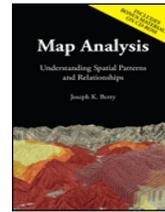


## Topic 8 – Spatial Modeling Example (Further Reading)



Map Analysis book

### (Extending Optimal Path Analysis)

['Straightening' Conversions Improve Optimal Paths](#) — discusses a procedure for spatially responsive straightening of optimal paths (November 2004)

[Use LCP Procedures to Center Optimal Paths](#) — discusses a procedure for eliminating “zig-zags” in areas of minimal siting preference (March 2006)

[Connect All the Dots to Find Optimal Paths](#) — describes a procedure for determining an optimal path network from a dispersed set of end points (September 2005)

### (Understanding Accumulation Surfaces)

[Building Accumulation Surfaces](#) — reviews how proximity analysis and effective distance is used to construct accumulation surfaces (October 1997)

[Analyzing Accumulation Surfaces](#) — describes how two surfaces can be analyzed to determine the relative travel-time advantages (November 1997)

[Determining Optimal Path Corridors](#) — describes a technique for determining the set of  $n^{\text{th}}$  best paths between two points (December 1997)

[Analyzing Stepped Accumulation Surfaces](#) — describes a technique for forcing an optimal path through a series of points (January 1998)

[<Click here>](#) for a printer-friendly version of this topic (.pdf).

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### (Extending Optimal Path Analysis)

## 'Straightening' Conversions Improve Optimal Paths

(GeoWorld, November 2004)

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Previous discussions (July through August 2003 BM columns) have addressed the basics of optimal path routing. The basic *Least Cost Path (LCP)* method described consists of three basic steps: *Discrete Cost Map*, *Accumulated Cost Surface* and *Optimal Route* (see Author's Notes).

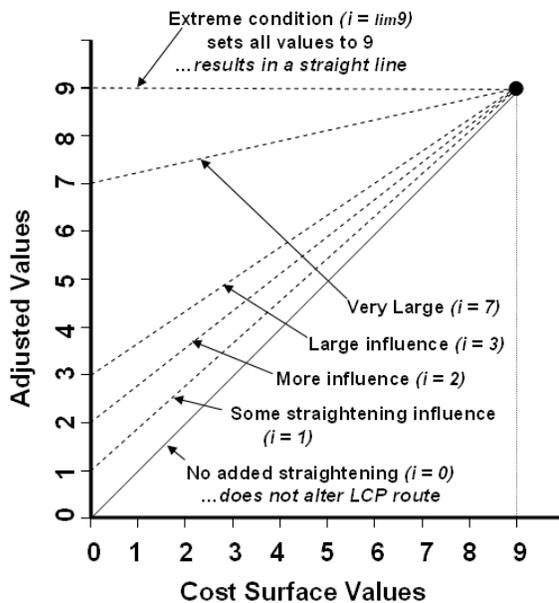
In addition, an optional fourth step to derive an *Optimal Corridor* can be performed to indicate routing sensitivity throughout a project area.

Key to optimal path analysis is the discrete cost map that establishes the relative “goodness” for locating a route through any grid cell in a project area. However, the traditional LCP approach only considers the landscape/terrain conditions between start and end points without direct consideration of differences in the feature itself, such as material costs.

For example, the same optimal solution is generated for both low and high priced pipe in constructing a pipeline. In the case of higher priced pipe, a straighter route can save millions of dollars. What is needed is a modified LCP procedure that considers feature characteristics as well as landscape/terrain conditions in optimizing route layout.

Simple geometry-based techniques of line smoothing, such as spline function fitting, are inappropriate as they fail to consider intervening conditions and can result in the route being adjusted into unsuitable locations as it is “smoothed.” The following discussion describes a robust technique for “straightening” an optimal path that works within the LCP framework.

The approach modifies the discrete cost map by making disproportional increases to the lower map values. This has the effect of straightening the characteristic minor swings in routing in the more favorable areas (low values) while continuing to avoid unsuitable areas.



**Traditional LCP Routing:** identifies the optimal path (shortest effective distance) considering a map of relative landscape/terrain “cost” for locating a route anywhere within a project area.

**Problem:** need to modify the traditional LCP procedure to consider feature cost where the routing procedure favors straighter paths for different feature types (e.g., relative pipe cost in routing a pipeline).

**Solution:** the equation  $Y = i + ((9 - i) / 9) * X$  is used to update the discrete cost map by preferentially raising the “good” locations (lower values). The result is an increasingly straighter path as the straightening factor is increased:  $i = 0$ , no added influence to  $i = 1$ , some straightening influence through  $i = \lim 9$ , total influence (straight line).

Figure 1. The LCP Straightening Equation progressively adjusts the relative importance of straightening the optimal path solution.

The left side of figure 1 graphically depicts the modified LCP approach. The solid diagonal line indicates a 1 to 1 conversion that does not change the discrete cost values (1= very good to 9=

very bad for locating a route). The dashed lines indicate conversions that incrementally increase the cost values with the greatest adjustments occurring at the lower values.

The general conversion equation based on an anchored straight line is—

$$Y = i + ((9 - i) / 9) * X$$

...and can be extended to the LCP Straightening Equation—

$$Adj\_Cost\_Map = i + ((9 - i) / 9) * Cost\_Map$$

...where all of the values on the discrete cost map are converted. The adjusted cost map is used to derive an accumulated cost surface from the starting location, that in turn is used to derive the optimal path from the ending location.

Figure 2 shows several examples of applying the LCP Straightening Equation. The top portion of the figure identifies the derivation of the optimal path using the standard LCP procedure. Note how the traditional solution has numerous “hooks, curves and deflections” responding to subtle differences in the shape of the accumulation surface based on the lower values of the discrete cost map.

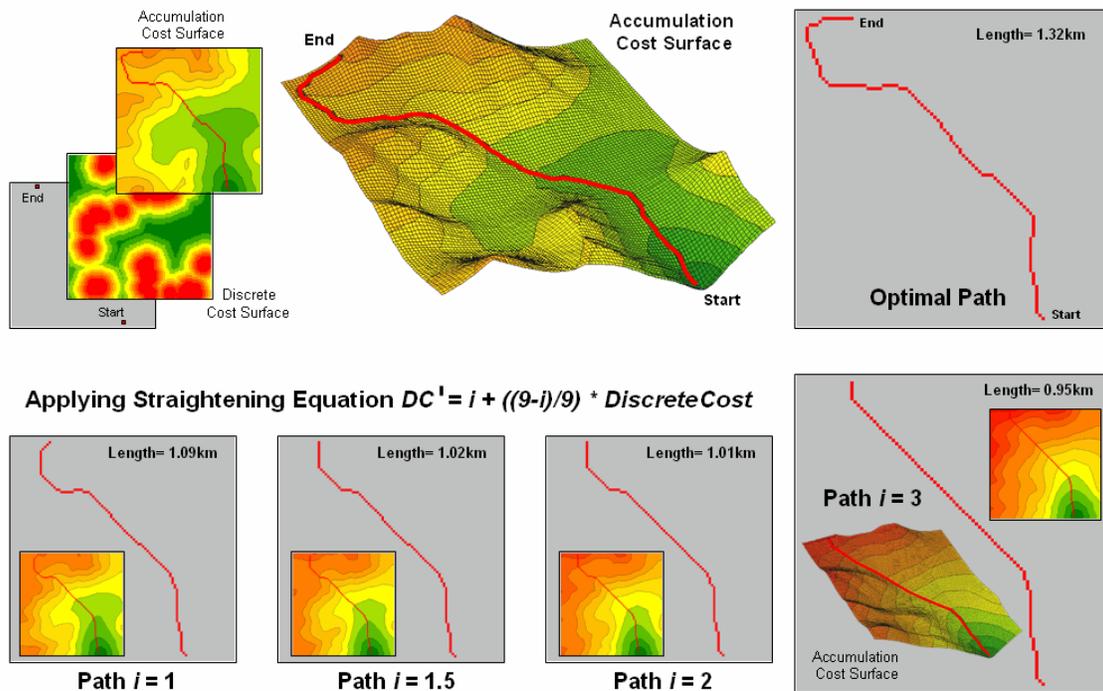


Figure 2. Examples of Applying the LCP Straightening Equation.

