps until the desired grid spacing and smoothness are reached. As with kriging, the estimated values often exceed the range of the original data values and things can go berserk in areas without sample values.

The inset (c) of figure 1 contains the plots for the minimum curvature technique. Note that it's the smoothest of the three plots and displays a strong edge effect along the unsampled border areas.

You likely concede that GIS is sticky, but it also seems a bit fishy. The plots show radically different renderings for the same data set. So, which rendering is best? And how good is it? That discussion is reserved for later (see "Justifiable Interpolation," February 1997 describing the "Residual Analysis" procedure for assessing interpolation performance).

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