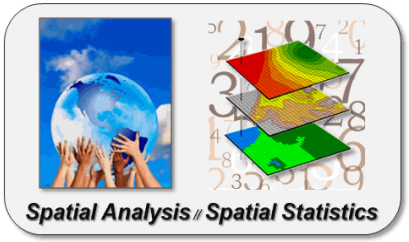
*Spatial*STEM:

A Mathematical/Statistical Framework for Understanding and Communicating Map Analysis and Modeling



<u>Premise</u>: There is a "map-*ematics*" that extends traditional math/stat concepts and procedures for the quantitative analysis of map variables (spatial data)

This presentation provides a fresh perspective on interdisciplinary instruction at the college level by combining the philosophy and approach of *STEM* with the <u>spatial reasoning and analytical power of grid-based Map Analysis and Modeling</u>

This PowerPoint with notes and online links to further reading is posted at

www.innovativegis.com/basis/Courses/SpatialSTEM/Workshop/

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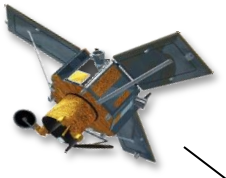
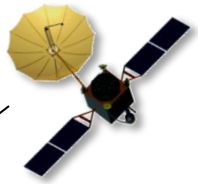
(Nanotechnology) **Geotechnology** *(Biotechnology)*

Geotechnology is one of the three "mega technologies" for the 21st century and promises to forever change how we conceptualize, utilize and visualize spatial relationships in scientific research and commercial applications *(U.S. Department of Labor)*



Remote Sensing
(measure and classify)

Geographic Information Systems ***(map and analyze)***



Global Positioning System ***(location and navigation)***

GPS/GIS/RS

The Spatial Triad

Computer Mapping (70s)
Spatial Database Management (80s)

Technological Tool

Mapping *involves precise placement (delineation) of physical features*

(graphical inventory)

Descriptive Mapping

is

Where **What**

Why **So What** and **What If**

Prescriptive Modeling

Map Analysis (90s)
Multimedia Mapping (00s)

Analytical Tool

Modeling *involves analysis of spatial patterns and relationships*

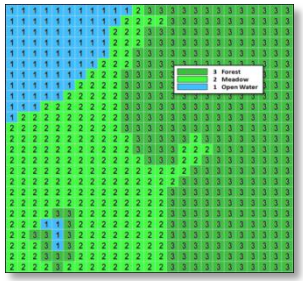
(map analysis/modeling)

(Berry)

A Mathematical Structure for Map Analysis/Modeling

Technological Tool — Geotechnology → RS – GIS – GPS — **Analytical Tool**

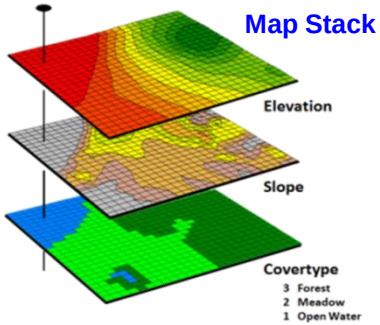
Mapping/Geo-Query *(Discrete, Spatial Objects)* (*Continuous, Map Surfaces*) **Map Analysis/Modeling**



…organized set of numbers

Geo-registered
Analysis Frame → Matrix
of Numbers

"**Map-*ematics***"



Maps as Data, not Pictures

*Vector & **Raster** — Aggregated & **Disaggregated***
*Qualitative & **Quantitative***

Spatial Analysis Operations

Grid-based
Map Analysis
Toolbox

Spatial Statistics Operations

GISer's Perspective:

Reclassify** and **Overlay
Distance** and **Neighbors

Mathematician's Perspective:

Basic GridMath & Map Algebra
Advanced GridMath
Map Calculus
Map Geometry
Plane Geometry Connectivity
Solid Geometry Connectivity
Unique Map Analytics

The SpatialSTEM Framework

Traditional math/stat procedures can be extended into **geographic space** to stimulate those with diverse backgrounds and interests for…

"***thinking analytically with maps***"

GISer's Perspective:

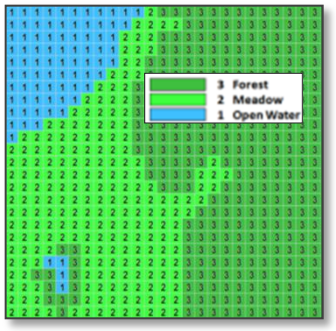
Surface Modeling
Spatial Data Mining

Statistician's Perspective:

