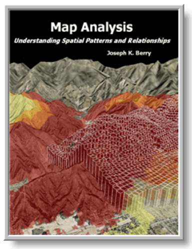
Beyond Mapping III

Topic 3: Considerations in Sampling Design Patterns and Relationships



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(*Back to the Table of Contents*)

What's the Point?

(*return to top of Topic*)

GIS's roots lie in computer mapping and spatial database management; however, for better or worse, more map-ematical applications are rapidly emerging. One such extension involves an excursion into the surrealistic realm of point sampling. For example, a soil nutrient map of the relative amounts of phosphorous throughout a farmer's field can be derived from a set of soil cores (a.k.a., point samples). Whereas the bulk of digital maps arise by the direct encoding of point, line and areal features, point sampling uses a series of dispersed discrete samples to characterize a continuous distribution of a mapped variable. The nature of the maps derived by the two approaches is radically different— a set of "discrete objects" for encoded maps versus a "continuous statistical estimate" for a spatially interpolated map.

The reliability of an encoded map primarily depends on the accuracy of the source document and the fidelity of the digitizer— which in turn is a function of the caffeine level of the prefrontal-lobotomized hockey puck pusher (sic). It's at its highest when GPS is used for "feet-down" digitizing as you stroll throughout a field and trip over a physical object. The reliability of a map based on point sampling, however, depends on the existence of spatial dependency within the data, the sampling design employed, and the interpolation algorithm applied. Several previous articles discussed interpolation algorithms (January through March, 1994), so now its time to turn our attention to spatial dependency and sampling design.

Spatial dependency within a data set simply means that "what happens at one location depends on what is happening around it" (formally termed positive spatial autocorrelation). It's this idea that forms the basis of statistical tests for spatial dependency. The Geary Index calculates the squared difference between neighboring sample values, then compares their summary to the overall variance for the entire data set. If the neighboring variance is a lot less than the overall, then considerable dependency is indicated. The Moran Index is similar, however it uses the products of neighboring values instead of the differences. A Variogram plots the similarity among locations as a function of distance.

Although these calculations vary and arguments abound about the best approach, all of them are reporting the degree of similarity among point samples. If there is a lot, then you can generate maps from the data; if there isn't much, then you are more than wasting your time. A pretty map can be generated regardless of the degree of dependency, but if dependency is minimal the map is just colorful gibberish… so don't bet the farm on it.

OK, let's say the data set you intend to map exhibits ample spatial autocorrelation. Your next concern is establishing a sampling frequency and pattern that will capture the variable's spatial distribution— sampling design issues. There are four distinct considerations in sampling design: 1) stratification, 2) sample size, 3) sampling grid, and 4) sampling pattern.

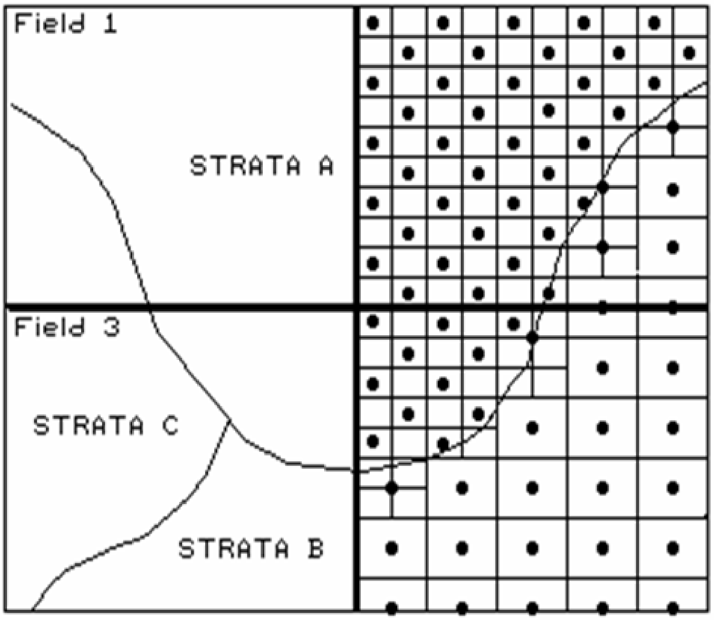

Figure 1. Variable sampling frequencies by soil strata for two fields.