The lower portion of the figure shows several straightened optimal paths. They were derived by applying the conversion equation to the discrete cost values, then completing the LCP procedure. Note the smoothing of the accumulation surfaces as larger straightening factors (*i*) are used. This is similar to the “tension” factors used in geometric line smoothing techniques; however an

important difference is that in this case, the straightening is spatially responsive to the unique pattern of suitable and unsuitable areas. A visual comparison of the two 3D plots shows a general smoothing of the accumulated cost surface but retention of the major trends in the surface (peaks and valleys).

Also note that the length of the route is progressively shorter for the straightened optimal paths. This information can be used to estimate pipe cost savings  $((1.32 - 0.95) / 1.32) * 100 = 28\%$  savings for Path 3). Decision-makers can balance the benefits of the savings in materials with the costs of traversing less-optimal conditions along the straightened route as compared to the non-straighten optimal path.

The traditional unadjusted optimal path is best considering only landscape/terrain conditions, whereas the adjusted path also favors “straightness” wherever possible. In many applications, that’s an improvement on an optimal path (as well as an oxymoron).

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*Author’s Note:* see [www.innovativegis.com/basis/present/Oil&Gas\\_04/](http://www.innovativegis.com/basis/present/Oil&Gas_04/), A Web-based Application for Identifying and Evaluating Alternative Pipeline Routes and Corridors, Berry, J.K., M.D. King and C. Lopez, for a paper describing the basic LCP procedure applied to pipeline routing.

## Use LCP Procedures to Center Optimal Paths

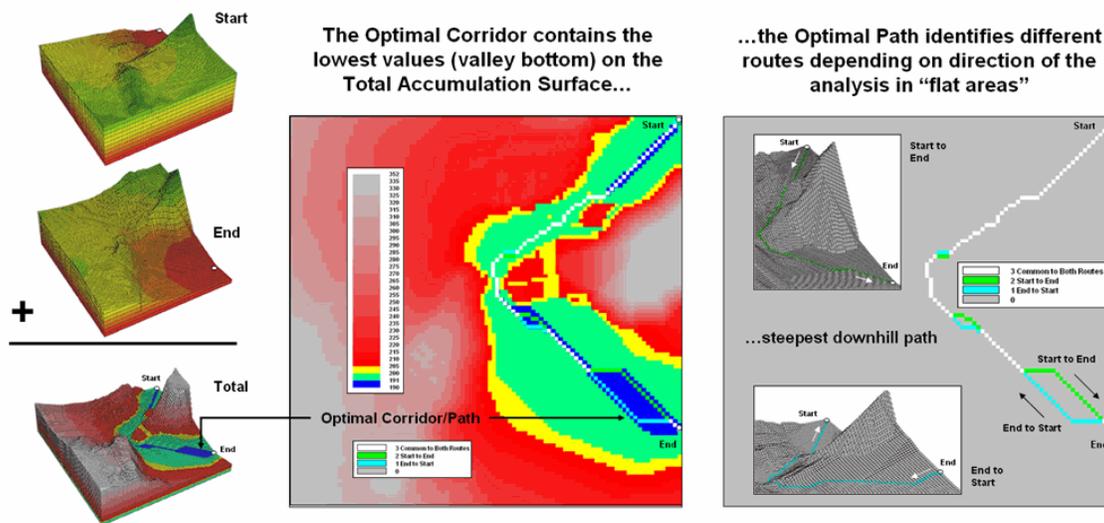
(GeoWorld, March 2006)

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Earlier sections have dealt with basic Least Cost Path (LCP) procedures for identifying optimal routes considering a set of map layers establishing siting preferences. The previous section discussed an improvement on the basics involving the straightening the often twisted and contorted routes derived using LCP. The discussion in this section will extend the improvements to tackle “centering” a route.

For example, routing pipelines involves identifying the optimal path between two points considering an overall cost surface of the relative siting preferences throughout a project area. This established procedure works well in areas where there are considerable differences in siting preferences but generates a “zig-zag” route in areas with minimal differences. The failure to identify straight routes under these conditions is inherent in the iterative (wave-like) grid-based distance measurement analysis considering only orthogonal and diagonal movements to adjacent cells.

The left side of Figure 1 shows the Optimal Corridor for a proposed route. The central (blue) zone identifies the “valley bottom” of the Total Accumulation Surface that contains the optimal path. The middle zone (green) and outer zone (yellow) identify areas that contain less optimal but still plausible solutions.



*Figure 1. “Flat areas” on the valley bottom of the Total Accumulation Surface (blue) are the result of artificial differences in optimal paths induced by grid-based distance measurement restriction to orthogonal and diagonal movements.*

The right side of the figure shows what causes the flat areas. The white route identifies the common locations for the optimal paths generated for both ways (Start-to-End and End-to-Start paths). These locations identify the true optimal path in areas with ample preference guidance. However in areas with constant cost surface values, the grid-based procedure seeks orthogonal and diagonal movements instead of a true bisecting line. For the area in the lower-right portion of the project area, the Start to End path (bright green upper route) shifts over and diagonally down. In the same area, the End to Start path (light blue lower route) shifts over and diagonally up.

The true optimal path traversing this area of minimal difference in siting preferences is a line splitting the difference between the two directional routes. Figure 2 depicts the steps for delineating a centered route through these areas. The first step generates a Total Accumulation Surface in the basic LCP manner described in the earlier discussions.

This map is normally used to identify optimal path corridors, however in this instance it is used to isolate the flat areas in need of centering. A binary map of the flat areas is created in step 2 by reclassifying areas to zero that are from the minimum value on the surface to the minimum value + .707. The .707 value is determined as one-half a diagonal grid space movement of 1.414 cells—the “zig-zag distance” causing the flat areas.

The third step determines the interior proximities for the flat areas, with larger values indicating the center between opposing edges. The proximity values then are converted to adjustment weights from nearly zero to 1.0 (step 4). This process involves inverting the proximity values then normalizing to the appropriate range by using the equation—

$$\text{Weight} = ((0.00 - \text{ProxValue}) + \text{maxProxValue}) / (\text{maxProxValue} + .01)$$

The result is a map with the value 1.0 assigned to all locations that do not need centering and increasing smaller fractional weights for areas requiring centering. This map then is multiplied by the original cost surface (step 5) with effect of lowering the cost values where centering is needed. The result is analogous to cutting a groove in the cost surface for the flat areas that forces the optimal path through the centered groove (step 6).