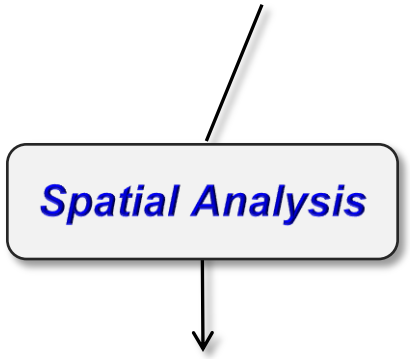
Basic Descriptive Statistics
Basic Classification
Map Comparison
Unique Map Statistics
Surface Modeling
Advanced Classification
Predictive Statistics

Spatial Analysis Operations *(Geographic Context)*

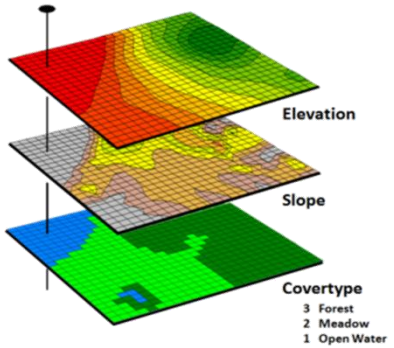
GIS as "Technical Tool" *(Where is What)* vs. "**Analytical Tool**" *(Why, So What and What if)*



Grid Layer



Map Stack



Spatial Analysis extends the basic set of discrete map features (points, lines and polygons) to map **surfaces** that represent continuous geographic space as a set of contiguous grid cells (matrix), thereby providing a **Mathematical Framework** for *map analysis* and *modeling* of the

Contextual Spatial Relationships within and among grid map layers

Map Analysis Toolbox

✓ *Unique spatial operations*

Mathematical Perspective:

Basic GridMath & Map Algebra *(+ - * /)*

Advanced GridMath *(Math, Trig, Logical Functions)*

Map Calculus *(Spatial Derivative, Spatial Integral)*

Map Geometry *(Euclidian Proximity, Effective Proximity, Narrowness)*

Plane Geometry Connectivity *(Optimal Path, Optimal Path Density)*

Solid Geometry Connectivity *(Viewshed, Visual Exposure)*

✓ ***Unique Map Analytics*** *(Contiguity, Size/Shape/Integrity, Masking, Profile)*

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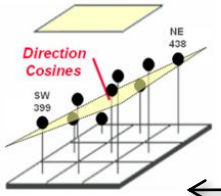
Spatial Analysis Operations *(Math Examples)*

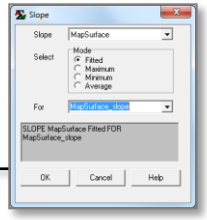
Advanced Grid Math — *Math, Trig, Logical Functions*

Map Calculus — *Spatial Derivative, Spatial Integral*

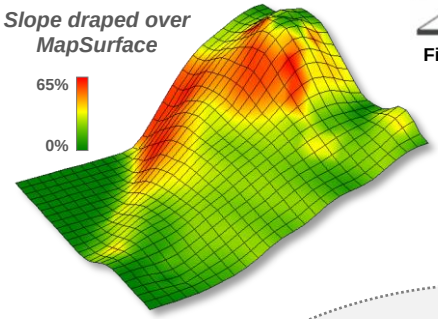
Spatial Derivative

…is equivalent to the slope of the ***tangent plane*** *at a location*



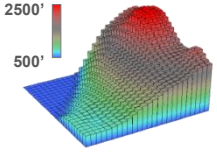


SLOPE MapSurface ***Fitted*** *FOR MapSurface_slope*



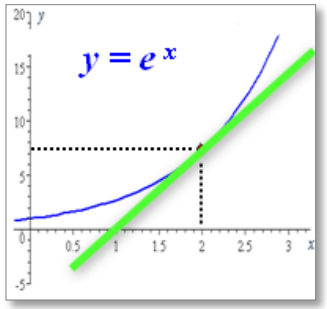
$D_z$xy Elevation

MapSurface



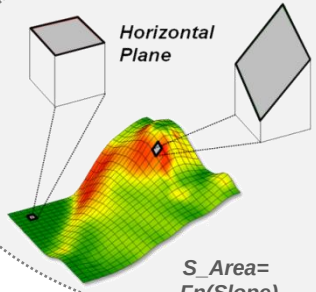
Surface

Curve



The ***derivative*** *is the instantaneous "rate of change" of a function and is equivalent to the slope of the* ***tangent line*** *at a point*