The first three considerations determine the appropriate groupings for sampling (stratification), the sampling intensity for each group (sample size), and a suitable reference grid (sampling grid) for expressing the sampling intensity for each group (see figure 1). All three are closely tied to the spatial variation of the data to be mapped. Let's consider mapping phosphorous levels within a farmer's field. If the field contains a couple of soil types, you might divide it into two "strata." If previous sampling has shown one soil strata to be fairly consistent (small variance), you might allocate fewer samples than another more variable soil unit, as depicted in the accompanying figure.

Also, you might decide to generate a third stratum for even more intensive sampling around the soil boundary itself. Or, another approach might utilize mapped data on crop yield. If you believe the variation in yield is primarily "driven" by soil nutrient levels, then the yield map would be a good surrogate for subdivision of the field into strata of high and low yield variability. This approach might respond to localized soil conditions that are not reflected in the traditional (encoded) soil map.

Historically, a single soil sampling frequency has been used throughout a region, without regard for varying local conditions. In part, the traditional single frequency was chosen for ease and consistency of field implementation and simply reflects a uniform spacing intensity based on how much farmers are willing to pay for soil sampling.

Within GIS/GPS technology, variable sample frequency is a modern alternative. Whether you are sampling soil nutrients, water pollution or people's opinions with the intent of making a map, the sampling frequency should consider the spatial variation in the data, not just the automation of a traditional non-spatial sampling intensity. In addition, the sampling pattern should be spatially based… but that's another story.

Designer Samples

(GeoWorld, January 1997, pg. 30)

(return to top of Topic)

Last issue we briefly discussed spatial dependency and the first three steps in point sampling design—stratification, sample size and sampling grid. These considerations determine the appropriate areas, or groupings, for sampling (stratification), the sampling intensity for each group (sample size) and a suitable reference grid (sampling grid) for locating the samples. The fourth and final step "puts the sample points on the ground" by choosing a sampling pattern to identify individual sample locations.

Traditional, non-spatial statistics tends to emphasize randomized patterns as they insure maximum independence among samples… a critical element in calculating the central tendency of a data set (average for an entire field). However, "the random thing" can actually hinder spatial statistics' ability to map field variability. Arguments supporting such statistical heresy involve a detailed discussion of spatial dependency and autocorrelation, which (mercifully) is postponed to another issue. For current discussion, let's assume sampling patterns other than random are viable candidates.

Figure 1 identifies five systematic patterns, as well as a completely random one. Note that the regular pattern exhibits a uniform distribution in geographic space. The staggered start does so as well, except the equally spaced Y-axis samples alternate the starting position at one half the sampling grid spacing. The result is a "diamond" pattern rather than a "rectangular" one. The diamond pattern is generally considered better suited for generating maps as it provides more inter-sample distances for spatial interpolation. The random start pattern begins each column "transect" at a randomly chosen Y coordinate within the first grid cell, thereby creating even more inter-sample distances. The result is a fairly regularly spaced pattern, with "just a tasteful hint of randomness."