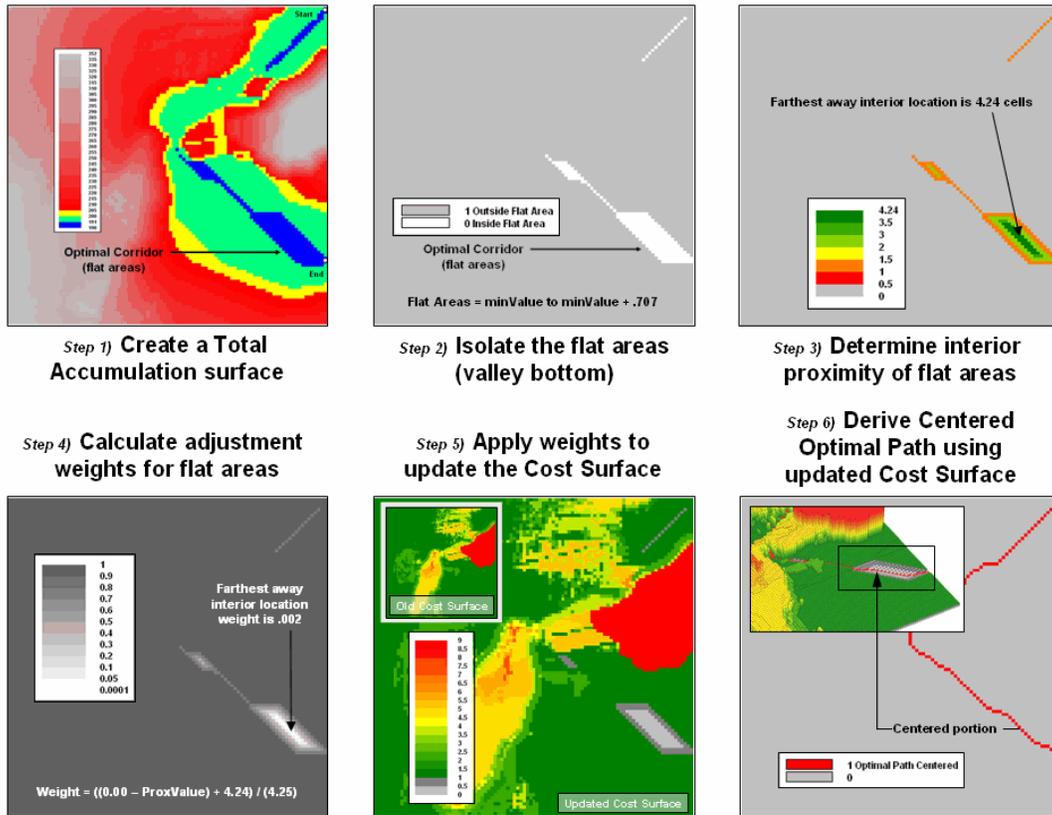


Figure 2. Procedural steps for centering an optimal path in areas with minimal differences in siting preference.

Figure 3 shows the results of the centering procedure. The red line bisects the problem areas and eliminates the direction dependent “zig-zags” of the basic LCP procedure in areas of minimal siting preference.

In practice, an automated procedure for eliminating zig-zags might not be needed as the optimal route and corridor identified are treated as a Strategic Phase solution for comparing relative advantages of alternative routes. A set of viable alternative routes is further analyzed during a Design Phase with a siting team considering additional, more detailed information within the alternative route corridors. During this phase, the zig-zag portions of the route are manually centered by the team—the art in the art and science of GIS.

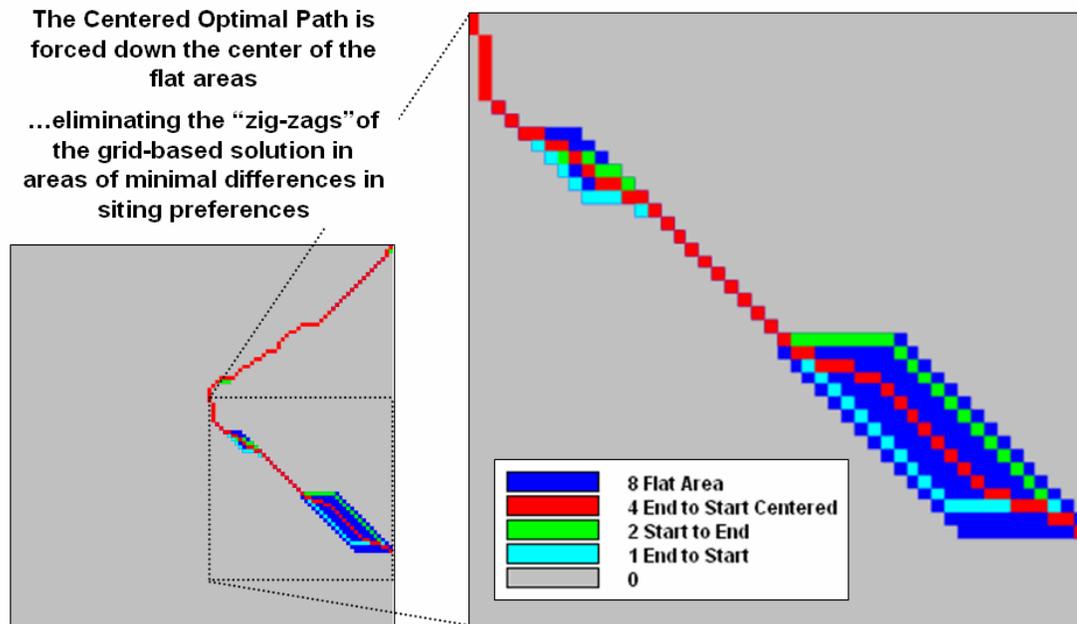


Figure 3. Comparison of directional optimal paths and the centered optimal path.

## Connect All the Dots to Find Optimal Paths

(GeoWorld, September 2005)

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Effective Distance forms the foundation for generating optimal paths. As discussed in the previous sections, the “Least Cost Path” method for determining the optimal route of a linear feature is a well-established grid-based GIS technique. It consists of three basic steps: *Discrete Cost Map*, *Accumulated Cost Surface* and *Optimal Route*. An additional step to derive an *Optimal Corridor* can be performed to indicate routing sensitivity throughout a project area.

The derivation of the Accumulated Cost Surface is the critical step. It involves calculating the effective distance from a starting location to all other locations considering the “relative preference” for favoring or avoiding certain landscape conditions.

For example, the left side of figure 1 shows a set individual cost maps for establishing a gathering network of pipelines for a natural gas field. The thinking is that routing should—

- 1) avoid areas within or near sensitive areas,
- 2) avoid areas that are far from roads, and
- 3) avoid areas of steep terrain, and 4) avoid areas of unsuitable soils.

These considerations first are calibrated to a common preference scale (0= no-go, 1= favor through 9= avoid) and then averaged for an overall Discrete Cost Map (favor green, avoid red and can't cross black).

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In turn, the discrete cost map is used to calculate effective distance (minimal cost) from the collection point to all accessible locations in the project area. The result is the Accumulated Cost Surface shown in 2D and 3D on the right side of figure 1. Note that the surface forms a bowl with the collection point at the bottom, increasing total cost forming the incline (green to red) and the inaccessible locations forming pillars (black).