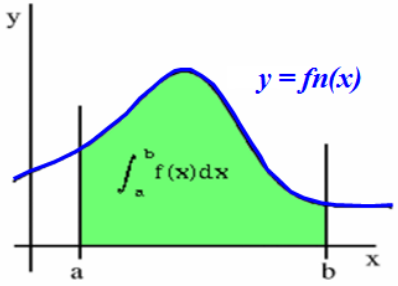
Advanced Grid Math



Surface Area

…increases with increasing inclination as a Trig function of the ***cosine of the slope angle***

S_area= cellsize / cos($D_z$xy Elevation)

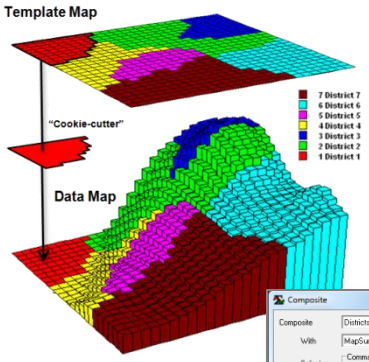


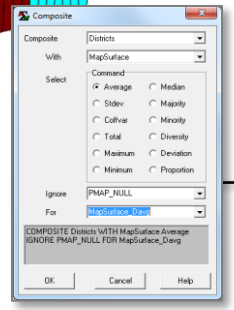
The ***integral*** *calculates the* ***area*** *under the curve for any section of a function.*

∫ *Districts_Average* Elevation

Spatial Integral

…summarizes the values on a surface for specified map areas (Total= ***volume*** *under the surface)*



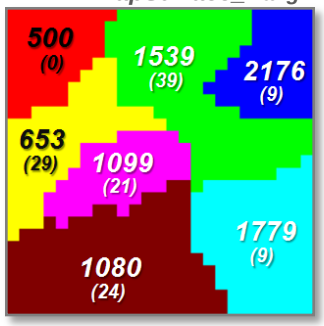


Surface

Curve

COMPOSITE Districts WITH MapSurface ***Average*** *FOR MapSurface_Davg*

MapSurface_Davg



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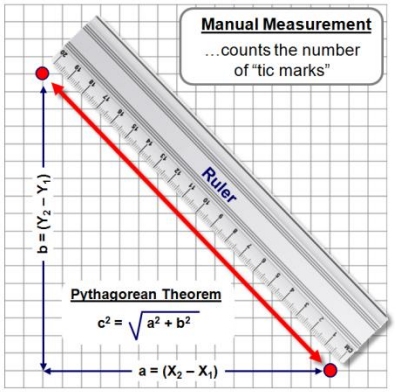
Spatial Analysis Operations *(Distance Examples)*