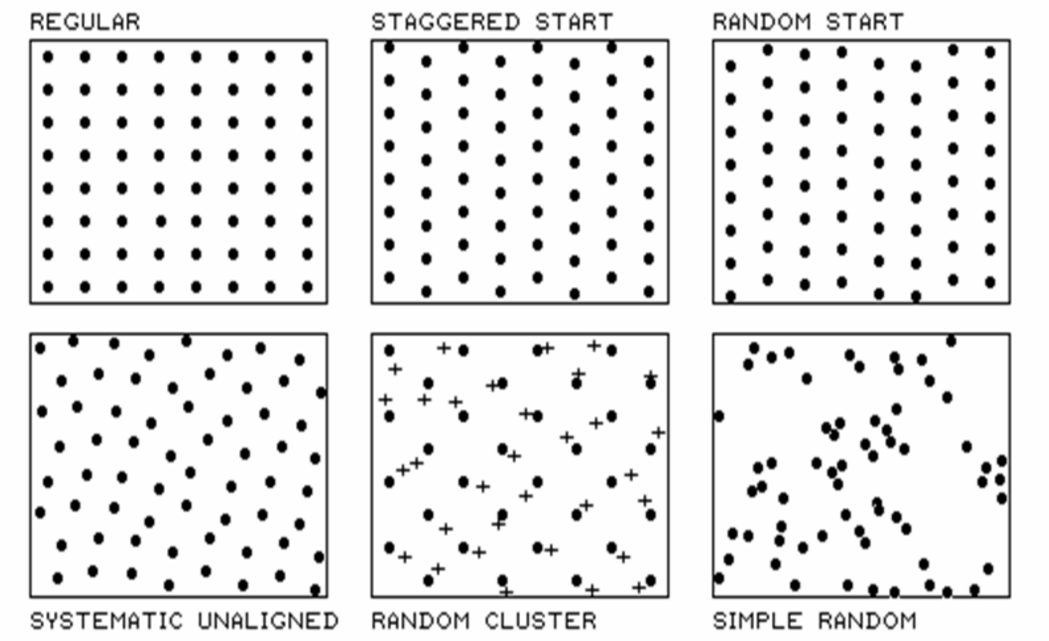

Figure 1. Basic spatial sampling patterns.

Systematic unaligned also results in a somewhat regularly spaced pattern, but exhibits even more randomness as it is not aligned in either the X or Y direction. A study area (i.e., farmer's field) is divided into a sampling grid of cells equal to the sample size. The pattern is formed by first placing a random point in the cell in the lower-left corner of the grid to establish a pair of X and Y offsets. Random numbers are used to specify the distance separating the initial point from the left border (termed the X-offset) and from the bottom border (termed the Y-offset).

For the bottom row of the sampling grid, the X-offset is held constant while Y is randomly varied. For the left-most column, the Y-offset is held constant while X is varied. Sample points are then placed in the remaining grid cells insuring that the X-offsets are the same along each row and the Y-offsets are the same along each column. The result is a set of sample points that are roughly equally spaced, but out of alignment.

The "dots" in the random cluster pattern establish an underlying uniform pattern (every other staggered start sample point in this example). The "crosses" locate a set of related samples that are randomly chosen (both distance and direction) within the enlarged grid space surrounding each regularly placed each dot. Note that the pattern is not as regularly spaced as the previous techniques, as half of the points are randomly set, however, it has other advantages. The random subset of points provides a foothold for a degree of unbiased statistical inference, such as a t-test of significance differences among population means.

The simple random pattern uses random numbers to establish X and Y coordinates within the entire study area. It allows full use of statistical inference (whole field non-spatial statistics), but the “clumping” of the samples results in large “gaps” thereby limiting its application for mapping (site-specific spatial statistics).

So which pattern should be used? Generally speaking, the Regular and Simple Random patterns are the worst for spatial analysis. If you have trouble locating yourself in space (haven’t bought into GPS yet?), then Random Start might get the nod. If you can freely navigate in space, then Systematic Unaligned might be best. But if you want to perform statistical inference, then Random Cluster might be considered. Non-generally speaking, you should use whichever pattern works best with your data and your objectives… how to tell which is best is left for another time.

Depending on the Data

(return to top of Topic)

Historically, maps have reported the precise position of physical features for the purpose of navigation. Not long after emerging from the cave, early man grabbed a stick and drew in the sand a route connecting the current location to the best woolly mammoth hunting grounds, neighboring villages ripe for pillaging, the silk route to the orient and the flight plan for the first solo around the world. The basis for the navigational foundation of mapping lies in referencing systems and the expression of map features as organized sets of coordinates. The basis for modern GIS, however, lies in the concept of spatial dependency.