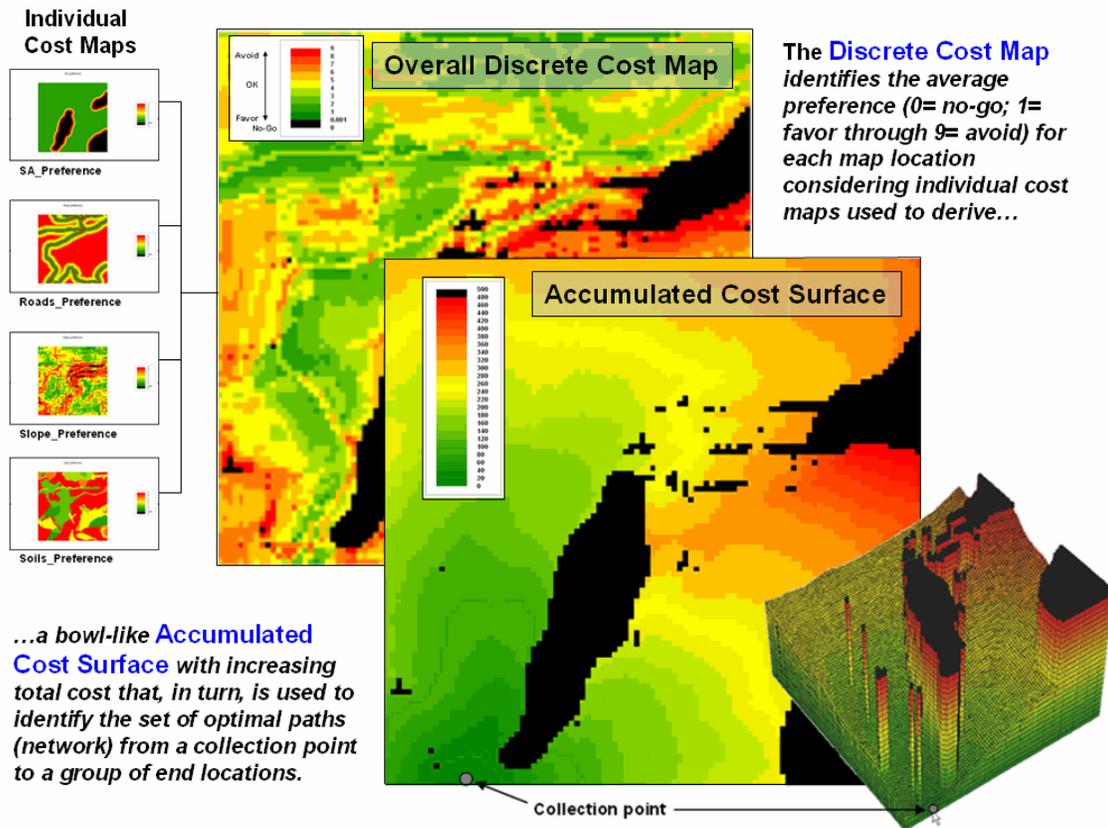


Figure 1. Relative preferences are used to identify the minimal overall cost of constructing a route from a collection point to all accessible locations in a project area.

Normally, a single end point is identified, positioned on the accumulated surface and the steepest downhill path determined to delineate the optimal route between the starting and end points. However in the case of a gathering network, a set of dispersed end points (individual wells) needs to be considered. So how does the computer simultaneously mull over bunches of points to identify the set of Optimal Paths that converge on the collection point?

Actually, the process is quite simple. In an iterative fashion, the optimal path is identified for each of the wells. What is different is the use of a summary grid that counts the number of paths passing through each map location. The map in figure 2 shows the result where red dots identify the individual wells, blue paths individual feeder lines and cooler to warmer tones an increasing number of commonly served wells.

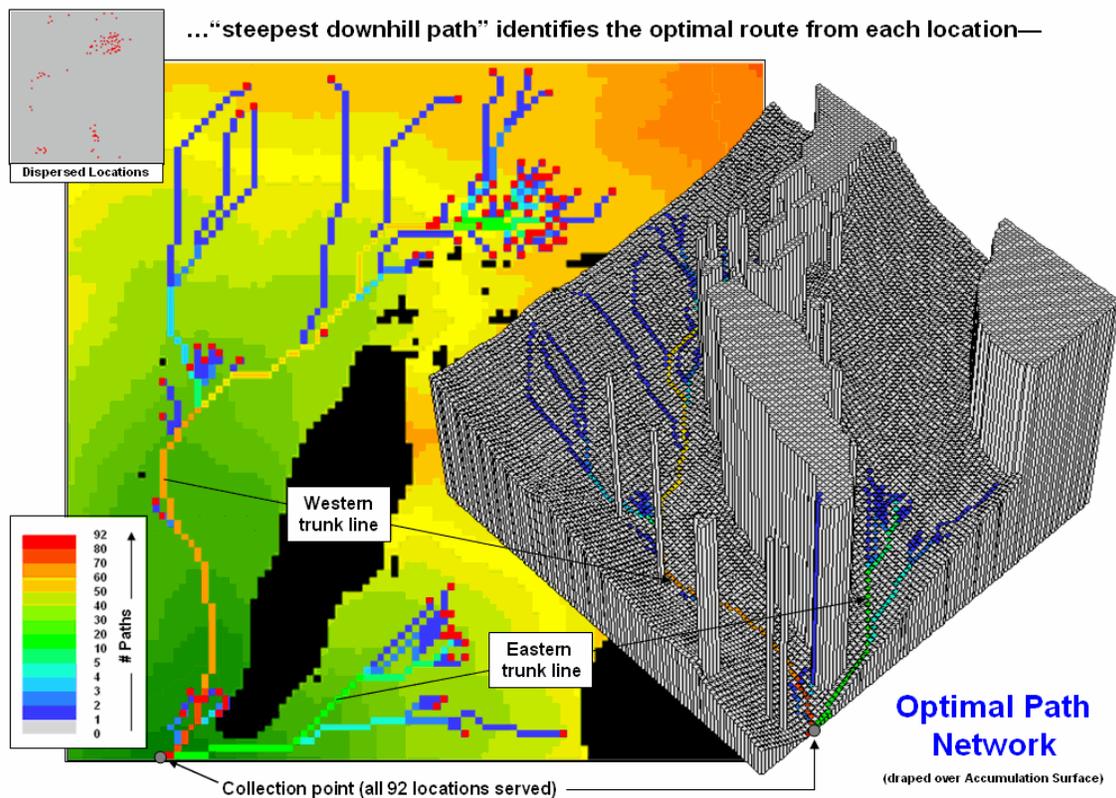


Figure 2. Optimal path density identifies the number of routes passing through each map location.

The green, yellow and red portions of the network identify trunk lines that service numerous end points (>10 wells). The collection point services all 92 wells (77 from the western trunk line and 15 from the eastern trunk line).

Figure 3 shows the gathering network superimposed on the optimal corridors for the western and eastern trunk lines. The corridors indicate the spatial sensitivity for sighting the two trunk lines— placement outside these areas results in considerable increase in routing costs.

In the example, a pressurized network is assumed. However, a gravity-feed analysis could be performed by analyzing a terrain surface (relative elevation) and hydraulic flows. The process is analogous to delineating watersheds then determining the optimal network trunk lines within each area.

Another extension would be to minimize the length of the feeder lines. For example, the three lines in the upper-left corner might be connected sooner to minimize pipeline materials costs. There are a couple of ways to bring this into the analysis— 1) re-run the model using the trunk line as the starting location, and/or 2) use a straightening factor as discussed in the previous section.