Map Geometry *— (Euclidian Proximity, Effective Proximity, Narrowness)*

Plane Geometry Connectivity *— (Optimal Path, Optimal Path Density)*

Solid Geometry Connectivity *— (Viewshed, Visual Exposure)*

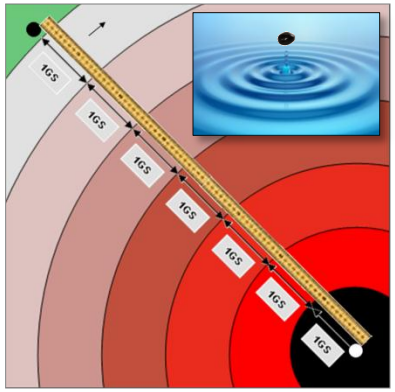
Distance



Mathematical Measurement
…solves the Pythagorean Theorem

Shortest straight line between two points…

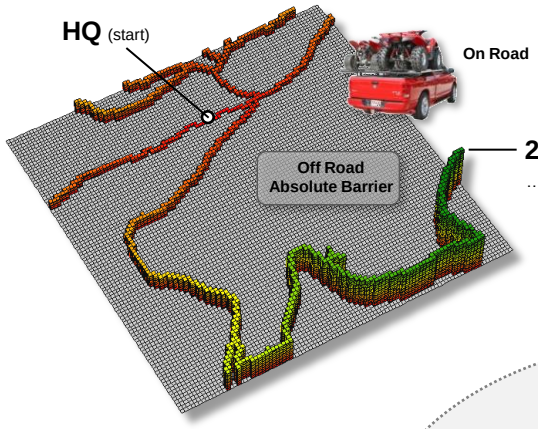
Euclidean Proximity



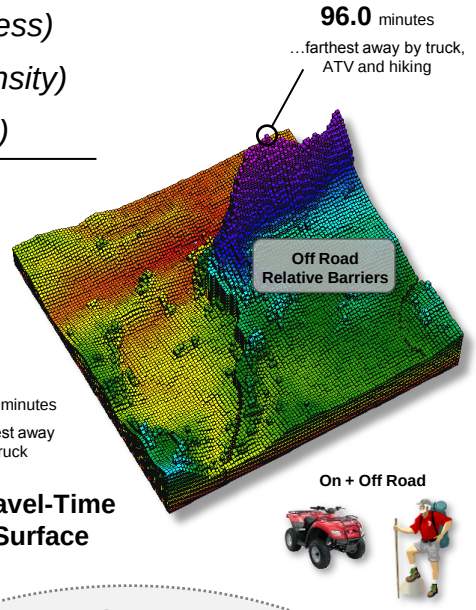
Splash Algorithm
…like tossing a rock into a pond— ripples

…from a point to everywhere…

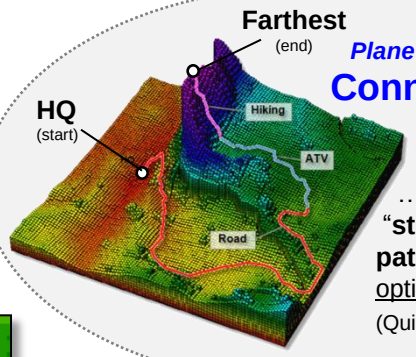
Effective Proximity



…not necessarily straight lines (movement)



Travel-Time Surface



Plane Geometry
Connectivity

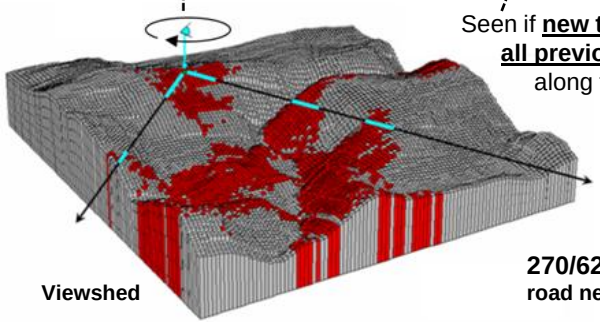
Truck = 18.8 min
ATV = 14.8 min
Hiking = 62.4 min

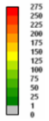
…like a raindrop, the "**steepest downhill path**" identifies the optimal route
(Quickest Path)

Solid Geometry **Connectivity**

Tan = Rise/Run

Seen if **new tangent exceeds all previous tangents** along the line of sight

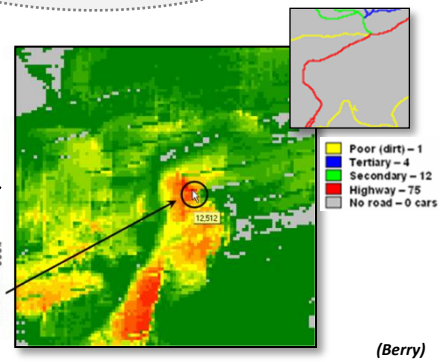




270/621= 43% of the entire road network is connected

← **Counts**
Viewers

Sums
Viewer
Weights →



Highest Weighted Exposure

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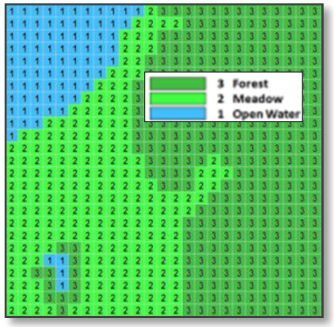
Spatial Statistics Operations *(Numeric Context)*

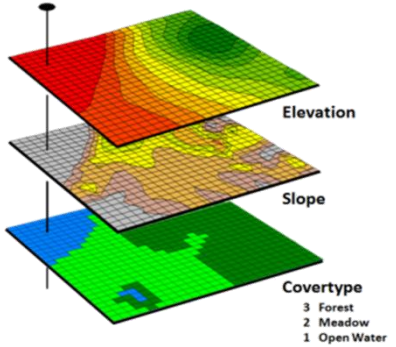
GIS as "Technical Tool" *(Where is What)* vs. "**Analytical Tool**" *(Why, So What and What if)*



Grid Layer

Spatial Statistics

Map Stack



Spatial Statistics seeks to map the **variation** in a data set instead of focusing on a single typical response (central tendency), thereby providing a **Statistical Framework** for *map analysis* and *modeling* of the