GIS technology embraces the traditional aspects of mapping. In fact it greatly extends the access and ease of use in generating custom views of the mosaic of elements comprising our real world. But its revolutionary impacts involve how we perceive, conceptualize and communicate thoughts on the linkages and relative importance of spatially explicit relationships. The emphasis has moved from the mapping question of "where is what," to the social, economic, environmental (etc.) questions of "so what" and "what should we do where?"

The technical focus has been enlarged to include a growing set of procedures for discovering and expressing the dependencies within and among mapped data. Spatial dependency identifies relationships based on relative positioning. Certain trees tend to occur on certain soil types, slopes and climatic zones. Animals tend to prefer specific biological and contextual conditions. Particularly good sales prospects for luxury cars tend to cluster in a few distinct parts of a city. In fact, is anything randomly placed in geographic space? A rock, a bird, a person, a molecule?

There are two broad types of spatial dependency: 1) spatial variable dependence and 2) spatial relations dependence. Spatial variable dependency stipulates that what occurs at a map location is related to:

- the conditions of that variable at nearby locations (termed spatial autocorrelation); and/or

- the conditions of other variables at or around that location (termed spatial correlation).

Spatial autocorrelation forms the backbone of all interpolation techniques. They relate neighboring sample points to predict a variable response at unmeasured locations. If the neighboring points are spatially independent, there is little justification for generating a map surface. For example, if the elevation is 100 feet here and you note that it is 50 over there, there has to be at least one location of 75 feet in between. That's because elevation forms a highly autocorrelated gradient in geographic space. True, it might be that the 75 foot location is part way down the precipice a foot in front of you, but it has to exist. The continuum is there, it's just that the functional form isn't a simple linear transition between the points.

Contrast an elevation surface with a map of roads. If you're standing on a heavy duty, road-type 4 (watch out for buses) and note a light duty road-type 1 over there, it is absurd to assume that there is a road-type 2.5 somewhere in between. Two spatial autocorrelation factors are at play— formation of a spatial gradient and existence of partial states. Neither make sense for the occurrence of the discrete map objects forming a road map (termed a choropleth map).

Both factors make sense for an elevation surface (termed an isopleth map). But spatial autocorrelation isn't black and white, present or not present; it occurs in varying degrees for different map types and spatial variables. The degree to which a map exhibits intra-variable dependency determines the nature and strength of the relationships one can derive about its geographic distribution.

In a similar manner, inter-variable dependency affects our ability to track spatial relationships. Spatial correlation forms the basis for mapping relationships among maps. For example, in the moisture limited ecosystems of Colorado, spruce and fir forests most often occur on northern slopes, while sparse pine forests tend to occur on the dry southern exposures. Soil conditions and depth play a big part in forest vitality, as well the frequency of catastrophic events, such as fire. Animal, bird, insect and micro-organism populations are strongly dependent on terrain and forest conditions. Boy scouts and resource managers have known these general "rules" of spatial coincidence for years. Scientists have written about them for years. What has changed are the "tools" for deriving, verifying and applying more detailed and spatially explicit relationships.

Historically, scientists have used sets of discrete samples to investigate relationships among field plots on the landscape, in a manner similar to Petri dishes on a laboratory table— each sample is assumed to be spatially independent. GIS technology provides the geo-referencing necessary to align sets of map variables and the analytic tools to investigate correlation among their spatial patterns. For example, a map of animal activity can be statistically related to a set of habitat determinant maps, such as terrain form and vegetative cover, on a pure coincidence basis (point-by-point).