**Optimal Corridors** identify the nearly optimal areas for locating trunk lines serving all locations

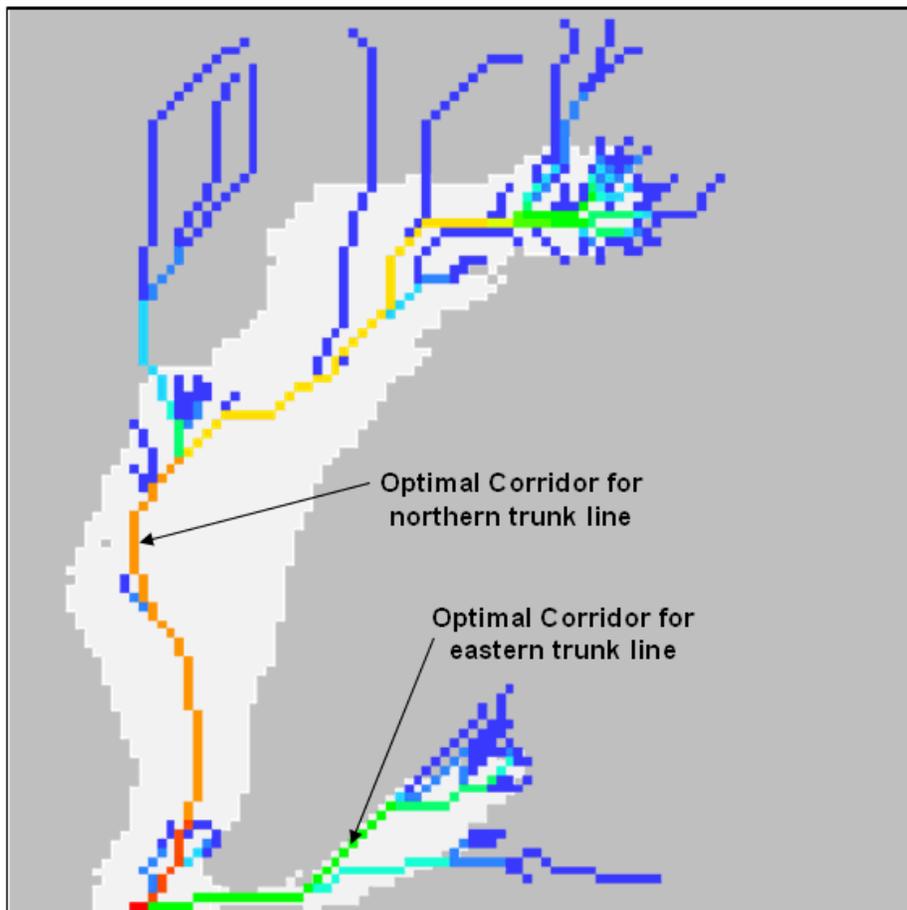


Figure 3. The optimal corridors for routes serving numerous end points indicate the spatial sensitivity in siting trunk lines.

The ability to identify the number of optimal paths from a set of dispersed end points provides the foothold for deriving an optimal feeder line network. It also validates that modern map analysis capabilities takes us well-beyond traditional mapping to entirely new concepts, techniques and paradigms spawned by the digital map revolution.

*(Understanding Accumulation Surfaces)*

## **Building Accumulation Surfaces**

*(GeoWorld, October 1997)*

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“You can’t get there from here,” is often the flippant response when you ask directions. In many cases it is a perfectly valid answer, as movement in geographic space is rarely as straight forward

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as a straightedge. Often there are several possible contorted paths that twist and turn in route to a destination. However, from the perspective of a ruler there is only one—the “shortest, straight line connecting two points.” Several Beyond Mapping columns (see Author’s Note) have addressed the concepts and procedures behind GIS’s capability to derive effective distance and optimal paths that respect intervening barriers. This column begins a series on the nature and analysis of accumulation surfaces, the basis of modeling real world movement.

Before we get too far, a quick review might be prudent. Recall that there are primarily two ways to represent distance in a GIS; the Pythagorean Theorem which mathematically mimics a ruler and the “splash” algorithm which mimics a drunken sailor weaving through a maze of barriers. There’s a good chance you remember Pythagorean’s equation of

$$c^2 = a^2 + b^2$$

from the brow-beating you received about right triangles in high school geometry. The GIS simply uses its X and Y coordinates in solving the equation for simple distance between two points.

The splash technique tracking simple proximity is a bit less familiar, but conceptually easy. Imagine what happens to a still pond when you toss in a rock— splash, then one ripple away, then another, and another, until there’s a whole set of concentric rings about the starting point. If the conditions are the same throughout the pond, the effect is similar to nailing a ruler at the starting point and spinning it, while scribing the set of circles formed by dragging the ruler’s tic marks. In a raster GIS, the width of each “wavelength ring” is one grid space. The result isn’t a single value identifying the length of the shortest, straight line between two points; it’s a map of values identifying all of the shortest, straight line distances between everywhere and the starting point.

Now imagine tossing a handful of rocks— splash, one ripple, two ripple, three ripple, and more radiate out from each of the starting points. When two wave-fronts meet, they stop, with the point of interference identifying the halfway location between starting points. The same holds true if you toss in a set of sticks or pieces of plywood, with the ripple patterns conforming to the irregular shapes of the objects. The end result of all the simulated splashing and crashing in a GIS is a map of the shortest, straight line distance from everywhere to the nearest starting location; be it a point, line or polygon.

Inset (a) in figure 1 is a 3-D plot of a simple proximity surface radiating from a single point. The lowest point on the surface contains the value 0 denoting it is “0 grid spaces away” from the start. Note that the surface is shaped like a bowl with increasing values (1 away, 2 away, etc.). The farthest location is in the upper right corner at a distance of 60 grid spaces \* 100 meters/grid space = 6000 meters. The slight depressions along the orthogonal and diagonal are a result of the slight directional variations in distances computed by the “splash” algorithm.

But continuous, straight line movement forming a perfect proximity bowl is rarely the case in the real world. Effective proximity respects movement around and through barriers— not “as the crow flies,” but as the crow might walk. Suppose there’s a lake in the way. Inset (b) identifies the absolute barrier itself as being infinitely far away (fear/reality of drowning). It assigns a value of 69 grid spaces to the farthest accessible location, indicating that the distance is a 900

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meters farther as a result of going around the lake. The set of all map values indicate the “shortest, but not necessarily straight” distances between the starting point and all of the other locations.

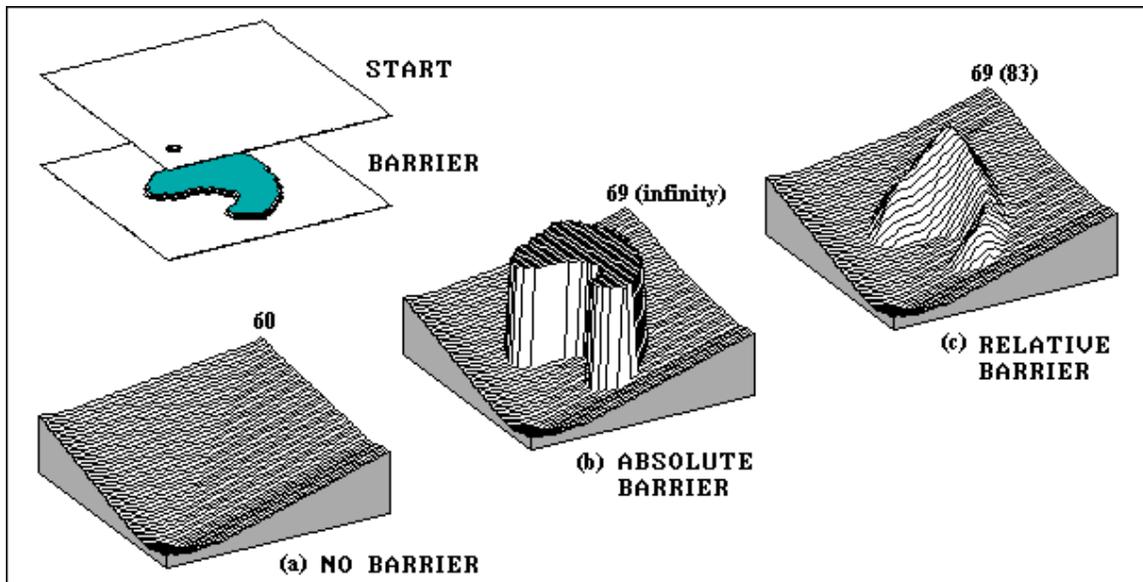


Figure 1. Accumulation surfaces showing the effects of relative and absolute barriers in mapping proximity.