Numerical Spatial Relationships within and among grid map layers

Map Analysis Toolbox

✓ *Unique spatial operations*

Statistical Perspective:

Basic Descriptive Statistics *(Min, Max, Median, Mean, StDev, etc.)*

Basic Classification *(Reclassify, Contouring, Normalization)*

Map Comparison *(Joint Coincidence, Statistical Tests)*

✓ ***Unique Map Statistics*** *(Roving Window and Regional Summaries)*

✓ ***Surface Modeling*** *(Density Analysis, Spatial Interpolation)*

Advanced Classification *(Map Similarity, Maximum Likelihood, Clustering)*

Predictive Statistics *(Map Correlation/Regression, Data Mining Engines)*

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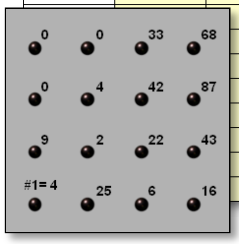
Spatial Statistics *(Linking Data Space with Geographic Space)*

Geo-registered Sample Data

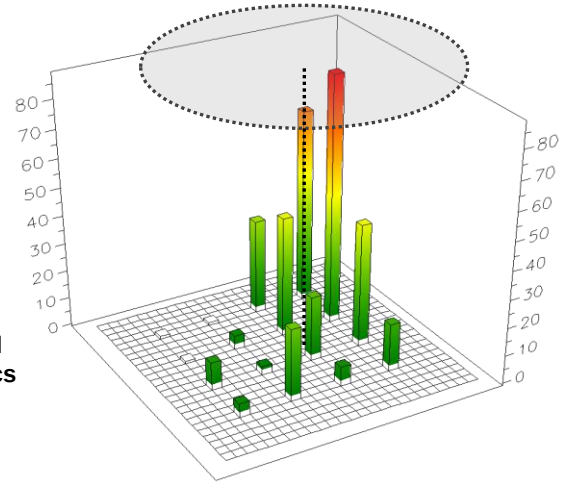
Sample	X	Y	P1	P2
#1	1000	1000	11	4
2	1000	1500	19	9
3	1000	2000	8	0
4	1000	2500	0	0
5	1500	1000	27	25
6	1500	1500	12	2
7	1500	2000	14	4
8	1500	2500	2	0
		1000	10	6
		1500	17	22
		2000	34	42
		2500	22	33
		1000	20	16
		1500	28	43
		2000	42	87
		2500	34	68



Non-Spatial Statistics

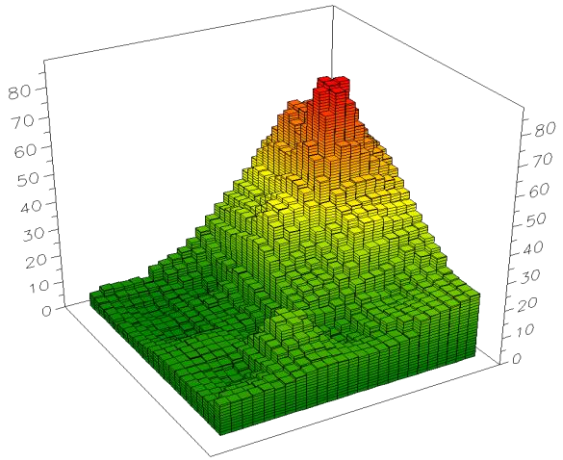
Spatial Statistics

Roving Window (weighted average)



Discrete Sample Map

Spatial Distribution

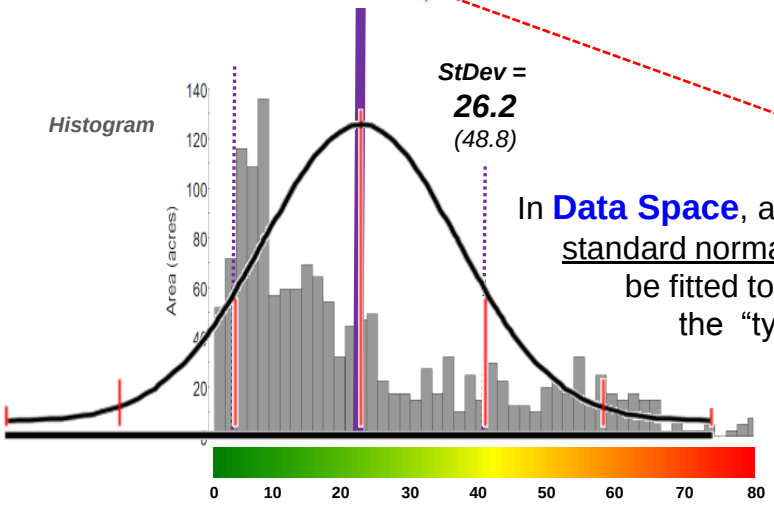


Continuous Map Surface

Surface Modeling techniques are used to derive a continuous map surface from discrete point data– fits a *Surface* to the data (maps the variation).