Introduction of neighboring conditions, such as proximity to water and cover type diversity, expands the simple alignment analysis to one of spatial context. The derived relationship can be empirically verified by generating a map predicting animal activity for another area and comparing it to known animal activity within that area. Once established, the verified spatial relationship can be used directly by managers in their operational GIS. In fact the cultural alignment of science and management activities brought about by GIS technology (i.e., same "tool" used by both scientists and managers) might prove to be as significant as the technical capability to align map layers for analysis. At minimum, it should expedite the linkages among basic science, applied research and technology transfer.

The other broad type of spatial dependency involves the nature of the relationship itself. Spatial relations dependency stipulates that relationships among mapped variables can be:

- constant throughout space and time (termed spatial homogeneity); or
- variable as a function of space and/or time (termed spatial heterogeneity).

Very few relationships exhibit pure spatial homogeneity by remaining constant over space and time. Even the "laws" of thermal dynamics have conditional boundaries— water freezes at zero degrees centigrade… as long as it is pure and at sea level. Pour in some salt and carry it to top of a mountain and it will freeze at a different kinetic temperature. Similarly, most spatial relationships exhibit spatial heterogeneity and vary to some degree with space and time.

For example, a habitat unit across a river might be considered disjoint and inaccessible to a non-swimming and flightless animal. However, if the river freezes in the winter, then the spatial relationships defining habitat needs to change with the seasons. Similarly, a forest growth model developed for Colorado might be inappropriate for application in Oregon. It's likely the basic map layers and logic structuring the model are identical, but the relative weights assigned to the growth functions vary with geographic regions and the model must be "tuned" for local variants.

The complexities of spatial dependence are not unique to resource models. GIS modeling of product sales, traffic flows, and voter patterns exhibit varying degrees of spatial variable and relations dependencies. Awareness of these concepts and their effects are as important to modern GIS as understanding data structures and geo-referencing procedures— possibly even more important?

Uncovering the Mysteries of Spatial Autocorrelation

(return to top of Topic)

This article violates all norms of journalism, as well as common sense. It attempts to describe an admittedly complex technical subject without the prerequisite discussion of the theoretical linkages, provisional statements, and enigmatic equations. I apologize in advance to the statistical community for the important points left out of the discussion… and to the rest of you for not leaving out more. The last article identified spatial autocorrelation as the backbone of all interpolation techniques used to generate maps from point sampled data. The term refers to the degree of similarity among neighboring points. If they exhibit a lot similarity, or spatial dependence, then they ought to derive a good map; if they are spatially independent, then expect pure, dense gibberish. So how do we measure whether "what happens at one location depends on what is happening around it?"

Previous discussion (GeoWorld, December 1996) introduced two simple measures to determine whether a data set has what it takes to make a map— the Geary and Moran indices. The Geary index looks at the differences in the values between each sample point and its closest neighbor. If the differences in neighboring values tend to be less than the differences among all values in the data set, then spatial autocorrelation exists. The mathematical mechanics are easy (at least for a tireless computer)— 1) add up all of the differences between each location's value and the average for the entire data set (overall variation), 2) add up all of the differences between values for each location and its closest neighbor (neighbors variation), then 3) compare the two summaries using an appropriately ugly equation to account for "degrees of freedom and normalization."

If the neighbors differences are a lot smaller than the overall variation, then a high degree of positive spatial dependency is indicated. If they are about the same, or if the neighbors variation is larger (a rare "checkerboard-like" condition), then the assumption that "close things are more similar" fails… and, if the dependency test fails, so will the interpolation of the data. The Moran index simply uses the products between the values, rather than the differences to test the dependency within a data set. Both approaches are limited, however, as they merely assess the closest neighbor, regardless of its distance.

That's where a variogram comes in. It is a plot (neither devious nor spiteful) of the similarity among values based on the distance between them. Instead of simply testing whether close things are related, it shows how the degree of dependency relates to varying distances between locations. Most data exhibits a lot of similarity when distances are small, then progressively less similarity as the distances become larger.