However, in winter the lake freezes and can be crossed, though at a much slower pace on the slippery ice. It represents a relative barrier that impedes movement, but doesn't totally restrict movement. Inset (c) shows the accumulation surface assuming you walk 5 times slower on the ice. The results show that it's still 6900 meters to the opposite corner by going around the lake. However, if you gingerly trek to the center of the lake, it is equivalent to traveling 8300 meters on open land.

Previous Beyond Mapping columns described how the computer finds the “steepest, downhill path” along an accumulation surface to locate the “not necessarily straight” optimal path. It's analogous to a rain drop's route along the surface, which effectively retraces the wave front that got there first. In fact, the rain drop paths from all locations identify the “shortest, but not necessarily straight set of lines connecting the origin to everywhere.” Information you with a ruler, or Pythagoras with a calculator, could never derive. But it's chicken-feed compared to what insights you get by analyzing the surfaces themselves— as you will see in the next section.

**Author's Note:** see *GIS World* issues September 1989 through May 1990 for discussion of the fundamental concepts in measuring effective distance and October 1992 through December 1992 for discussion of the procedures involved. These columns have been compiled into Topics 3 and 9, respectively, in *Beyond Mapping: Concepts, Algorithms and Issues in GIS* (Berry, 1993; *GIS World Books*).

## Analyzing Accumulation Surfaces

(GeoWorld, November 1997)

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The previous section described the nature of accumulation surfaces and how they are built. Recall that the “splash” algorithm measures distance from a starting location like waves that spread out when a rock is tossed into a pond. The result is a 3-dimensional surface with increasing distance reflected by the increasing Z values stored in a matrix of grid cells. If absolute and relative barriers to movement are introduced, these surfaces form unique shapes with ridges and peaks similar to terrain surfaces.

However, unlike terrain surfaces, accumulation surfaces are always increasing (no “false-bottoms”) from point, line and areal features designated as starting locations. Areas with absolute barriers are identified as infinitely far away and form sheer walls on an accumulation surface. Relative barriers form hills and ridges as they identify areas that are passable, but at an increased “cost” (e.g., more time) per grid space. The valleys emanating from the starting locations locate corridors of minimal resistance to movement along the accumulation surface.

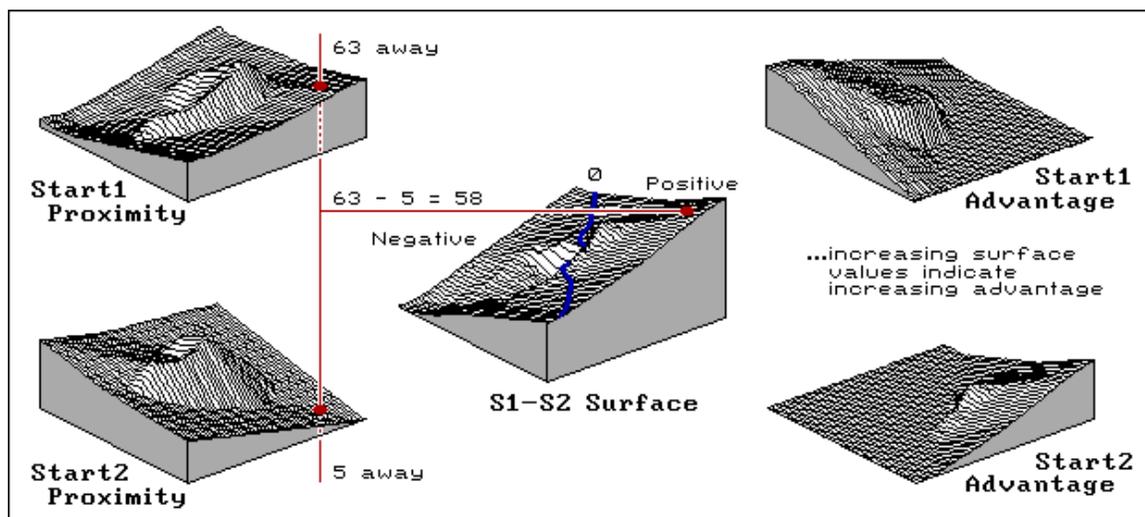


Figure 1. The difference between two proximity surfaces identifies the relative geographic advantage between two locations.

For example, the two surfaces on the extreme left of figure 1 characterize movement from opposing corners through the horseshoe-shaped relative barrier described in the previous section. The proximity from Start1 generally increases from left to right, while the increase from Start2 is in the opposite direction. Both surfaces show an abrupt increase when the relative barrier is encountered, however the shape of the resulting “hill” is different due to the different directions of the distance waves and the shape of the barrier.

Since the horseshoe ends of the barrier face Start1, the waves easily move into the center before interacting with the increased impedance of the barrier. The formation of a ridge indicates that some of the waves moved around the barrier, then penetrated the barrier from the back side. Any location along the ridge is equally distant from the start by moving to either side of hill.

The locations on the back side of the ridge, however, are optimally connected to Start1 by moving down the hill to the right and around the barrier... a counter-intuitive move. Neither ruler nor Pythagoras' Theorem suggest that you must initially move away from Start1 to begin the optimal route connecting the location to Start1. That's because they simply assume that all movement is in a straight line connecting two points— an extremely limiting assumption in the real world of complex barriers.

The vertical line intersecting both surfaces identifies a map location that is 63 grid spaces from Start1 and only 5 from Start2. Since these “shortest, but not necessarily straight” distances are stored at the same column, row position in the two proximity matrices, they can be easily retrieved and their difference computed ( $63 - 5 = 58$ ). If this is done for all locations, a difference surface (S1-S2 Surface inset in the middle of the figure) is generated identifying the relative advantage between Start1 and Start2 access for all locations throughout the project area.

The “0” line identifies locations that are equally distant between the two starting locations. It's similar to the “perpendicular bisector” you might remember from high school geometry, except it is bent and twisted reflecting the effect of the intervening barrier on actual movement. The sign of the difference indicates who has the advantage— negative values identify locations where Start1 has an advantage; positive values indicate a Start2 advantage. The magnitude of the value identifies the strength of the advantage. The two surfaces on the right isolate the relative advantages for both starting locations. Similar advantage surfaces can be derived for additional starting locations, keeping in mind that the “starters” can be any combination of point, line or areal features.