Standard Normal Curve

Average = 22.6

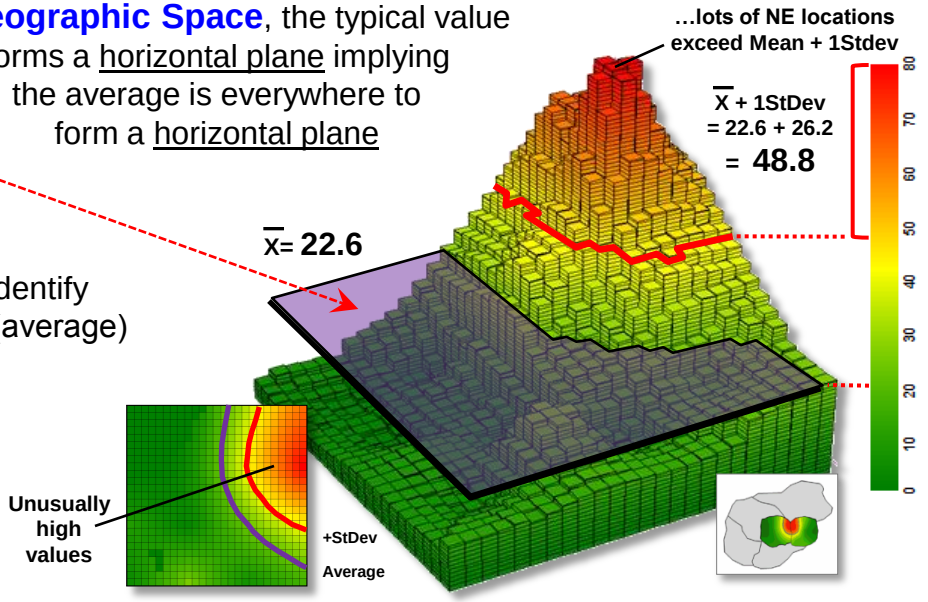


StDev = 26.2 (48.8)

In **Data Space**, a standard normal curve can be fitted to the data to identify the "typical value" (average)

Numeric Distribution

In **Geographic Space**, the typical value forms a horizontal plane implying the average is everywhere to form a horizontal plane



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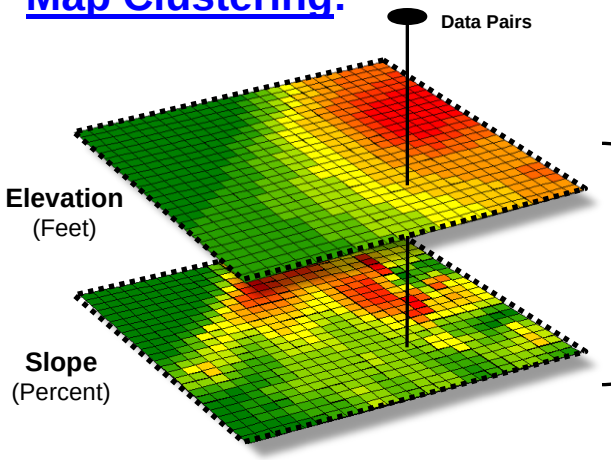
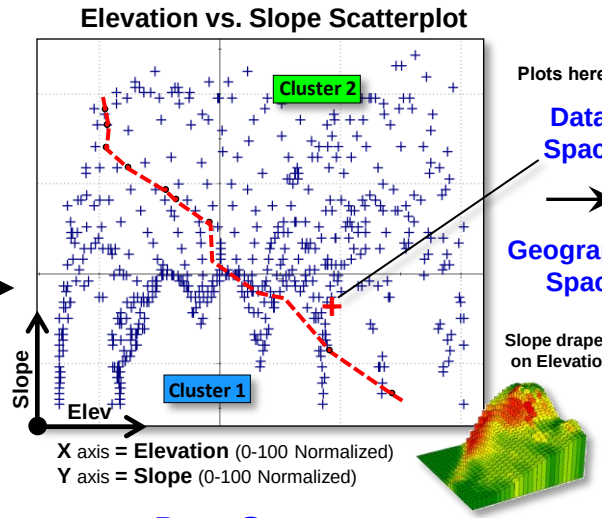
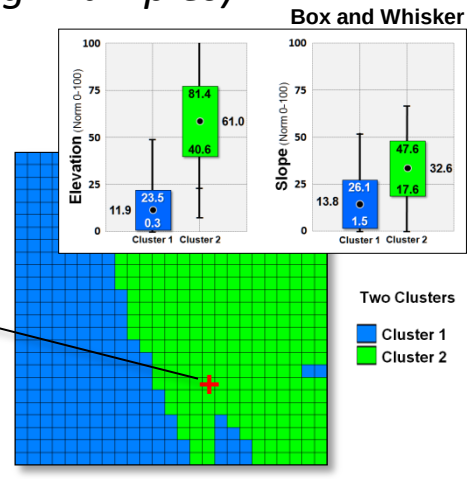
Spatial Statistics Operations *(Data Mining Examples)*