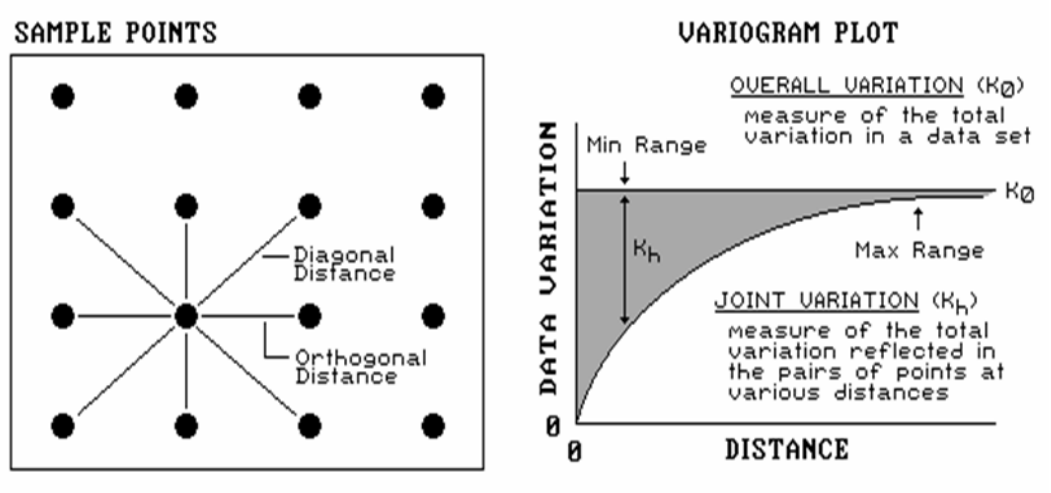

Figure 1. Plot of the similarity among sample points as a function of distance (shaded portion) shows whether interpolation of the data is warranted.

In Figure 1, you would expect more similarity among the neighboring points (shown by the lines), than sample points farther away. Geary and Moran consider just the closest neighbors (orthogonal distances of above, below, right and left for the regular grid sampling design). A variogram shows the dependencies for other distances, or spatial frequencies, contained in the data set (such as the diagonal distances). If you keep track of the multitude of distances connecting all locations and their respective differences, you end up with a huge table of data relating distance to similarity. In this case, the overall variation in a data set (termed the variance) is compared to the joint variation (termed the covariance) for each set of distances. For example, there is a lot of points that are "one orthogonal step away" (four for the example point). If we compute the difference between the values for all the "one-steppers," we have a measure reminiscent of Moran's "neighbors variation"— differences among pairs of values.

A bit more "mathematical conditioning" translates this measure into the covariance for that distance. If we focus our attention on all of the points "a diagonal step away" (four around the example point), we will compute a second similarity measure for points a little farther away. Repeating the joint variation calculations for all of the other spatial frequencies (two orthogonal steps, two diagonal steps, etc.), results in enough information to plot the variogram shown in the figure.

Note the extremes in the plot. The top horizontal line indicates the total variation within the data set (overall variation; variance). The origin (0,0) is the unique case for distance=0 where the overall variation in the data set is identical to the joint variation as both calculations use essentially the same points. As the distance between points is increased, subsets of the data are scrutinized for their dependency (joint variation; covariance). The shaded portion in the plot shows how quickly the spatial dependency among points deteriorates with distance.

The maximum range position identifies the distance between points beyond which the data values are considered to be independent of one another. This tells us that using data values beyond this distance for interpolation is dysfunctional (actually messes-up the interpolation). The minimum range position identifies the smallest distance (one orthogonal step) contained in the data set. If most of the shaded area falls below this distance, it tells you there is insufficient spatial dependency in the data set to warrant interpolation.

True, if you proceed with the interpolation a nifty colorful map will be generated, but it'll be less than worthless. Also true, if we proceed with more technical detail (like determining optimal sampling frequency and assessing directional bias in spatial dependency), most this column's readership will disappear (any of you still out there?).

*(**return to top of Topic**)*

*(**Back to the Table of Contents**)*