In the example, a +58 denotes a location with a strong advantage for Start2 access... it would be stupid to trek all the way to Start1. If you were a thirsty animal (or pub patron), why would you travel the extra distance? The liquid libations would have to be a lot better, or the ambiance and other thirsty organisms much more to your liking. If that were the case, then the relative attractiveness of starter locations can be incorporated into the derivation of the accumulation surfaces (termed “gravity” modeling).

Accumulation surface analysis provides valuable information for a wide array of applications. Natural resource managers use the technique to identify “home ranges” and “corridors of movement” based on the arrangement of landscape features. Instead assuming a simple distance of “within a two mile radius” of an animal's burrow, an effective distance home range based on absolute (e.g., river) and relative (e.g., cover type preferences) barriers can modeled.

Similarly, a retail market analyst can model the “home range” of a particular breed of shopper by characterizing the road network (e.g., speed limit) and ancillary attractions (e.g., areas of interest). Or in-store shopper habitat can be modeled using the aisles like roads and department fixtures as attractions; even “blue-light specials” could be modeled as dynamic features in a real time system. Traffic flows, whether in-store, across town or in the wild are similar beasts from a GIS analysis perspective. The absolute and relative barriers might change, but the basic application model (GIS procedure) remains the same. Next time we will consider other accumulation surface analyses... what do you think would happen if you added two accumulation surfaces? What information would the summation surface provide?

**Author's Note:** for “hands-on” experience in deriving and analyzing accumulation surfaces, see exercises TMAP2, TUTOR5, TUTOR6 and TU-RESP in the Tutorial Map Analysis Package (tMAP) software (Berry, 1993; GIS World Books). (DOS-based TMAP is no longer available and was replaced in 1998 by MapCalc Tutorial software available for free download from [www.innovativegis.com](http://www.innovativegis.com)).

## Determining Optimal Path Corridors

(GeoWorld, December 1997)

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The first section in this series described the construction and fundamental nature of accumulation surfaces. The second section discussed an analysis procedure for mapping relative geographic advantage that involved subtracting two surfaces. This time we will investigate the interpretation of slope and aspect of an accumulation surface and what information is derived if we add the surfaces.

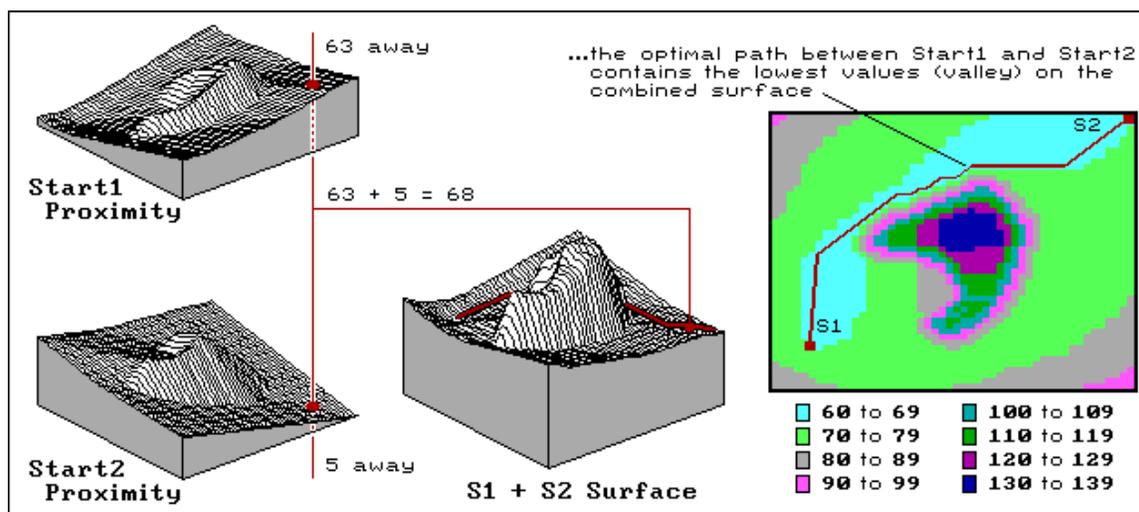


Figure 1. The sum of two proximity surfaces identifies the optimal path between two locations as the lowest values, while increasing values identify the “opportunity cost” of forcing a path through any location.

Recall that as distance increases from a location(s) the values can be plotted as a 3-dimensional surface, like those shown in the left portion of figure 1. The lowest point on the surface identifies the starting location(s) as zero units away from itself. All other locations contain increasing distance values forming the characteristic “bowl-shape” of an accumulation surface.

Effective distance (as opposed to “simple,” straight-line distance) can be derived by introducing absolute and relative barriers to movement. The “hill” in the Starter1 and Starter2 proximity surfaces reflect the increased impedance of a horseshoe-shaped barrier in the center of the map area. Each grid space on a friction map is coded with the “relative cost” of traversing that location. Note that the increased impedance is translated into the steeper slopes for the barrier area. Therefore a slope map of an accumulation surface unmasks the relative ease of optimal travel through each grid space.

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The notion of “optimal movement” embedded in an accumulation surface is important. The “splash” algorithm used to build the surface considers movement from the eight surrounding cells to each location. The accumulated distance and the relative impedance for each of the eight potential “steps” is evaluated. The least costly step, in terms of total movement, is assigned. Therefore an aspect map of an accumulation surface unmasks the direction of optimal movement through each grid space.

That’s a lot of spatially-specific information— the rate and direction of optimal movement throughout a map area. It allows us to relax the assumption that “everything moves in a straight line and with equal impedance.” In fact, things rarely move as simply as they respond to the complex patterns of absolute and relative barriers existing in the real world. Slope and aspect maps of an accumulation surface allow us to track the conditions of that complex movement at each map location.

As described in the first article, the optimal path from any location to the origin is identified as the steepest downhill route over the surface. In the second article, we found that subtracting two accumulation surfaces located the bisecting line between the origins as 0 (equidistant). The sign of the distance value on the difference map indicated which origin was closer, and its magnitude indicated how much closer.

For example, a wildfire response model might generate a proximity map for two fire stations considering both on and off-road movement. Subtracting the maps locates the effective dividing line between the two stations. In retail marketing, the halfway line is extended into a broad band indicating a “combat zone” for customers. Areas outside the band have distinct proximity advantages, while the real battles are waged in the combat zone where there the differences are marginal.

If subtracting accumulation surfaces creates useful information, what do you think happens when you add them? The surface in the center of the figure is a summation surface for the example data. The highlighted location is 63 from Start1 and 5 from Start2, therefore it is a total of 68 units away from both. In other words, the best path connecting the two origins which passes through that location has a total length of 68. In fact, the values on the summation surface identify the length of the “best” path forced through any given map location.

The optimal path between the two locations (identified by the line in both the 2-D and 3-D views) contains the set of locations having the lowest values (a valley connecting the origins). The saw-toothed appearance of the optimal path is an artifact of arithmetic rounding, the nature of the splash algorithm and minimal friction outside the barrier in the center. Values above the valley floor indicate the length of the best, but sub-optimal paths forced through any location.

The difference between the lowest value on the summation surface and the value at any other location identifies the “opportunity cost” of forcing a route through that location. The 2-D display shows fixed intervals of increasing opportunity cost— you would be crazy to force a route through the darker tones (a mountain of opportunity cost). Next time we will look at constructing a “stepped accumulation surface” which enables you to determine the optimal path

connecting a series of predetermined stops along the way. As a sneak preview, it involves minimizing successive accumulation surfaces... whew!