Map Clustering:





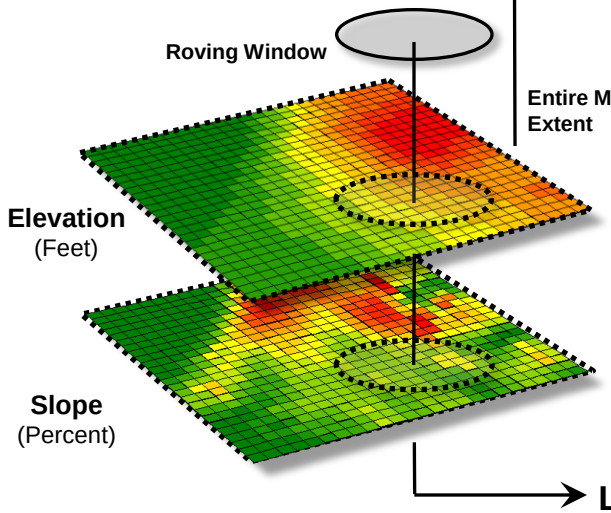


Advanced Classification *(Clustering)*

Data Space

Geographic Space

Map Correlation:



Spatially Aggregated Correlation

Scalar Value – one value represents the overall non-spatial relationship between the two map surfaces

…1 large data table
with 25rows x 25 columns =
625 map values for **map wide summary**

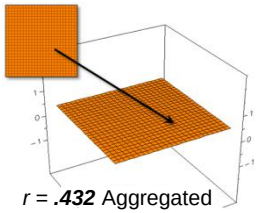
$$r = \frac{n\Sigma xy - (\Sigma x)*(\Sigma y)}{\mathrm{Sqrt}([n\Sigma x^2 - (\Sigma x)^2]*[n\Sigma y^2 - (\Sigma y)^2])}$$

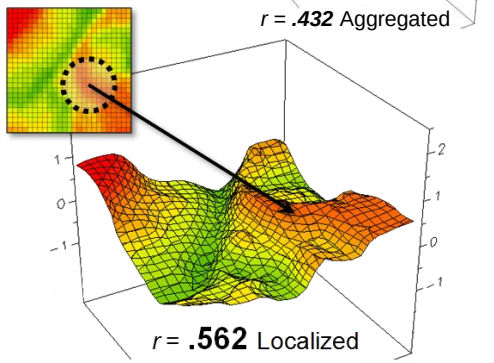
…where **x** = Elevation value and **y** = Slope value and **n** = number of value pairs

…625 small data tables
within 5 cell reach =
81map values for **localized summary**

Localized Correlation

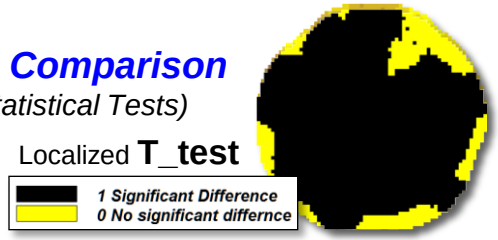
Map Variable – continuous quantitative surface represents the localized spatial relationship between the two map surfaces





Map Comparison
(Statistical Tests)