## Analyzing Stepped Accumulation Surfaces

(GeoWorld, January 1998)

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Hopefully you have survived the last three sections on accumulation surfaces. The discussion has covered the fundamental nature of accumulation surfaces (increasing distance waves), procedures for determining relative geographic advantage (subtract), ways to uncover direction and speed of optimal movement (slope and aspect), and a technique for determining the corridor of optimal movement (add). If that wasn't enough, now we get to extend the discussion to a "stepped accumulation surface" and "optimal path zones."

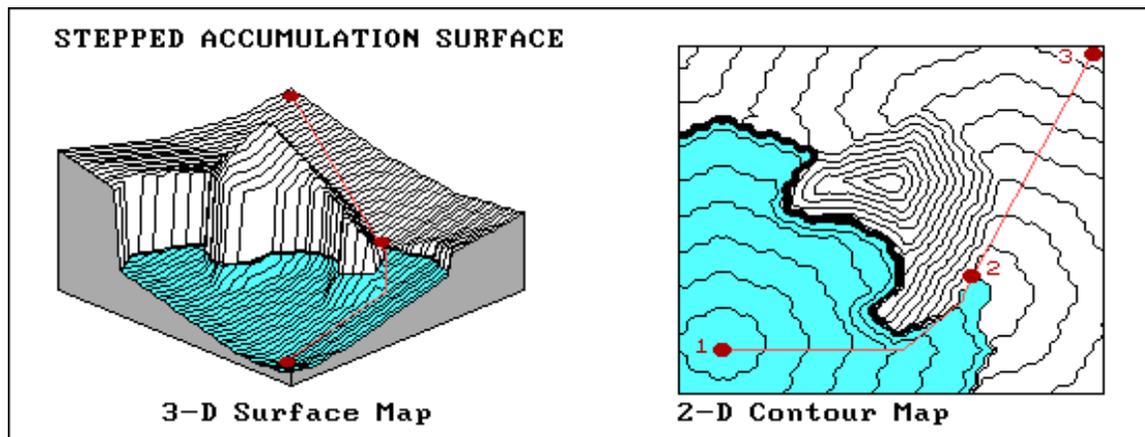


Figure 25.4. Stepped Accumulation Surface. Proximity from the first point is calculated (shaded) until the second point is reached, then proximity from that point is calculated (non-shaded) forming the next tier; the individual optimal paths along each of the stepped proximity surfaces forms the overall optimal route.

Suppose you know several places you would like to visit, but don't have a particular route in mind. If you know the order you would like to visit them, then a directed stepped accumulation surface is for you. Simply generate an effective proximity surface from the first location like those discussed in the previous articles— splash, one ripple, two ripple, three ripple and more radiate out from your starting point. The familiar bowl of proximity identifies the effective distance to all other locations. All you have to do is "stream" down the bowl from your second point of interest to identify the optimal path.

Now, construct a proximity surface from the second point to everywhere, and stream your third stop down it for the second leg of your journey. The left side of Figure 1 shows a stepped accumulation surface for the first two segments of a "directed hike" through the demonstration area. The shaded area shows the portion of the proximity surface from the start until the

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concentric ripples encountered the second point. The non-shaded area picks up the count by adding the proximity from there to all other locations. The result is a two-tiered surface similar to a spiral staircase. If you stream down the stepped accumulation surface from the third stop, it first flows down the top tier to the second stop, then continues down to the first point (right side of Figure 1).

Additional stops are considered by repeating the successive construction of “proximity bowls” and streaming down them for sequential optimal path segments. An undirected procedure allows the computer to determine a spatially efficient ordering of the points. You simply identify a starting location, the points you need to visit and the computer calculates the optimal route connecting them. The solution spreads out from the first point until it encounters the closest visitation point, then streams down the truncated proximity surface for the first leg. The next tier spreads out until it encounters its closest point and streams down for the next leg. The process continues until all of the visitation points have been evaluated. If you intend to return to the starting point, the home leg considers the starting point and is evaluated last.

The undirected, stepped accumulation surface technique (whew ...quite a mouthful) is similar to the classic “traveling salesman” problem in network analysis. However, it provides a solution in continuous space, respecting the complex reality of absolute and relative barriers. This is important if the traveling salesman doesn’t have a car, or if the mover isn’t constrained to a bunch of lines, like a herd of elk, or shoppers in a store.

In a recent project (see Author’s Note), we encoded the floor plan for a retail superstore (98,000 1-foot grids), translating the fixtures into absolute barriers and congested areas into relative barriers. Shelving locations were identified on each fixture and linked to product codes. The checkout records for each market basket were used to “place” each item purchased on the appropriate shelf. These visitation points were evaluated for the “plausible” path used to collect the items between the front door and the cash register at the exit.

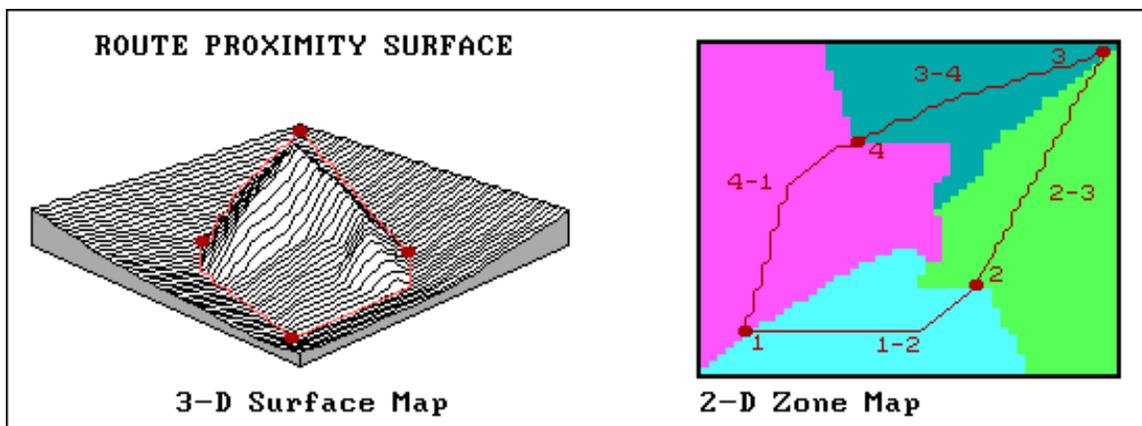


Figure 2. Route Proximity Surface. Proximity from the optimal route identifies the distance to the closest segment of the route for every location in a project area; the influence zone of each segment along the route identifies which segment is the closest.

Granted, a shopper could do “a random walk” to collect the items, but a shopper with a mission who knows the store, would be foolish to veer off the calculated route. Also, the consideration of several thousands of paths over a period of time converges on a map of relative in-store shopper activity. The analysis was summarized in hourly time-steps, displayed as normalized thematic maps, and animated to show the ebb and flow of shopper activity throughout the day.

The right side of Figure 2 shows the zones of influence for each leg of the optimal route. The locations within each zone are optimally connected to a particular segment. So, how might one use such information? In the shopper example, the distance from a shopper’s path can be interpreted as geographic impedance that must be overcome to veer off stride. Influence zones locate areas along a common portion of a shopper’s route. Overlaying these data with in-store departments, sales density surfaces and item categories produces a lot of information about shelving for retail marketing types. How might you use accumulation surfaces? ...it’s up to your innovative mind.

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**Author’s Note:** see discussion on “analyzing in-store shopping patterns” in February through April 1998 BM columns.

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