Localized **T_test**



Predictive Statistics *(Correlation)*

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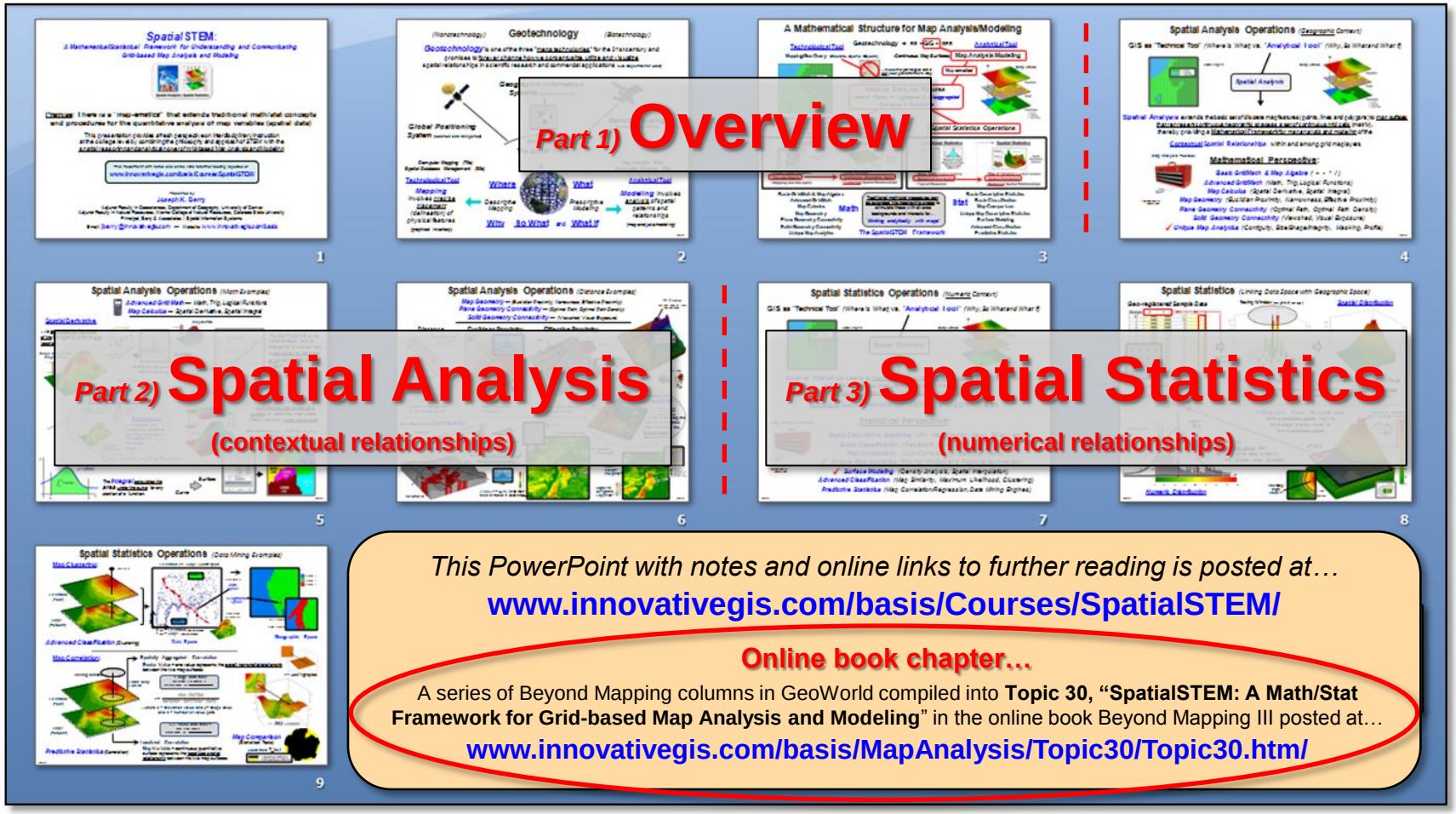
PowerPoint and Online Book Chapter on SpatialSTEM



Further Reading

Making a Case for SpatialSTEM — *a 15-page white paper describing a framework for grid-based map analysis and modeling concepts and procedures*
http://www.innovativegis.com/basis/Papers/Other/SpatialSTEM/SpatialSTEM_case.pdf

SpatialSTEM: Extending Traditional Mathematics and Statistics to Grid-based Map Analysis and Modeling — *white paper describing an innovative approach for teaching map analysis and modeling fundamentals within a mathematical/statistical context*
http://www.innovativegis.com/basis/Papers/Other/SpatialSTEM/SpatialSTEM_extendedcase.pdf

Further *Spatial*STEM Readings — *a comprehensive appendix to the SpatialSTEM "extended readings" with URL links to over 125 additional readings on the grid-based map analysis/modeling concepts, terminology, considerations and procedures described in the papers on SpatialSTEM*
http://www.innovativegis.com/Basis/Courses/SpatialSTEM/sSTEMreading.pdf

Part 4) Future Directions